Using Effective Teaching Strategies and Personality Type to Enhance the Mathematics Classroom: A Handbook for Intermediate Math Teachers

Connie Herbert, B.Sc., B. Ed.
Department of Graduate and Undergraduate Studies in Education

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Faculty of Education, Brock University
St. Catharines, Ontario

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Abstract

This project addressed the need for more insightful, current, and applicable resources for intermediate math teachers in Canadian classrooms. A need for a handbook in this division seemed warranted by a lack of government resource support. Throughout an extensive review of the literature, themes and topics for the handbook emerged. The handbook was designed to not only provide educators with examples of effective teaching strategies within the mathematics classroom but to also inform them about the ways in which their personal characteristics and personality type could affect their students and their own pedagogical practices. Three teaching professionals who had each taught in an intermediate math class within the past year evaluated the handbook. The feedback received from these educators was directly applied to the first draft of the handbook in order to make it more accessible and applicable to other math teachers. Although the handbook was written with teachers in mind, the language and format used throughout the manual also make it accessible to parents, tutors, preservice education students, and educational administrators. Essentially, any individual who is hoping to inspire and educate intermediate math students could make use of the content within the handbook.
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# Table of Contents

Abstract ............................................................................................................................... ii  
Acknowledgements ............................................................................................................ iii  

## CHAPTER ONE: INTRODUCTION TO THE STUDY .................................................... 1  
  Background of the Problem ............................................................................................ 1  
  Purpose of the Study ....................................................................................................... 5  
  Objectives ........................................................................................................................ 6  
  Theoretical Framework ................................................................................................... 6  
  Rationale for the Study .................................................................................................. 10  
  Scope and Limitations of the Study .............................................................................. 11  
  Outline of Remainder of the Document ........................................................................ 12  

## CHAPTER TWO: REVIEW OF RELATED LITERATURE ........................................... 14  
  The Effective Teacher ................................................................................................... 14  
  Math Strategies .............................................................................................................. 28  
  Personality Traits .......................................................................................................... 39  
  Summary of the Literature Reviewed ........................................................................... 46  

## CHAPTER THREE: METHODOLOGY .......................................................................... 48  
  Rationale for the Handbook .......................................................................................... 48  
  Personal Relevance ....................................................................................................... 49  
  Development of the Handbook ..................................................................................... 50  
  Evaluation of the Handbook .......................................................................................... 52  
  Limitations .................................................................................................................... 53  
  Overview of the Handbook ........................................................................................... 54  

## CHAPTER FOUR: THE HANDBOOK ............................................................................ 55  

## CHAPTER FIVE: SUMMARY, IMPLICATIONS, AND CONCLUSIONS ......................... 163  
  Summary of the Handbook ......................................................................................... 163  
  Positive Elements of the Handbook from Formative Feedback .................................. 164  
  Suggestions for Improvement from Formative Feedback ........................................... 165  
  Practical Uses of the Handbook .................................................................................... 166  
  Implications for Theory ............................................................................................... 167  
  Implications for Practice ............................................................................................. 168  
  Future Research .......................................................................................................... 169  
  Concluding Remarks ................................................................................................... 170  

References ........................................................................................................................ 172  

Appendix: Intermediate Math Handbook Questionnaire ................................................. 187
CHAPTER ONE: INTRODUCTION TO THE STUDY

The purpose of this project was to develop a handbook for intermediate mathematics teachers in grades 7 and 8. Often a handbook is valuable for teachers as a resource and way of provoking thinking and deepening of effective practices. This handbook is unique in that it does not solely focus on teaching strategies but also takes into consideration the personal characteristics of the teacher and how personal characteristics impact pedagogy. The resulting handbook from this project is intended to offer support to intermediate mathematics teachers; however, much of the information presented in the handbook can be applied to teachers in all subject areas and grade levels looking to reflect upon their current teaching practices.

Background of the Problem

Each and every child deserves a quality education. Both the Canadian government and media have expressed the need for improved education and more competent, effective educators (Rushton, Morgan, & Richard, 2007; Waldrip, Fisher, & Dorman, 2009). Unfortunately, many students progress through the education system without a steady exposure to effective teachers. Any person who ventured through the education system would agree that not all teachers measure up in the same way. There are those teachers who motivate, engage, and master their craft, while others continue to remain unsuccessful to this day. According to a report summarizing the proceedings of the International Summit on the Teaching Profession, countries across the globe struggle to create world-class education systems simply because they find it difficult to recruit, develop, and maintain effective teachers (Asia Society, 2011).
The best teachers, effective teachers, are needed in our Canadian schools to successfully educate the minds of tomorrow. In an increasingly technological world, students must be educated for jobs that have not even been invented yet. The skills that students are learning today are easy to automate, digitize, and outsource (Asia Society, 2012). Students require an education that goes far beyond passive instruction and basic literacy skills and involves such components as active engagement, higher order critical thinking skills, self-monitoring, and complex communication skills in order to meet the increasing demands of the information age (Asia Society, 2012; Waldrip et al., 2009).

According to a report from the Asia Society (2011):

The roles of teachers and the demands placed on them are also changing rapidly. As systems seek to prepare their students with the kinds of skills required in a science and technology-driven, innovation-oriented economy, they need teachers who can prepare students with the kinds of higher-order cognitive skills to become knowledge workers, not factory workers; who can help every child succeed, not just the “easy to teach”; who can address the increasing diversity of many school systems; and who can adapt to and harness new technologies. (pp. 7–8)

Twenty-first century classrooms require effective teachers: professionals who are open, flexible, and have the capability to adapt to the new and changing demands that are constantly being placed upon them (Rushton et al., 2007). Although there is an obvious need for effective teachers, the question of how to define and measure a quality teacher is still an unresolved, pressing issue.
Historically, researchers have found it very difficult to identify and define effective teachers and their teaching practices (Waldrip et al., 2009). Since the late 1800s, researchers have attempted to predict which teaching applicants would become successful teachers (Lessen & Frankiewicz, 1992). Early researchers focused on identifying individual or clusters of personality traits of successful and not successful teachers (Barr, 1948; Charters & Waples, 1929; Halpin & Croft, 1963; Hamachek, 1969; Muijs & Reynolds, 2002; Walker, 1969). Between 1970 and 1990, effective teaching research moved away from the study of personality and looked at specific teaching practices as predictors of student achievement (Lessen & Frankiewicz, 1992; Muijs & Reynolds, 2002). This process–product model was considered extremely important because it suggested that teachers could be trained to increase student achievement (Lessen & Frankiewicz, 1992). Student gains were correlated with such practices as time on task, academic engagement, academic learning time, pacing, use of feedback, and teacher expectations (Anderson, Evertson, & Brophy, 1982; Brophy & Good, 1970; Fisher et al., 1980; Gage & Berliner, 1984). In recent years, studies have focused on both personality traits and teaching practices and how they are linked to student achievement (Allington & Johnston, 2000; Gentry, Steenbergen-Hu, & Choi, 2011; Rushton et al., 2007; Stronge, & Hindman, 2003; Thornton, Peltier, & Hill, 2005; Waldrip et al., 2009).

Teaching practices within mathematics vary greatly between schools and even classrooms within the same school. In order to establish some form of consistency, the Ontario government has designed the curriculum standards *The Ontario Curriculum, Grades 1-8: Mathematics* (2005b), which, in principle, should guide decisions related to practice. Additionally, the Ontario government has developed instructional guides for
mathematics teachers. *A Guide to Effective Instruction in Mathematics: Kindergarten to Grade 6* (2006a) includes five volumes and is further expanded upon in 10 other guides (i.e., five guides for grades K-3 and five guides for grades 4-6) addressing the five curriculum strands: Number Sense and Numeration, Measurement, Geometry and Spatial Sense, Patterning & Algebra, and Data Management & Probability. Unfortunately, similar instructional guides for teachers in grades 7 and 8 have not been produced by the Ontario government. Intuitively, teachers within the intermediate grades require similar support in order to ensure their practices are relevant, meaningful, and helpful for their students.

According to Fred van Leeuwen, General Secretary of Education International, teaching is a science and an art, where such characteristics as passion and communication are not easily measured (Asia Society, 2011). Similarly, a report published by the Organisation for Economic Co-Operation and Development (2005) noted that teachers are often assessed by their qualifications, experience, and tests of academic ability, while other important aspects such as the ability to convey ideas, create effective learning environments, enthusiasm and creativity, and working effectively with colleagues and parents are overlooked. Personal characteristics are extremely important for teachers to consider when reflecting on their current practices because they can affect classroom management style, responses to teacher-team meetings and parent–teacher conferences, reactions to students’ misbehaviours, student achievement, and the occurrence of burnout (Ayme, Ferrand, Reynes, & Borteyrou, 2009; Kokkinos, 2007; Lessen & Frankiewicz, 1992; N. K. Martin, Yin, & Baldwin, 1998; Thornton et al., 2005). In order for teachers
to be successful in their schools, it is essential that they are reflective of not only their pedagogy but also their personal characteristics.

Personality profiles are yet another quality upon which teachers (including preservice) should be encouraged to reflect. According to Rushton et al. (2007), “the knowledge and understanding of one’s [personality] type is pertinent to success in public education” (p. 440). An analysis of personality traits can offer teachers much insight into their strengths, weaknesses, learning styles, and ability to succeed in the education setting (Thornton et al., 2005). When teachers become cognizant of their personality type, they can use this information to make career choices, enhance their pedagogical practices, increase job satisfaction, prevent attrition, accommodate the various learning styles of their students, adapt to changes in the workplace, find constructive values in people who possess opposite preferences, and prevent potential student–teacher conflicts (Fairhurst & Fairhurst, 1995; Lawrence, 1993; Lessen & Frankiewicz, 1992; Rushton et al., 2007; Sprague, 1997; Thornton et al., 2005). In order for teachers to be successful, embrace change, and persevere into the future, they must be aware of their personality types (Rushton et al., 2007).

