Addendum for:

Armstrong MJ, 2014. "Modeling short-range ballistic missile defense and Israel's Iron Dome system". *Operations Research* 62 #5, 1028-1039.

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Section 2

Development of Equation 1

If salvos are small, i.e., $mp_m < n$, then:

$$\text{Hits/salvo} = mp_m(1 - p_n)$$

This represents accurate missiles that survive interception attempts.

If salvos are large, i.e., $mp_m > n$, then:

$$\text{Hits/salvo} = n(1 - p_n) + (mp_m - n)$$

The first term represents accurate missiles that survive interception attempts. The second term represents accurate missiles where no attempt occurs. These can be rearranged as follows.

$= n(1 - p_n) + (mp_m - n)$

$= n(1 - p_n) + (mp_m - n)(1 - p_n) + (mp_m - n)p_n$

$= mp_m(1 - p_n) + (mp_m - n)p_n$

$\boldsymbol{= mp_m(1 - p_n) + [\ mp_mp_n - np_n\ ]^+}$ {This is the published form.}

$= mp_m(1 - p_n) + p_n[\ mp_m - n\ ]^+$ {This may be simpler.}

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Section 3 (and Appendix A.1)

Derivative of Equation 2 with respect to p_m

$Loss = (A/m)\{\ mp_m(1 - p_n) + [\ mp_mp_n - np_n\ ]^+\ \}(v/h)$

For small salvos: $\partial Loss / \partial p_m = (1 - p_n)(Av/h) > 0$

For large salvos: $\partial Loss / \partial p_m = (Av/h) > 0$

The small salvo version in the article has an error: $(1 - p_m)$ should be $(1 - p_n)$

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Section 5.3

Development of the number of hits with false negatives and false positives

If salvos *appear* small, i.e., $mp_{fp} + mp_m < n$, then:

$$\text{Hits/salvo} = mp_{fn} + mp_m(1 - p_n)$$

The first term represents false negatives, i.e., missiles that don't appear accurate but are. The second represents accurate missiles that survive interception attempts.

If salvos *appear* large, i.e., $mp_{fp} + mp_m > n$, then:

$$\text{Hits/salvo} = mp_{fn} + (n)(p_m / (p_m + p_{fp}))(1 - p_n) + [\ mp_m - (n)(p_m / (p_m + p_{fp}))\ ]$$

The first term represents false negatives. The second represents accurate missiles that survive interception attempts. The third represents accurate missiles where no interception attempt occurs. These can be rearranged as follows.

$$= mp_{fn} + n(p_m/(p_m + p_{fp}))(1-p_n) + [mp_m - n(p_m/(p_m + p_{fp}))](1-p_n) + [mp_m - n(p_m/(p_m + p_{fp}))]p_n$$

$$= mp_{fn} + n(p_m/(p_m+p_{fp}))(1-p_n) + mp_m(1-p_n) - n(p_m/(p_m+p_{fp}))(1-p_n) + mp_m\, p_n - n(p_m/(p_m+p_{fp}))p_n$$

$$= mp_{fn} + mp_m(1-p_n) + [\ mp_m p_n - n(p_m/(p_m+p_{fp}))p_n\ ]^+$$

$$= mp_{fn} + mp_m(1-p_n) + p_m p_n[\ m - n/(p_m+p_{fp})\ ]^+ \quad \text{\{This may be simplest.\}}$$

$$= mp_{fn} + mp_m(1-p_n) + (p_m/(p_m+p_{fp}))[\ ((p_m+p_{fp})/p_m)mp_m p_n - np_n\ ]^+$$

$$= mp_{fn} + mp_m(1-p_n) + (p_m/(p_m+p_{fp}))[\ (p_m/p_m)mp_m p_n + (p_{fp}/p_m)mp_m p_n - np_n\ ]^+$$

$$\boldsymbol{= mp_{fn} + mp_m(1 - p_n) + (p_m/(p_m + p_{fp}))[\ mp_m p_n + mp_{fp} p_n - np_n\ ]^+} \quad \text{\{This was the intended form.\}}$$

E.g., suppose the attacker fires 10 missiles with 10% false negatives, 40% accurate, and 20% false positives. The defense has 4 interceptors, each with a 75% success rate.

$$\begin{aligned}\text{Hits} &= 10(.1) + 10(.4)(1-.75) + (.2/(.2+.1))[10(.4)(.75) + 10(.2)(.75) - 4(.75)] \\ &= 1 + 10(.4)(.25) + (2/3)[3 + 1.5 - 3] = 1 + 1 + 1 = 3\end{aligned}$$

The version in the article has 3 errors: $mp_{fn}p_n$ should be mp_{fn} ; $-mp_m p_n$ should be $mp_m p_n$; and mp_{fp} should be $mp_{fp}p_n$

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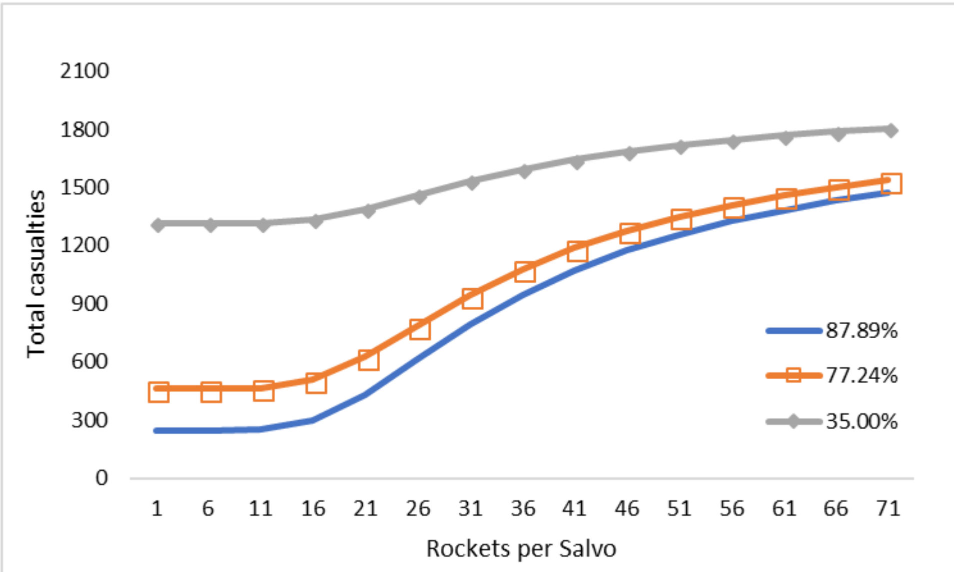
Section 6.2

Below is a corrected version of Figure 2.

Figure 2. Mean casualties for 3 interception rates, using accuracy rate 31.81%.



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Thank you to those readers who asked questions and provided feedback.