**Purpose of the Study**

The purpose of the handbook developed in this study is to encourage intermediate math teachers to be reflective and aware of how their personal characteristics, pedagogy, and personality types affect their current teaching practice. The focus of the handbook stretches beyond pedagogical practices to provide teachers with a well-rounded wealth of knowledge for reflection. There are many education handbooks that address either math strategies or teaching personalities, but none that look at both. This novel combination of
characteristics within one handbook is extremely important to the field of education because it encourages teachers to reflect upon their own personality traits and pedagogical skills as a lens to better understand how to meet the needs of the students within their classrooms.

**Objectives**

The following questions have been addressed in the extensive review of the literature located in Chapter Two. These questions were developed in order to identify information that focuses on important aspects of the problem. The three questions investigated in the literature include:

1. What characteristics comprise an effective teacher?
2. What are the specific teaching strategies used by effective mathematics teachers?
3. How does personality influence the effectiveness of mathematics teachers?

**Theoretical Framework**

Teacher effectiveness is an elusive term that is often difficult to define. Scholars do not agree on a single definition for this term; instead, they typically provide an explanation that is based on their own understandings of this concept (Aaronson, Barrow, & Sander, 2007; Collinson, Killeavy, & Stephenson, 1998; Gentry et al., 2011; Waldrip et al., 2009). Sometimes teachers are considered effective because of the teaching practices they use within their classrooms, while others are considered effective because of the personality traits they possess. Other teachers use these characteristics in combination to ensure that their teaching is effective.

In order for teachers to determine their personality traits, this handbook will make use of the Myers-Briggs Type Indicator (MBTI). According to Cranton and Knoop
(1995) “the MBTI is the most widely used personality measure. . . . Almost all research endeavors that include psychological type as a variable have used the MBTI” (Background to the Assessment of Type section, para. 3). This instrument is based on Carl Jung’s theory of psychological types and was developed by Katharine C. Briggs and her daughter Isabel Briggs-Myers to assess typology (McCutcheon, Schmidt, & Bolden, 1991; Quenk, 2009). Today, it is widely believed that typology can be used to capitalize on natural preferences and describe personality differences (Quenk, 2009). However, it is interesting to examine how Carl Jung’s theory evolved into the now popular MBTI.

In 1921, Carl Jung, a renowned Swiss psychologist, published a German book entitled *Psychological Types*, which was translated into English in 1923 (Quenk, 2009). This book focuses on typological research and describes two attitude types (extraversion and introversion) and four psychological functions (thinking, feeling, sensing, and intuition); (Cranton & Knoop, 1995; Lawrence, 1993). At first, Jung believed that the two attitude types, extraversion and introversion (opposite attitudes of energy), sufficiently explained the differences he observed among people; however, he later determined that the two attitude types could exist only in conjunction with the function types (Cranton & Knoop, 1995; Jung, 1976; Quenk, 2009). He categorized the four functions into two groups, perception and judgement. Jung claimed that sensation (now sensing) versus intuition were considered opposite functions of perception, and thinking versus feeling were considered opposite functions of judgement (Cranton & Knoop, 1995; Quenk, 2009). From here, Katharine Briggs and Isabel Briggs-Myers spent 20 years reading and studying Jung’s work in order to develop a way of assessing personality differences (Quenk, 2009).
Based on Jung’s theory of psychological types, Briggs and Briggs-Myers created the MBTI—a 93-item, self-report, paper-and-pencil questionnaire (McCutcheon et al., 1991; Quenk, 2009; Rushton et al., 2007; Sprague, 1997). The MBTI is different from Jung’s theory because judgement and perception are no longer used to categorize functions but instead are considered a new dimension (judging versus perceiving) used to distinguish opposite attitudes toward the outside world (Lawrence, 1993; Quenk, 2009). The questions within the assessment are presented in a forced-choice format in order to ensure preferences are clearly elicited for each of the four dichotomous poles (extraversion and introversion, sensing and intuition, thinking and feeling, and judging and perceiving); (McCutcheon et al., 1991; Quenk, 2009; Sprague, 1997). According to Myers (as cited in McCutcheon et al., 1991) the poles can be summarized as follows:

1. For EI, Extraverts (E) focus on the outer world of people, the external environment, while Introverts (I) focus on their own inner world, preferring to understand the world before experiencing it.

2. For SN, Sensing individuals (S) become practical by accepting and working with what is “given” in the here and now, while Intuitive individuals (N) value imaginations, inspiration, new possibilities, and ways of doing things.

3. For TF, Thinking individuals (T) make decisions by analyzing and weighing all of the evidence, while Feeling individuals (F) make decisions based on person-centered values.

4. For JP, Judgement individuals (J) prefer order and control in their lives, while Perceptive individuals (P) prefer flexibility and spontaneity.
Upon completion of the questionnaire, four preferences are identified and psychological type is represented by four letters (e.g., INFJ, ESTP). Overall, there are 16 possible letter combinations, or personality profiles, that can be distinguished from the four dichotomous pairs (Cranton & Knoop, 1995; Lawrence, 1993; McCutcheon et al., 1991; Quenk, 2009; Rushton et al., 2007; Sprague, 1997).

The information derived from the MBTI can provide a better understanding of self by informing individuals of the type of person they are and how they approach the world (Cranton & Knoop, 1995; Lawrence, 1993; C. Martin, 1997). In particular, “a knowledge of type explains one’s strengths and weaknesses, habits and motivations, peculiarities and idiosyncrasies, characteristic styles of behavior and relations to others, and differences from and similarities to others” (Cranton & Knoop, 1995, Implications and Applications section, para. 4). In some cases, these insights can help people gain confidence and feel enlightened and free (Cranton & Knoop, 1995; Lawrence, 1993). Many people struggle to understand why they are unable to do everything with the same amount of success and can feel a sense of relief when they are able to comprehend their own strengths and weaknesses (Lawrence, 1993). While teachers can utilize personality type to better understand the way in which they prefer to learn and teach, communicate with others, deal with conflict, lead and manage others, make decisions, and solve problems, organizations can make use of type to ensure that the requirements of a position fit well with the natural inclinations of a potential candidate (Cranton & Knoop, 1995). An understanding of personality type can certainly add much insight into the study of effective teachers.
Rationale for the Study

The handbook produced in this report will benefit the educational community because it adds to the literature on effective teaching and highlights the need for more dialogue related to effective teaching in Canadian classrooms. Effective teachers are highly versatile and positively affect student achievement gains regardless of which materials or programs they are provided (Allington, 2002). According to Stronge, Ward, Tucker, and Hindman (2008), even after just 1 year of being exposed to a high-performing teacher, students will remain ahead of their peers for at least the next few years. These teachers can find success with students demonstrating either low or high abilities, whereas low-ranking teachers are ineffective in achieving success with both types of students (Aaronson et al., 2007; Sanders & Rivers, 1996). Similarly, highly effective teachers in one content area (reading, math, science, or social studies) produce high student achievement gains in all four of the content areas, whereas ineffective teachers in one content area tend to be ineffective in all four content areas (Stronge et al., 2008).

Teachers who are currently working in schools can be proactive in reflecting upon and analyzing their own teaching strategies and personality traits. As teachers compare their own traits to the characteristics and strategies presented in this handbook, they can make changes to their pedagogical practices in order to increase job satisfaction and prevent attrition (Lessen & Frankiewicz, 1992, Thornton et al., 2005). According to Fairhurst and Fairhurst (as cited in Rushton et al., 2007), “knowing ones temperament and personality is important for teachers so they can recognize the differences between their personality types and their students’ learning styles” (p. 434). Teachers can work to
ensure their personality traits do not hinder the learning of their students by catering to their different needs and viewing the world through their students’ various lenses (Rushton et al., 2007). Similarly, Lawrence (1993) claims:

At times, all students need the support of being with a teacher who is like them in type, because understanding comes more easily between similar types. At times, every student needs the challenge of being with a teacher of a different type. Finding the right balance between support and challenge for students is an important task for those who plan instruction. If a faculty has a mixture of types, and knows about the characteristic strengths of different types, then students can be better served. (p. 20)

It is the responsibility of the teacher to ensure all of the needs are being met in his or her classroom. As teachers develop a better understanding of themselves, they will be better able to meet the needs of their students.

**Scope and Limitations of the Study**

The scope of this project is restricted to intermediate math teachers because a need for a handbook in these grade divisions seemed warranted by a lack of government resource support. The decision to specifically focus the handbook on mathematics was not only made to narrow and focus the information in the handbook but also because the researcher already had a large amount of experience and interest in this field of study.

The MBTI is a self-report questionnaire that may be accessed only after a purchase has been made online. Unfortunately, the handbook cannot include a copy or reproduction of the questionnaire for teachers to complete at their own will due to copyright infringement laws. To combat this issue, both the authentic Myers-Briggs
assessment site and a noncredited site that offers a similar personality assessment for free will be included in the handbook for teachers to access if they so choose. In addition, the handbook will provide detailed explanations of the 16 different personality types in order to offset the inability to include the actual questionnaire within the handbook.

In the creation of the handbook, every effort is made to include as many characteristics and traits of effective teachers as possible; however, there may be categories or specific traits that represent effective teachers that are not addressed or are simply overlooked in this handbook. As it is extremely difficult to include absolutely every essential quality and characteristic of an effective teacher, the handbook is based on the characteristics that have already been identified by current researchers as significant and worthy of implementation.

**Outline of Remainder of the Document**

Chapter Two provides an extensive overview of the literature that surrounds this report. First, the review explores the concept of an effective teacher and rationalizes the importance of effective teachers in our school systems. The chapter continues by defining the term “effective teacher” and uses five emergent themes to further unpack this definition. Afterwards, the strategies employed specifically by math teachers to improve the performance of their students and help motivate them in the math classroom are discussed in great detail. The final section of this chapter takes a comprehensive look into the way in which personality traits can be used to predict, identify, and describe effective teachers.

Chapter Three describes the methodology and procedures used in order to collect the data for the handbook. Specifically, this chapter describes the research methodology
and design, data collection and recording, data processing and analysis, methodological assumptions, limitations, methods for establishing credibility, and ethical considerations. The chapter ends by restating the main purpose of this study.

Chapter Four presents the results found in this report—namely the handbook. The handbook is presented in the exact format that would be printed for copy and distribution.

Chapter Five summarizes the entire report as a whole. The purpose, methodology, and evaluation are presented in detail. Then the implications of this handbook for administrators, teachers, and students are discussed. Conclusions are made at the end of this chapter in order to summarize the entire report and provide recommendations to researchers.
CHAPTER TWO: REVIEW OF RELATED LITERATURE

Chapter Two provides a critical review of the existing literature relevant to this project. The chapter begins with a brief explanation of the importance of effective teachers. The chapter continues with an exploration into the term “effective teacher” by investigating five different themes. These themes depict how effective teachers: (a) know and take a personal interest in their students; (b) set high expectations for themselves and for their students; (c) make content and learning meaningful and relevant to the future and respect students’ choices; (d) have a clear passion for their students, teaching, and for their content; and (e) respect and trust their students. The focus of the chapter then shifts to a discussion of the strategies that are specifically used by effective mathematics teachers within the classroom. Finally, a discussion of teacher personality traits is presented as an additional way for effective teachers to reflect upon their own practices.

The Effective Teacher

Effective teachers set the standard for all other educators. In fact, several studies (e.g., Allington & Johnston, 2000; Barber & Mourshed, 2007; Sanders & Rivers, 1996; Stronge, 2010; Wright, Horn, & Sanders, 1997) demonstrate that the quality of the teacher practice is the single most important factor associated with the variation in student learning and achievement; other factors include contextual variables like class size and classroom heterogeneity. Thompson, Warren, Foy, and Dickerson (2008) noted that “outstanding teachers play a significant role in closing the achievement gap in student achievement” (p. 122). Regardless of students’ family background or academic proficiency, effective teachers ensure that their students’ needs are being met. As
students feel more secure in the classroom, their confidence increases and, in turn, they try harder and work better. The influence of an effective teacher on students is extremely significant and typically long-lasting. Stronge et al. (2008) claim that even after just 1 year of being exposed to a high-performing teacher, students will remain ahead of their peers for at least the next few years.

Effective teachers are highly versatile in their instructional approaches and “produce better [student] achievement regardless of which curriculum materials, pedagogical approach, or reading program they use” (Allington, 2002, p. 742). These teachers can find success with students demonstrating either low or high abilities, whereas low-ranking teachers are ineffective in achieving success with both types of students (Aaronson et al., 2007; Sanders & Rivers, 1996). Similarly, highly effective teachers in one content area (reading, math, science, or social studies) produce high student achievement gains in all four of the content areas, whereas ineffective teachers in one content area tend to be ineffective in all four content areas (Stronge et al., 2008). The McKinsey report by Barber and Mourshed (2007) stated that

students placed with high-performing teachers will progress three times as fast as those placed with low-performing teachers. . . . Evidence suggests that even in good systems, students that do not progress quickly during their first years at school, because they are not exposed to teachers of sufficient calibre, stand very little chance of recovering the lost years. (p. 12)

The need for effective teachers cannot be overstated. Not only do these teachers create learning environments where students achieve gains even beyond their expected level of growth, but they also help students who have been exposed to ineffective teachers in the
past make excellent academic gains (Allington & Johnston, 2000; Wright et al., 1997).

So, how do we define an effective teacher?

It is difficult to quantify teaching expertise or to create a concrete definition of an “effective teacher.” Among researchers, differing definitions of what an effective teacher is and what an effective teacher does can be found (Aaronson et al., 2007; Collinson et al., 1998; Gentry et al., 2011; Waldrip et al., 2009). Collinson et al. (1998) define effective teachers simply as those who produce results and make professional accomplishments that can be looked upon as models by their peers. Hativa, Barak, and Simhi (as cited in Gentry et al., 2011) contend that effective “teachers differ from their colleagues, and particularly from novice teachers, in the complexity and sophistication of their thinking about teaching, in their cognitive schemata and pedagogical reasoning skills, in their decision making, and in their teaching-related knowledge” (p. 112).

Conversely, Aaronson et al. (2007) claim that “education background characteristics, including certification, advanced degrees, quality of college attended, and undergraduate major, are loosely, if at all, related to estimated teacher quality” (p. 129). Effective teachers are difficult to both identify and describe; this leads to most research being in the form of small case studies, where a set of skills and techniques that good teachers embrace in their practice are described in detail (Waldrip et al., 2009). Notably absent from much of the discourse on defining “effective” teachers is a lack of student voice. Students’ perceptions and experiences of effectiveness must be considered an important construct of quality teachers.

Gentry et al. (2011) created a definition of effectiveness based on students’ perceptions of their teachers and from comments provided by effective teachers.
themselves. In their study, Gentry et al. sampled more than 11,000 students using the My Class Activities (MCA) and Students Perceptions of Classroom Quality (SPOCQ) survey instruments. Third through 12th grade students rated their teachers on appeal, challenge, choice, enjoyment, interest, meaningfulness, and self-efficacy. Teachers who scored within the top 5% to 10% were identified as exemplary, gifted individuals. These 18 teachers came from nine different schools in eight districts and four states. Observations, survey questions, and interviews of the selected teachers provided a richer understanding of their distinguishing attributes. According to Gentry et al. effective teachers:

1. Know and take a personal interest in their students.
2. Set high expectations for themselves and for their students.
3. Make content and learning meaningful and relevant to the future and respect students’ choices.
4. Have a clear passion for their students, teaching, and for their content (p. 116).
5. Respect and trust their students (p. 121).

Each study about effective teachers that was examined for this literature review used at least one of these characteristics to define effective teaching (Allington, 2002; Allington & Johnston, 2000; Collinson, 1996; Collinson et al., 1998; Deutsch, 2005; Gentry et al., 2011; Haberman, 1995; Palmer, 1997; Spencer and Spencer, 1993; Stronge & Hindman, 2003; Waldrip and Fisher, 2003; Wentzel, 1997; Wubbels, Levy, & Brekelmans, 1997). Therefore, these five traits are representative of many different scholars’ definitions of effective teaching. The following sections will depict each characteristic in further detail.
Teachers Know and Take a Personal Interest in Their Students

As teachers interact with their students, a connection is made that can provide students with feelings of comfort, assurance, friendship, and security. When students feel safe and cared for within their classrooms they are more inclined to take risks, speak their mind, and participate in new activities (Wentzel, 1997). Developing this sense of inclusion and community within the classroom takes a high degree of interpersonal knowledge. According to Collinson (1996), effective teachers work at developing a high level of interpersonal knowledge in order to develop successful relationships with students. These relationships are extremely significant because they provide information about students’ interests and abilities, which teachers can use to inform and construct their pedagogies.

Spencer and Spencer (1993) explain that effective teachers take time to listen to their students in order to know their moods, feelings, background, interests, and needs. When teachers invest this time in getting to know their students, they are not only able to create lesson and unit plans that are more interesting and personally relevant to their students’ lives, but they are also able to demonstrate that they truly care about the feelings and well-being of their students. Gentry et al. (2011) found that students responded positively when teachers demonstrated a general liking towards them, which can increase on-task classroom behaviour. There are many ways that effective teachers can show a genuine interest in developing relationships with their students. For instance, Gentry et al. (2011) found that effective teachers “engaged in activities such as coaching, academic competitions, attending student events, directing the musical, advising student council, competing in art fairs, and supporting their students’ extracurricular endeavors”
Being involved in the lives of students beyond the classroom will help any teacher be more in tune with his or her students’ needs.

**Teachers Set High Expectations for Themselves and for Their Students**

Many teachers model the type of behaviour they would like to see in their classrooms, so it is only suitable that teachers would have high expectations not only for their students, but also for themselves. Collinson (1996) noted that effective teachers “challenge themselves and their students to find better and more interesting ways to think and learn” (p. 12). Teachers who understand that there is always something new to discover or more to learn constantly push themselves to be the most relevant and up-to-date with their teaching skills and their subject matter content (National Council of Teachers of Mathematics [NCTM], 2000). Effective teachers use their need to stay relevant as a way to demonstrate to students the importance of being a lifelong learner.

In addition to being continuous learners, effective teachers have a strong work ethic (Collinson, 1996). According to Gentry et al. (2011), connecting with students and finding ways to educate them effectively is a challenge that effective teachers enjoy. Not only do effective teachers find creative ways to educate their students, but they also try to make each day better than the last. Haberman (1995) states that effective, or “Star” teachers are continuously searching for the best way to help their students learn. He goes on to say that a teacher’s job is never fully complete because there is always something more than a student can learn in any activity (Haberman, 1995). Attempting to reach every student on a different level each day may sound exhausting; however, these teachers are smart enough to protect themselves from burning out. Effective teachers
make sure to have support systems and networks in place which they can turn to for extra help when needed (Haberman, 1995).

Not many would argue that teachers should set high expectations for themselves, but should they do the same for their students? According to Waldrip and Fisher (2003), students were comfortable with a high level of strict behaviour and wanted their teachers to be somewhat stricter because they felt that less strict teachers would negatively affect their learning. Similarly, Gentry et al. (2011) noted that students liked and trusted teachers who were demanding and not easy because it showed that they cared. Effective teachers did not just set high, unrealistic expectations; instead they set high expectations and helped their students to achieve them. These teachers “challenged their students to achieve and continually sought ways to support their students in their efforts to achieve, often pushing them to work harder, think harder, take risks, and plan for their futures” (Gentry et al., 2011, p. 121).

**Teachers Make Content and Learning Meaningful and Relevant to the Future and Respects Students’ Choices**

Effective teachers do not regurgitate a common script, but instead they are responsive to the needs of their students (Allington, 2002). These teachers differentiate their students’ learning in order to ensure that each pupil is achieving success (Stronge & Hindman, 2003). The key to successful differentiation is respecting individual differences while at the same time maintaining the expectation that each student will do his or her best (Allington & Johnston, 2000; Collinson et al., 1998). Some students may require more help than others to achieve their best. Effective teachers take it upon themselves to ensure that each of their students is engaged in the learning activity and
feeling successful in his or her own way, despite having been previously labelled the problem student, the talented student, or the special education student (Haberman, 1995; Stronge & Hindman, 2003). When students learn in a way that is more specified to their needs, they are able to better understand the content of the lesson; in turn, the lesson becomes more meaningful to the student.

Effective teachers continuously encourage dialogue within the classroom in order to find out what students understand and to push that knowledge further (Collinson et al., 1998). Appropriate behaviour in these conversation communities must be explicitly taught and continuously modeled by the teacher (Allington & Johnston, 2000). When expectations are clear and skills for respectful, supportive, and productive talk are in place, students can feel confident in sharing their ideas. Within these conversations, students have the opportunity to voice their opinions, relate subject matter to real-life situations, correct their mistakes, clarify and explain problem-solving strategies, practice respectful critiquing, and solve problems related to the real world and to their futures (Allington & Johnston, 2000; Collinson, 1996; Collinson et al., 1998; Gentry et al., 2011). Students are interested in learning activities that can be directly applied to their lives, and they become more invested in their education when it can help them in the real world or in other subject areas (Waldrip et al., 2009). As teachers invite questions and critiques from students, they are demonstrating respect for their students’ thinking and validating students’ thoughts and opinions (Collinson et al., 1998).

One of the best strategies that effective teachers use to make curriculum content meaningful is involving students in the learning process. Teachers can actively involve students by giving them responsibility, allowing them to make suggestions, providing
opportunities for students to teach each other, and giving students the chance to make their own choices (Collinson, 1996; Waldrip & Fisher, 2003; Waldrip, Reene, Fisher, & Dorman, 2008). When students are given the opportunity to make their own choices, they enjoy class more and become more interested with the course material (Waldrip et al., 2009; Waldrip et al., 2008). As students progress through different grade levels, their opportunities for choice should reflect their new levels of knowledge and understanding. Gentry et al. (2011) explain that students can choose areas of interest for further exploration, the order of the curriculum to study, methods of preparing their work, the time needed to complete an assignment, and placements in the field including independent study, mentorships, and apprenticeships. Students must learn how to make appropriate choices and develop many other skills that are beyond the basic subject matter in order to be successful in life beyond the classroom (Collinson, 1996).

**Teachers Have a Clear Passion for Their Students, Teaching, and Their Content**

Most teachers enter the teaching profession because they want to make a difference in the lives of their students. According to Collinson et al. (1998), the ultimate goal for an effective teacher is to bring out the best in his or her students. Similarly, Gentry et al. (2011) noted that teachers described their students as their primary concern and gauged their accomplishments based on their students’ lives. For many teachers, students are the most important aspect of their career. Effective teachers ensure their students’ needs are being met by knowing them as individuals with specific skills and interests, by constantly monitoring their progress, and by helping them understand concepts which are particularly challenging (Gentry et al., 2011; Thompson et al., 2008). Waldrip and Fisher (2003) discovered that students of effective teachers described them
as teachers who were willing to help, knew if their students understood the content, listened to their students, did not become angry quickly, and took extra time to explain the material with which the students struggled.

In order for teachers to differentiate their instruction and help students with their individual needs, these teachers must know their students extremely well. Effective teachers learn about their students’ interests, hobbies, strengths, and weaknesses in order to better meet their needs within the classroom. In addition, students feel as though their teacher genuinely cares about them when their teacher knows each student as an individual and not just as part of a class (Stronge & Hindman, 2003). According to Collinson et al. (1998), effective teachers are honest and open with their students but are careful not to cross that professional line. Teachers may share anecdotes about their family in order to help create a bond with students and provide examples of life lessons for the class; but any of their personal problems that exist outside of the classroom should definitely not be addressed in front of their students (Collinson et al., 1998).

When teachers are passionate about their profession and teaching curriculum content, this enthusiasm is passed on to their students. These students are excited to learn about subject matter because their teacher makes it interesting and engaging. Teachers who were deemed the best and most successful by their students embraced a student-centered educational praxis, presented their subject matter in meaningful ways, communicated content to their students in an effective manner, and used stimulating, motivating, and inspirational techniques to initiate or increase student interest in a particular subject area (Bishop, 1968; Sparks & Lipka, 1992; Waldrip et al., 2009). When teachers motivate and inspire students to learn, students develop their own sense of
interest and appreciation for the subjects being taught (Bishop, 1968). This development of intrinsic motivation can lead students to learn more quickly and competently with less teacher guidance and support (Alberta Education, 2010).

Collinson (1996) explains that effective teachers are “skilful at encouraging complex functioning, learning, creativity, and flexibility to create intellectual, interactive classroom cultures” (p. 10). These effective teachers understand that students must be interested and excited about what they are learning in order to be on task and working hard within the classroom. When students are engaged with their learning, they are less likely to be disruptive and take time away from learning. Beyond simply presenting information in meaningful ways, teachers can get students more invested in what they are learning by welcoming their input, giving them choices, allowing them to make decisions, and giving them opportunities to teach each other (Allington & Johnston, 2000; Collinson, 1996; Gentry et al., 2011; Waldrip et al., 2008). Teachers who are truly passionate about teaching realize that the best way to teach is to relinquish at least some control of the classroom to their students.

Every teacher must have some background knowledge about the core material she or he is teaching. Collinson (1996) claimed that expert knowledge (subject matter knowledge, curricular knowledge, and pedagogical knowledge) is “a necessary foundation for good teaching” (p. 2). In addition, teachers who possess subject-matter competence are respected and deemed successful by their students (Bishop, 1968; Cruickshank & Haefele, 2001). However, it is important to remember that expert knowledge, or professional knowledge, is only one aspect of a triad of knowledge and is insufficient on its own to produce excellent teachers (Collinson, 1996).
Unfortunately, many research studies claim subject matter competency to be the most important or only indicator of good teaching and trivialize the complex nature and high demands of the teaching profession simply to rote skills (Thompson et al., 2008). Palmer (1997) contends that good teachers are able to recognize the complexities of the subjects and students they teach while at the same time understanding their own self-knowledge. He emphasizes the fact that good teachers connect with students and use this connection to introduce students to new and interesting subjects (Palmer, 1997).

Effective teachers not only know their subject matter, but they also make the learning process engaging, fun, and relevant. Gentry et al. (2011) found that effective teachers helped their students learn content by creating a welcoming classroom climate, using humour within the classroom, and having fun with their students.

Besides having an extensive knowledge of curriculum content, effective teachers also know that subject matter is large, complex, and ever-changing (Palmer, 1997). In order to truly be professionally knowledgeable, effective teachers must commit to being lifelong learners and develop a keen sense of intrapersonal knowledge (Collinson, 1996). In describing teachers who made the greatest impact in their educational careers, the participating students in Bishop’s study indicated that an obvious desire to constantly update their personal knowledge and understanding of curricular content were key (Bishop, 1968). In an effort to expand and challenge their own perspectives, effective teachers engage in team teaching and collaboration with other adults (e.g., other teachers and/or parent volunteers); (Collinson, 1996). According to Howey and Collinson (1995), these efforts to work in partnership with other adults are extremely beneficial because it allows teachers to develop different but effective ways to teach and learn, make more
balanced decisions, and receive constructive criticism that encourages them to take risks within their classrooms.

**Teachers Have Respect and Trust with Their Students**

Being respected by one’s students and sharing a mutual feeling of trust with them does not spontaneously happen without any effort on the part of the teacher. Teachers must work at developing effective communication techniques and strong interpersonal skills in order to build positive relationships with their students (Wubbels et al., 1997). These relationships are essential for creating a classroom climate where students engage in respectful interaction and feel comfortable sharing their ideas, knowing they will be considered valuable and important (Deutsch, 2005; Stronge & Hindman, 2003). However, these relationships cannot be forced and must develop out of a genuine care and respect for students.

Effective teachers exhibit respect for their students by requesting information and criticism from them, treating them fairly, knowing them on an individual basis, altering instruction for their intellectual differences, communicating that teachers are learners too, keeping student information confidential, and sharing personal stories from which students learn life lessons (Bishop 1968; Collinson et al. 1998; Stronge & Hindman, 2003; Waldrip et al., 2009). When teachers maintain respect for their students and expect the same in return, they are building a foundation of mutual respect within their classrooms and, ultimately, demonstrating to their students that they care (Collinson et al., 1998).

Eliciting trust within the classroom is similar to developing respect, because the teacher must still work at knowing his or her students on a more intimate, personal level.
Students who expressed feelings of trust with their teachers were in classes where the teachers viewed relationship-building with their students as important (Gregory & Ripski, 2008). Trust is also developed when teachers demonstrate a trustworthy demeanour towards their students. Effective teachers genuinely listen and respond to students in a positive manner, honestly admit to their mistakes, limitations, and fallibility, share anecdotes about their personal lives, provide students with classroom responsibilities and opportunities to make decisions, encourage questioning in the learning process, and create learning environments where their students feel successful in order to develop trust with their students (Collinson et al., 1998; Gentry et al., 2011; Haberman, 1995; Spencer & Spencer, 1993; Wubbels et al., 1997). The more trust that teachers invest in their students, the more their students will trust them in return.

Feelings are extremely complicated and different for every person, but teachers must try their best to connect with their students and show them they are understood and appreciated. Strong relationships with students are based on sensitivity, empathy, and praise (Rey, Smith, Yoon, Somers, & Barnett as cited in Gregory & Ripski, 2008; Wubbels et al., 1997). Although it takes a lot of time and effort to build student–teacher relationships within the classroom, trust does develop and students, in turn, become more cooperative, more engaged with course activities, and more willing to accept the rules and norms of the classroom, plus requests from the teacher (Gregory & Ripski, 2008). Additionally, teachers who work at building relationships are more likely to earn the trust of students who feel distance from their teacher by such social determinants as race and social class (Gregory & Ripski, 2008). In view of the fact that there are so many benefits
that come from reciprocal respect and trust within the classroom, teachers should consider relationship-building as one of their top priorities.

The following five themes (a) know and take a personal interest in your students; (b) set high expectations for yourself and for your students; (c) make content and learning meaningful and relevant to the future and respect students’ choices; (d) have a clear passion for your students, teaching, and for your content; and (e) respect and trust your students, can be applied to any teacher looking to improve his or her practice. However, there are more specific strategies that can be used to enhance the learning activities in particular subject areas. It is important to recognise that the strategies used within a language arts classroom can vary considerably from the strategies used in a mathematics or science classroom. The following section will exclusively investigate mathematics strategies.

**Math Strategies**

There is a wide variety of effective math strategies that teachers can utilize within their classrooms on a daily basis in order to improve the performance of their students. This section of the chapter offers a description of some these strategies, included within the following categories: the use of manipulatives, physical movement and game play, 21st century tools and real-world applications, interdisciplinary approaches and mathematical literacy, and instructional design and flexible groupings. Strategies should be selected by the teacher throughout the school year, based on student and classroom needs, for particular math concepts. According to Loong and Herbert (2012), teachers of mathematics must attempt to cater to the needs and various learning styles of their
students in order to ensure that each of them is engaged, motivated, and interested in learning math.

The Use of Manipulatives

One of the most common and widely discussed strategies for enhancing the mathematics classroom is the use of manipulatives. Teachers should provide students with as many different manipulatives as possible because they encourage students to become actively engaged with their learning, accommodate a variety of learning styles, provide every student with an equal opportunity to participate, and help students develop a better understanding of numbers, patterns, geometry, and measurement (Lim & Kor, 2012; NCTM, 1989; Rapp, 2009). Typically, manipulatives refer to “concrete” or “hands-on” material like pattern blocks, unifix cubes, legos, drinking straws, paperclips, buttons, rulers, balances, geoboards and rubber bands, peg boards, beads and strings, checkers, coins, and tiles of various geometric shapes (Henke, Chen, & Goldman, 1999; NCTM, 1989; Rapp, 2009). However, manipulatives can also be represented in various technological formats.

The novelty of SMART Boards, graphing calculators, and virtual manipulatives can help to captivate students who do not understand or naturally gravitate towards concrete manipulatives. SMART Boards have an endless number of programs that students can use to manipulate shapes, objects, numbers, graphs, and pictures, thereby helping them to understand abstract concepts and making learning fun (Mueller, 2012). Although the SMART Board can be beneficial within the classroom, it is limited in that only a few students can access the material at once (Mueller, 2012). Some teachers may
prefer to use virtual manipulatives or graphing calculators so that each student can participate in the lesson at the same time.

Students can use spreadsheet programs on school computers to create and manipulate graphs, or they can use interactive sites like NCTM’s Illuminations, the National Library of Virtual Manipulatives (NLVM), and Shodor’s Interactivate to access virtual manipulatives (Chrisler, 2013; Johnson, Campet, Gaber, & Zuidema, 2012). These exercises are extremely beneficial because they provide students with instant feedback and integrate pictures, sounds, verbal cues, and symbolic notations in order to demonstrate connections between the various forms of representation (Johnson et al., 2012; Moyer-Packenham, Salkind, & Bolyard, 2008). These manipulatives can be used in combination with concrete manipulatives in order to provide students with comprehensive learning opportunities.

Physical Movement and Game Play

When teachers introduce a new unit or concept, students may be required to remain seated at their desks. Although this method of teaching has been popular in the past, it is not necessary. Students learn and work best when they are able to move around and explore their learning environment (Beaudoin & Johnston, 2011). There are two types of movements that teachers can incorporate into their mathematics classrooms: unrelated movement and purposeful movement (Beaudoin & Johnston, 2011). Unrelated movement does not necessarily connect to the content of the lesson, but it can still help students engage in the learning process (Beaudoin & Johnston, 2011). For instance, students may dance, exercise, create videos about math concepts, take turns using the SMART Board, participate in silent math discussions on chart paper, or explore the
school in a math scavenger hunt in order to participate in their learning (Beaudoin, & Johnston, 2011; Lapko, 2013; McKeny & Foley, 2012/2013; Mueller, 2012; Sliman, 2013).

Activities concentrating on purposeful movement would directly relate to mathematical content. For example, students could walk around the edge of a desk to understand perimeter, create numbers from body positioning, divide themselves into sets in order to demonstrate a mathematical process like addition, or display function transformations with the movement of a cut-out parabola (Beaudoin, & Johnston, 2011; Rapp, 2009). Both unrelated and purposeful movement can be beneficial; however, when movement is purposeful and specifically related to the content being taught, students demonstrate better academic gains and develop a more positive attitude towards the content of the lesson (Beaudoin, & Johnston, 2011).

In recent years, the Ontario government has deemed play-based learning to be a critical, central focus for teaching children at the elementary level (Ontario Ministry of Education, 2006c). According to Hammer (2011), an education reporter for The Globe and Mail, play is extremely important because it teaches children about multiple viewpoints and self-regulation, qualities that can lead to a more fulfilling life in the future. In order to motivate students within the mathematics classroom, teachers can support their students in creating their own math games, giving them a sense of ownership, or play games provided by the teacher, both of which help to build understanding and computational fluency for various math concepts (Legnard & Austin, 2012; Rapp, 2009; Silverman, 2002). Board games and card games that incorporate math skills include Candyland, Monopoly, Uno, Battleship, snakes and ladders, chess, Connect
Four, and checkers (Rapp, 2009). Teachers may also create their own math games which directly link to the curriculum by using the SMART Board or materials found within their classroom (McKeny & Foley, 2012/2013, Mueller, 2012). No matter what type of game is chosen, students will typically be eager to participate.

**21st Century Tools and Real-World Applications**

Using the computer to complete a math activity is a rather commonplace occurrence in most schools. However, there are many applications for which computers can be used besides simple drill-and-practice activities. For instance, students can utilize computer software (e.g., Math Circus) or websites such as coolmath4kids.com to play math games (Rapp, 2009); create and use blogs, wikis, podcasts, videoconferencing, and discussion forums to communicate with others around the world about their mathematical learning (Henke et al., 1999; Loong & Herbert, 2012; Tuttle, 2007). The quality of both the digital tools and mediums chosen and the student–digital interaction are significant factors contributing to learning. Teachers must know when and how to use this type of technology appropriately or their efforts can be counterproductive to student learning in mathematics (Henke et al., 1999).

Students can enhance their mathematical knowledge by doing virtual math and science experiments (Henke et al., 1999); use spreadsheet software (e.g., Microsoft Excel) to generate and manipulate a variety of graphs (Chrisler, 2013, Jensen, 2013); and explore the libraries of information on the internet to write a report about a mathematical theory or theorist (Henke et al., 1999). In addition, students can work collaboratively online via Prezi and Google Docs in order to write and edit mathematics presentations in real time (Settle, Abrams, & Baker, 2011; Yang, 2010) and pose questions to certified
math teachers using the Government of Ontario’s funded Homework Help website or to mathematicians using the Ask NRICH Website managed by the Centre for Mathematical Sciences at the University of Cambridge (Loong & Herbert, 2012). Computer literacy, especially in mathematics, is essential for students growing up today because our workforce, and essentially our world, is becoming more and more computer dependent than ever before.

Teachers using computers to assist students in mastering mathematical concepts must understand that students will not benefit from the technology if it is not used appropriately (Henke et al., 1999). Students often need a prelesson or scaffolding before attempting to master a concept or lesson on the computer. This scaffolding is especially important for younger students (Christler, 2013; Kay, 2012). According to Johnson et al. (2012), teachers should consider “the extent to which the online tool addresses the target math content, the ways in which the tool takes advantage of technology, and how to elicit responses that would yield meaningful insights about student understanding” (p. 202) before selecting an online tool to consolidate math learning. However, when online tools are carefully selected and utilized appropriately, students can benefit from visual scaffolding, immediate feedback, increased motivation and focus, and control over the learning process (Kay, 2012). These benefits contribute to more productive students and a better understanding of mathematical concepts.

Using mathematics in real-world contexts is imperative to student learning. Math problems addressing real-world issues help students make more sense of the world around them, construct a larger conceptual framework for solving problems, make connections between different types of problems, and develop skills like critical thinking,
reasoning, and creativity (Chrisler, 2013; Jensen, 2013; Orr & Suh, 2013; Rapp, 2009; Schifter, 2005). Activities that incorporate real-world ideas do not have to be overcomplicated; they can range from simple to complex. For instance, teachers can discuss fractions and division in terms of sharing food, such as pizza or cake (Schifter, 2005). In upper grades, students can use grocery items to explore mathematical ideas and create real-world problems based on their findings (Orr & Suh, 2013; Tuttle 2007).

Spreadsheet software can be used to demonstrate real-world connections throughout various grade levels. Younger students can use spreadsheets to enter data and create basic graphs like mathematicians, where older students can use the same software to take on the role of mathematics consultants and find which algebraic functions appropriately model data from real-life scenarios (Chrisler, 2013; Jensen, 2013). Lessons and projects linking mathematical concepts to real-world ideas can take on many different forms; however, teachers should not feel overwhelmed. These activities should be a fun change of pace and chosen on the basis of student need and interest.

**Interdisciplinary Approaches and Mathematical Literacy**

Creating math lessons that are motivating, interesting, and applicable to various learning styles can be a difficult task. In order for every student to make significant learning gains within the classroom, teachers should make an effort to integrate some of their math lessons with other subjects. Math can be incorporated into art by studying patterns in quilts (Kurz, 2013), finding shapes in flags (Schad, Georgeson, & Bunten, 2009), and posing mathematical questions about pictures (Orr & Suh, 2013). Literacy can be interwoven into math in many different ways as well. For instance, students can read books, write poetry, and create short stories inspired by mathematical concepts
In a more traditional sense, literacy can be used to describe numbers, number sense, and measurement (McKeny & Foley, 2012/2013).

One of the easiest subjects to combine with math is science. Essentially, almost every experiment in science involves a mathematical measurement, calculation, or description. A simple experiment about circuits can incorporate probability, counting, tables, charts, numbers, symbols, fractions, units of measurement, and math language (Hall, 2009). Alternatively, teachers can create larger group activities that combine multiple subjects or various disciplines in order for students to use their mathematical knowledge in a variety of different ways on the same task (Jensen, 2013). These strategies allow students to experience math concepts in a manner that is more recognizable and meaningful to their lives.

Students who are visual–spatial learners, struggle with language, have difficulty using fine motor skills, or are unmotivated by words may feel unsuccessful in math because they are unable to easily produce written output. Teachers can support these students by creating math activities where solutions are found with concrete objects such as pattern blocks, beads, or quilts (Holbert & Barlow, 2012/2013; Kurz, 2013). Students can demonstrate what they know by drawing pictures, recording their explanations, playing games on the SMART Board, taking pictures using a digital camera and narrating a photo story, dictating to a scribe, creating graphs with spreadsheet programs, or using computer presentation software like PowerPoint and Prezi (Chrisler, 2013; Holbert & Barlow, 2012/2013; Jensen, 2013; Mueller, 2012; Orr & Suh, 2013; Rapp, 2009; Settle et al., 2011; Tuttle, 2007). While students describe concepts or ideas, teachers can record their explanations and use them for marking purposes or professional development...
exercises (Schifter, 2005). The SMART Board is a great tool for recording student explanations because it can capture what the student is saying and the math work that is being performed by the student on the screen at the same time (Lapko, 2013). Oral communication is an important language skill and should be encouraged in the mathematics classroom.

**Instructional Design and Flexible Groupings**

Students are successful in mathematics classes when teachers actively present information in structured lessons (Reynolds & Muijs, 1999). The Government of Ontario has released several documents that encourage the implementation of the three-part lesson in mathematics (Literacy and Numeracy Secretariat, 2010, 2011a, 2011b; Ontario Ministry of Education, 2006a). The three parts describe the actions teachers should be taking before, during, and after a math lesson. In essence, these steps include: (a) preparing students for the lesson by activating prior knowledge or introducing a new concept; (b) allowing students time to work through the problem in pairs, small groups, or individually to find a solution; and (c) bringing students together to consolidate, summarize, and present their learning in creative ways (Literacy and Numeracy Secretariat, 2010, 2011a, 2011b; Ontario Ministry of Education, 2006a).

Using the steps of a three-part lesson plan as a guide, teachers can create unique lessons that help make concepts and information more memorable (Reynolds & Muijs, 1999). Examples that incorporate the three-part lesson into various math activities can be found in numerous articles (Chrisler, 2013; Legnard & Austin, 2012; Literacy and Numeracy Secretariat, 2010; Shimizu, 2009; Sliman, 2013). These activities can be modified or altered in order to accommodate any learning environment. Teachers are
encouraged to try the three-part lesson and decide whether the structured format is a good fit for their classrooms.

When teachers provide instruction, introduce a new task, or observe students working in groups, they should constantly be monitoring students for success. Some students will struggle with the material, while others will complete it at a brisk pace. Rapp (2009) suggests providing struggling students with more time to complete tasks so that they may process their thinking and demonstrate their mathematical skills. Extra time is especially important when questions are more cognitively demanding or require translation for such students as English language learners (Miller & Koesling, 2009; Reynolds & Muijs, 1999). Alternatively, Stepanek (1999) recommends using a strategy for high-ability students in the mathematics classroom called “Most Difficult First.” Students are given the opportunity to complete the five most difficult problems of their assignment first, and, if answered correctly, are allowed to skip the rest of the assignment and work on an alternative activity or enjoy free time (Stepanek, 1999). The pace of instruction is optimal when each student is actively engaged with his or her work and able to respond to questions accurately, not when every student is working at the exact same pace.

Solving mathematical problems can be an overly challenging task for some students within the classroom. These students can often give up quickly because the problems seem too hard or they are not sure where to start (Reynolds & Muijs, 1999). Allowing students to work in cooperative groups can often combat these problems. Students who work in groups learn to be flexible, provide and receive help through mathematical explanations, reflect on their own thinking, consider multiple viewpoints at
once, overcome insecurities, develop new problem-solving strategies, and respect the opinions of others (Henke et al., 1999; Jensen, 2013; Lapko, 2013; Reynolds & Muijs, 1999; Stepanek, 1999). In addition, when students learn from each other instead of the teacher, they are more motivated to learn and feel a sense of accomplishment when they complete a task (NCTM, 2000; Slavin, 1996).

Although group work has many benefits, like any other activity, it must be orchestrated with a great amount of thought and care in order to be successful. Students can become passive and off task if expectations are not clear (Reynolds & Muijs, 1999). Teachers must ensure that each person in the group is accountable and participating in some way (Lim & Kor, 2012; Reynolds & Muijs, 1999). For example, the teacher may assign roles for each group member (i.e., leader, note taker, etc.) and award points based on group participation (Lim & Kor, 2012). A computer program, such as Google Docs, can also help to increase individual accountability because the contribution made by each group member is highlighted in a different colour and can easily be tracked by the teacher in the revision history (Wood, 2011). There are endless possibilities when it comes to designing and implementing a group activity or project. Teachers should take advantage of this lesson style and proudly observe the capabilities of their students when left to their own devices.

Whole-class math discussions are excellent activities for introducing new concepts or debriefing students after small group work. From these discussions, students learn to listen to one another, share ideas, learn new strategies, make connections between similar ways of thinking, and give teachers the opportunity to observe and assess their comprehension (Cengiz, 2013; Reynolds & Muijs, 1999; Schifter, 2005; Sliman,
2013). Through whole-class discussions, students are able to add more problem-solving strategies to their repertoire. Activities that complement classroom discussions include student–student interactions (i.e., think-pair-share exercises), teacher–student interactions, and student or teacher presentations to the entire class (Henke et al., 1999). For instance, students may begin working on a small group assignment, present their findings to the class, and answer questions from classmates or the teacher, resulting in a discussion of ideas (Sliman, 2013). Teachers should anticipate responses and solutions from students and be ready with questions to push the conversation forward (Cengiz, 2013). Students (especially in upper grades) should be required to justify their answers and be challenged to find the most efficient ways to solve their problems (Cengiz, 2013). The main idea is that discussions should not end once a solution is found. In order for discussions to be meaningful, teachers must continue to challenge their students, ask questions, and encourage students to extend their thinking beyond a simple answer.

In an effort to remain relevant and effective, teachers must adopt several different teaching strategies and continuously evaluate their practice over the course of their career. Much of this effort is typically met with success. Unfortunately, one of the biggest factors influencing teacher effectiveness and student success is extremely difficult to alter—teacher personality. This topic requires a great deal of consideration and further exploration.

**Personality Traits**

In order for teachers to make informed decisions about their practice, they need to have a clear understanding of how they affect their students (Sanders & Horn, 1998). Effective teaching research provides invaluable information about successful teaching
strategies; however, this picture is incomplete without the consideration of teacher personal attributes and characteristics (Lessen & Frankiewicz, 1992).

The most widely used instrument for assessing personality characteristics is the Myers-Briggs Type Indicator (MBTI) assessment (Quenk, 2009; Rushton et al., 2007). This instrument is based on Carl Jung’s theory of psychological types, which identified three dichotomous dimensions for categorizing people based on their personality (Thornton et al., 2005). Jung (1976) discovered that contradictory types of attitude, extraversion (E) and introversion (I), could be characterized by four basic psychological functions: sensation (S), intuition (N), thinking (T), and feeling (F).

Jung grouped the psychological functions as perceptive or irrational (sensing and intuition) and judging or rational (thinking and feeling); (Cranton & Knoop, 1995). Each pair has one predominant and one inferior psychological function, depending on the person, because a function can never be eliminated completely (Jung, 1976). In addition, there is often an auxiliary function that supports the dominant function (Cranton & Knoop, 1995). Jung explains:

Naturally only those functions can appear as auxiliary whose nature is not opposed to the dominant function. For instance, feeling can never act as the second function alongside thinking, because it is by its very nature too strongly opposed to thinking. Thinking, if it is to be real thinking and true to its own principle, must rigorously exclude feeling. . . . Experience shows that the secondary function is always one whose nature is different from, though not antagonistic to, the primary function. Thus, thinking as the primary function can
readily pair with intuition as the auxiliary, or indeed equally well with sensation, but, as already observed, never with feeling. (pp. 405–406)

According to Cranton and Knoop (1995), a person should make a conscious effort to develop an auxiliary function if a dominant function is unsupported.

Throughout his book, Jung never explicitly states that judging and perceiving are a fourth pair of opposites; however, after Briggs and Myers spent years intensively reading his work, they concluded that this pairing was implied throughout his work and added the fourth pair to Jung’s system (Quenk, 2009). Instead of using the judging and perceiving dimensions to categorize the functions as Jung did, the MBTI measures this attitude as a separate dimension (Cranton & Knoop, 1995). Although the Jung/Myers theory of psychological types keeps with the notion that there are dominant and auxiliary mental functions, which must be opposing, it also claims that there is a tertiary function that still contributes to development but is relatively unconscious (Quenk, 2009). The fourth function, or inferior function, is said to be “‘inferior’ only in the sense of being last in its accessibility to conscious control” (Quenk, 2009, p. 14). In other words, the inferior function is the least used by a person and only slightly developed and mainly unconscious in that person (Quenk, 2009). However, the Jung/Myers theory maintains that all eight categories (extraversion, introversion, sensing, intuition, thinking, feeling, judging, perceiving) are used by every person at some point in time (Quenk, 2009).

The MBTI questionnaire invented by Katharine Briggs and her daughter, Isabel, Briggs Myers was created for one purpose—to assess typology. This instrument is used to identify the “consistent differences in the ways that normal people use their minds” (Quenk, 2009, p. 5), as described in the Jung/Myers theory of psychological types. As of
1998, the standard paper-and-pencil questionnaire (Form M) has included 93 forced-choice items which measure the four personality dimensions (Bayne, 1995; Quenk, 2009). This question format is imperative for determining the natural preferences of an individual towards a specific mental function or attitude (Quenk, 2009). According to Quenk (2009), “forcing respondents to choose between two legitimate ways of using their minds most directly and clearly elicits a preference” (p. 5). Once individuals have completed the MBTI, they are provided with four letters that describe their personality type (Cranton & Knoop, 1995). According to Rushton et al. (2007), these letters can be described as:

- **Extroversion (E) and Introversion (I).** Extraverted individuals obtain information through an orientation toward the outer world of people, events, or things. They enjoy meeting new people, thinking aloud, and being active. Introversion types seek the introspection of ideas, thoughts, and concepts. They prefer to process their thoughts internally before speaking, have few close friends, and often seek conversations that tend to be deeper in nature.

- **Sensing (S) and Intuition (N)** relates to individuals’ preferences in how they receive and make sense of information or data from the external world. Sensing types are more aware of their senses in relation to their environment, are often factually based, focus on practical concrete problems, and generally believe that if something works, it is best left alone. Individuals who have a tendency to understand the world through an Intuitive process prefer to live in a world of possibilities and options, often looking toward the future. They
also tend to focus on complicated abstract problems, seeing the big picture, sometimes at the expense of the details.

- **Thinking (T) and Feeling (F)** are considered the “rational processes” by which we come to certain conclusions and judgments regarding the information collected. Thinking types (T) prefer to focus on making decisions based on an impersonal objective position. Feeling types (F) have a tendency to respond well and easily to people’s values and are adept at assessing the human impact of decisions.

- **Judging (J) and Perceiving (P)** relates to how we “live our outward life.” Judging types prefer to live a structured, organized life. They also tend to be self-disciplined, enjoy making decisions, and thrive on order. Perceiving types prefer to live a lifestyle that is more flexible and adaptable. They tend to thrive on spontaneity, prefer to leave things open, require more information in order to make decisions, and often get things done at the last minute. (p. 434)

From these dichotomous pairs, 16 different letter combinations can be formed; representing 16 different personality profiles (Rushton et al., 2007). For example, one of these combinations could be expressed as ESTP, representing an extroverted, sensing, thinking, and perceiving personality type.

Several studies have found that the typical elementary and preservice teacher has an ESFJ or an ISFJ profile (Hinton & Stockburger, 1991; Lawrence, 1993; Macdaid, McCaulley, & Kainz, 1986; McCutcheon et al., 1991; Reid, 1999; Rushton et al., 2007; Sears, Kennedy, Kaye, & Gail, 1997; Thornton et al., 2005). These profile types seem quite fitting, especially for the primary grades, because they represent a patient,
nurturing, loyal, and devoted personality (Rushton et al., 2007). However, introverts prefer a quiet and orderly learning environment, whereas extroverts are more likely to welcome noise and movement (Lawrence, 1993). The overall teaching style of the ISFJ teacher, depicted by Fairhurst and Fairhurst (1995), involves:

[the] use [of] pencil-and-paper drills, workbook assignments, and quiet deskwork to teach their lessons. They often use course outlines and contracts with students. Their students are encouraged to memorize facts and will be exposed to audiovisuals as a way of teaching them about reality. ISFJ teachers also like short periods of teacher-led questions and answers, and brief lectures. (p. 61)

Most often ISFJ teachers are called “stabilizers” or “traditionalists,” because they appreciate clearly defined rules, enjoy consistency, and have a strong desire to keep things under control (Rushton et al., 2007). These teachers do prefer to stay set in their ways; however, if they feel a change is necessary and are provided with a detailed explanation, they can change over time.

Effective teachers possess a very different combination of personality characteristics. In a study conducted by Rushton et al. (2007), outstanding teachers were found to have a personality profile composed of Extroversion, Intuition, Feeling, and Perceiving (ENFP) types. The nature of an ENFP type is generally enthusiastic, highly creative, adaptable, and able to rely on improvisation instead of preparation (Lawrence, 1993; C. Martin, 1997). As teachers, ENFPs create novel, stimulating lectures; encourage class discussions, team building, and cooperative learning; and are sensitive to the needs of their students (Fairhurst & Fairhurst, 1995; C. Martin, 1997; Rushton et al., 2007). More specifically, Fairhurst and Fairhurst (1995) claim that
ENFPs are energetic and enthusiastic teachers. They often stimulate students to seek out what is unknown and make it known. They promote imagination and creativity in their classrooms through many different kinds of activities. Their students usually feel that their ENFP teachers understand them and help them to deal with their personal problems. (p. 63)

These teachers are viewed as “idealists” or “advocates” because they are intolerant of routine, proud to be unique, look forward to the future, and are flexible, open-minded, and innovative problem solvers (Rushton et al., 2007). According to Sears et al. (1997), SFJ-type teachers will not be the future leaders in education. The risk-takers and visionaries who easily accept change and naturally adapt to new situations are more likely to be the leaders of tomorrow (Rushton et al., 2007).

Both current teachers and preservice teachers can benefit from finding their personality types. Teachers can reflect on how their type influences their classroom management styles (N. K. Martin et al., 1998), responses to teacher-team meetings and parent–teacher conferences (Thornton et al., 2005), reactions to students’ misbehaviours (Ayme et al., 2009), levels of student achievement (Lessen & Frankiewicz, 1992), and the occurrence of burnout (Kokkinos, 2007). Preservice college students can benefit from knowing their personality types because they tend to “make significant life decisions with limited data” (Thornton et al., 2005, p. 490). Feedback about personality types could affirm career choices, suggest alternative career paths, or help preservice students to choose an appropriate age level to teach (Lessen & Frankiewicz, 1992; Thornton et al., 2005).
Using the MBTI questionnaire to assess typology can be extremely beneficial; however, one must remain cognizant of the tool’s limitations. For instance, Lawrence (1993) warns against using the MBTI as a screening tool because, very simply, people can manipulate their responses. In addition, the MBTI was not created or developed to measure competence or skillfulness and therefore should not be utilized by administrators for hiring purposes. Each type has its strengths and limitations, and one type may suit the needs of a couple of students in the class but leave the others bored (Lawrence, 1993). All 16 personality types are represented by the student population and, therefore, should be represented by the faculty as well. In order for teachers to be effective, they should be aware of their own personality types (Rushton et al., 2007). They can use this information to improve their practice and better connect with all of the students in their class. Students have diverse needs; therefore, teachers must learn how to adapt their personality type in order to meet these needs.

**Summary of the Literature Reviewed**

Effective teachers are essential for creating engaging classrooms and improving the academic achievement levels of their students (Allington & Johnston, 2000; Barber & Mourshed, 2007; Sanders & Rivers, 1996; Stronge, 2010; Wright et al., 1997). These teachers set an example for others to follow because they know and take a personal interest in their students; set high expectations for themselves and for their students; make content and learning meaningful and relevant to the future and respect students’ choices; have a clear passion for their students, teaching, and for their content; and respect and trust their students (Gentry et al., 2011). As teachers strive to be effective, they must take note of the subject matter that they are trying to impart upon their
students. Each subject will require different strategies to help students be successful and engaged with their learning. In mathematics, effective teachers provide students with manipulatives, incorporate physical movement into their lessons, allow students to play or create math games, offer opportunities for students to complete math work on the computer, incorporate math into other subjects, allow students to complete their math without writing, alter the pace of instruction, ensure math problems are related to the real world, allow students to work together in order to solve word problems, involve students in whole class discussions, and make use of the three-part lesson plan (Chrisler, 2013; Henke et al., 1999; Jensen, 2013; Rapp, 2009; Reynolds & Muijs, 1999; Tuttle, 2007).

Classroom practices can be modified and perfected by teachers over time. However, it is significantly more difficult to enhance personal characteristics. The personality of a teacher can predict classroom management styles (N. K. Martin et al., 1998), responses to teacher-team meetings and parent–teacher conferences (Thornton et al., 2005), reactions to students’ misbehaviours (Ayme et al., 2009), levels of student achievement (Lessen & Frankiewicz, 1992), and the occurrence of burnout (Kokkinos, 2007). Therefore, administrators should use personality information in combination with other interview practices to make informed judgements about which teachers to hire for particular positions; and teachers should use the knowledge of their own personality types to adjust their teaching style in order to better accommodate the needs of their students (Lawrence, 1993).
CHAPTER THREE: METHODOLOGY

The purpose of this chapter is to outline the research process that was involved in creating the handbook, *Using Effective Teaching Strategies and Personality Type to Enhance the Mathematics Classroom: A Handbook for Intermediate Math Teachers*. This chapter will discuss the rationale for the handbook, the personal relevance this study has to the author, the development of the handbook, the evaluation of the handbook, the critical review of the literature, the limitations of the handbook, and a restatement of the area of study. The complete handbook is presented in Chapter Four of this research paper.

**Rationale for the Handbook**

Teachers are constantly working to improve their professional practice. This handbook provides a valuable and needed resource for teachers to use within their classrooms. This handbook was specifically designed for intermediate math teachers for two reasons: (a) the author had a personal connection to this subject matter and age group, and (b) there was an obvious lack of support material for intermediate math teachers within Ontario looking to reflect and adjust their current teaching practices. The Ontario Ministry of Education has developed and published a wide variety of support material for mathematics teachers in grades K–6 (Ontario Ministry of Education, 2003, 2004, 2005a, 2006a, 2006b, 2007a, 2007b, 2007c, 2008a, 2008b, 2008c, 2008d). However, a supply of similar resources currently does not exist for intermediate math teachers (grades 7 and 8).

In light of the recent discovery that math scores have been declining across the country, the Canadian government has suggested improving math teacher training programs (Alphonso, 2013; Canadian Press, 2013). The Organisation for Economic Co-operation and Development (OECD), which conducts the Program for International
Student Assessment (PISA) every 3 years, has reported a downward trend in math scores in both 2009 and 2012 (Alphonso, 2013; Canadian Press, 2013). The handbook from this research paper can help teachers in Canadian classrooms to be reflective and ultimately improve upon their math teaching by looking at not only their teaching practices but also the relationship and impact of their personality types on their pedagogy.

**Personal Relevance**

The entire concept of the handbook was developed from the author’s personal experience in various intermediate mathematics classrooms throughout Ontario. From placements, teaching practicums, supply teaching, and long-term teaching positions in two different school boards and simply experiencing the education system firsthand as a student herself, the author has had the opportunity to witness the teaching style and technique of a variety of different math teachers. Very few teachers raised concern; however, the lack of consistency between teaching strategies and personality type was clearly evident. As a young professional entering the teaching profession herself, the author decided to find out how and why the inspirational teachers were so great.

Books, journal articles, courses, internships, workshops, and day-to-day work in the field of mathematics education have provided the author with experiences and information that directly influenced the content of this report and resulting handbook. In particular, each experience has played a role in validating and extending the list of math strategies included in the handbook. These must-haves for intermediate math teachers have been discussed and observed by the author firsthand in a variety of educational settings. In addition, the role of personality within the classroom has had a dramatic effect on students within each and every classroom that the author has visited. Some
teachers use their personality as a teaching tool to inspire and motivate their students, where others unfortunately continue to deliver dry lessons with even more tedious and uninteresting assessment opportunities. This handbook was designed with the intention to help other teachers improve their practice, but now also reflects the years of teaching experience directly from the author.

**Development of the Handbook**

The handbook began as a personal quest to discover the characteristics, or the essence, of outstanding intermediate math teachers within the teaching profession. As more information was gathered and a variety of classrooms visited, the handbook began to take shape. The first article that really communicated the ideas and themes presented throughout the literature came from Gentry et al. (2011). Not only did this article communicate examples of esteemed teaching practices, but the teachers selected for this study were based on student ratings. Students can provide significant and detailed information about effective teachers, so it is important that their voice be captured within this report. The five themes garnered from Gentry et al. (2011) depicted effective teachers as people who:

1. Know and take a personal interest in their students.
2. Set high expectations for themselves and for their students.
3. Make content and learning meaningful and relevant to the future and respect students’ choices.
4. Have a clear passion for their students, teaching, and for their content (p. 116).
5. Respect and trust their students (p. 121).
Almost every article from the literature review supported and reaffirmed these themes in some way, and the more the literature supported these themes, the more the author knew that she had made the correct choice in choosing them to represent the characteristics of effective teachers within the handbook.

A section of the handbook had to specifically address the math strategies that teachers could apply within their classrooms. This part of the handbook ensured that teachers had ideas, information, and actual lessons that they could take away and immediately implement within their classrooms. The practicality of this section of the handbook is what truly makes it useful to teachers. The strategies included in the handbook were carefully selected based on repetitive exposure. If a strategy had been presented as useful in the literature and had also been demonstrated within a classroom to positive student responses, it was included in the handbook.

The next section of the handbook focused on an element of teaching that goes beyond teaching strategies that can be learned and applied. This section of the handbook focused on personality traits and how they can have an effect on both teaching and learning, particularly in the mathematics classroom. Again, this section of the handbook was designed to be not only informative but also useful and applicable to teachers. Since free access to the Myers-Briggs Type Inventory (MBTI) was not offered, the handbook had to compensate for this problem. A link to a free, but unaccredited, online typology test was offered in the handbook as well as a self-report questionnaire developed by typology expert, Jane Kise.

The final section of the handbook offered supplemental material for math teachers in the form of websites, books, and online videos. Each source was accompanied by a
brief paragraph explaining the content and usefulness of the source in terms of the intermediate math classroom. In some cases, the description included an explanation of how the source was directly linked with the information presented in both the literature review and the handbook.

**Evaluation of the Handbook**

Several different sources contributed to the evaluation of the handbook. Both the author’s supervisor and second reader evaluated the practicability, usefulness, and readability of the content within the handbook. Suggestions for improvement were provided throughout the creation of the handbook and were applied both before and after other professionals reviewed the material within the handbook.

An evaluation of the first draft of the handbook was also completed by three teachers within a school board in southern Ontario. These teachers were connected to the author in a professional capacity, as they all worked within the same board. Each teacher was selected based on the fact that he/she was currently teaching or had recently (within the past year) taught intermediate mathematics. According to Creswell (2008), this participant selection technique is referred to as *purposeful sampling*. Individuals are selected in order to provide useful information to the study and either confirm or disconfirm specific preliminary findings. Hence, this type of purposeful sampling may be performed after the collection of data and facts has occurred (Creswell, 2008).

The teachers selected to evaluate the handbook were contacted via email and were provided with an electronic copy of the handbook as well as the accompanying evaluation questionnaire (see Appendix). The questions from the evaluation addressed the design, content, strategies, concepts, and feasibility of the handbook. Teachers were encouraged
to provide suggestions for improvement and were assured that their responses would be kept private and confidential. The feedback received from these educators was directly applied to the first draft of the handbook. Ultimately, these revisions contributed to the usefulness and practicality of the final version of the handbook in order to make it more assessable and applicable to intermediate math teachers.

**Limitations**

There are several limitations inherent in the development of the handbook. While the handbook was designed for use by intermediate math teachers, educators of this grade level were not directly asked about what content would be useful to them in a handbook. Although the author is an Ontario Certified Teacher and has observed many classrooms and lessons during the development of this handbook, teachers were not included in the decisions affecting the content within the handbook.

The background of the author and her own personal biases could possibly be reflected in the content of the handbook. Her upbringing was based in southern Ontario and thus the material presented in the handbook may be less applicable to teachers in different provinces and countries around the world. The age, gender, socioeconomic status, and ethnicity of the author could affect the content within the handbook and, in turn, the applicability of this handbook to wider audiences.

The task of defining an effective teacher and his/her practices is a very complex and difficult task. Within this study, specific personal characteristics, teaching strategies, and personality traits were identified as effective. Other researchers may use characteristics not mentioned in this study to classify effective teachers. However, the author of this study decided to focus on these three avenues of information because these
qualities were most often mentioned and observed in classrooms and research studies. In addition, the three avenues of focus were of personal preference to the researcher.

The evaluation of the handbook may also be limited in its effectiveness due to the relationship each evaluator had with the author of the handbook. Because each evaluator knew the author on a personal level, in some cases sharing a working environment with the author, feedback may have been focused more on positive aspects of the handbook rather than on providing constructive criticism.

**Overview of the Handbook**

The purpose of this study was to develop a handbook for teachers that embodied the characteristics of effective intermediate math teachers. This handbook is unique in that it not only presents effective teaching practices but also considers personal characteristics and personality type as factors affecting student achievement. Furthermore, this handbook goes beyond simply presenting great theories and ideas and actually provides concrete resources for teachers to implement within their classrooms immediately. Every lesson and activity in the handbook can be photocopied for ease of use within the classroom. The variety of strategies and opportunities for reflection presented in the handbook should make it an accessible resource for any teacher who is at any point in his or her teaching career. Both veteran and new teachers should be able to gain some useful insights when reading and applying this handbook to their classrooms.
CHAPTER FOUR: THE HANDBOOK

The following handbook was developed as a resource for intermediate math teachers looking to improve upon their current teaching practices. There are four main sections within the handbook: (a) personal characteristics of effective teachers, (b) math strategies utilized by effective teachers, (c) the effect of personality traits on the math teacher, and (d) additional resources that can accompany the information presented in the handbook. Teachers are encouraged to record and reflect on any observations that they make while reading the handbook. Additionally, teachers are permitted to photocopy and make use of any of the lessons and activities presented throughout the handbook.
Using Effective Teaching Strategies and Personality Type to Enhance the Mathematics Classroom

A Handbook for Intermediate Math Teachers

Connie Herbert
# Table of Contents

Introduction ........................................................................................................... 58

How to Use this Handbook .................................................................................. 59

The Effective Teacher .......................................................................................... 60
  Know and Take a Personal Interest in Your Students ........................................ 61
  Set High Expectations for Yourself and for Your Students ............................. 69
  Make Content and Learning Meaningful and Relevant to the Future and
  Respect Your Students’ Choices (Differentiation) ........................................... 74
  Have a Passion for your Students, Teaching, and for Curriculum Content ..... 82
  Have Respect and Trust With Your Students ..................................................... 87

Mathematics Strategies ....................................................................................... 90
  Manipulatives (Yes I know, I said the M word) .............................................. 91
  The SMART Board and Virtual Manipulatives ................................................. 96
  Physical Movement .......................................................................................... 103
  Math Games .................................................................................................... 106
  Computers ...................................................................................................... 110
  Subject Integration .......................................................................................... 114
  Non-Writing Activities .................................................................................... 117
  Altering the Pace of Instruction ...................................................................... 120
  Real-World Problem Solving .......................................................................... 122
  Flexible Groupings and Math Talk .................................................................. 125
  The Three-part Lesson Plan .......................................................................... 137

You’ve got to have...Personality! ......................................................................... 140
  Why Should You Know Your Personality Type? ............................................ 141
  The Myers-Briggs Type Indicator ................................................................... 143
  Sixteen Typological Types ............................................................................. 149
  What the Research Says ................................................................................. 151

Additional Resources ......................................................................................... 153
  Websites ......................................................................................................... 153
  Books ............................................................................................................. 155
  Online Videos ................................................................................................. 156

References ........................................................................................................... 157
Introduction

This handbook was designed to aid intermediate math teachers who are looking to improve upon their current teaching pedagogy. Teachers who are new to the profession and looking for a helpful resource, or those who have experience in the profession and have begun to consider a new approach to teaching, and even those very familiar in their role looking to update their current teaching practices, can all benefit from the information presented in this manual.

The information presented in this handbook is significantly different from other resources because it takes a very holistic approach to capturing the essence of an effective teacher. Not only are specific math strategies suggested in this guide as a means to improve instruction, but also personal teaching characteristics and personality type are presented as equally important factors in improving current pedagogical practices.

When reading through the manual, it is suggested that the reader keep a separate log of important thoughts and ideas that resonate with them. Hopefully the reader will even go so far as to implement or practise some of the new information or strategies they are learning. The best way to take on a new chapter in your teaching career is by jumping in and just giving it a try! Remember, if a method or strategy does not go well, continue to persevere and you may experience results you never thought possible. There are many different avenues for improvement to focus on in this guide. Give each one a fair attempt; you will be happy you did...and so will your students!
How to Use this Handbook

The information in this handbook can be applied in many different ways. Namely, teachers who are currently working at schools can take a proactive role to stay ahead of their peers by analyzing their own teaching strategies and personality traits. As teachers compare their own traits to the characteristics and strategies presented in this handbook, they can make changes to their pedagogical practices and ultimately improve upon the ways in which they teach.

In getting to know their personality types, teachers can recognize the differences between their personality types and their students’ learning styles (Fairhurst & Fairhurst as cited in Rushton, Morgan, & Richards, 2007). Teachers can work to ensure their personality traits do not hinder the learning of their students by catering to their students’ different needs and viewing the world through their students’ various lenses (Rushton et al., 2007).

At times, all students need the support of being with a teacher who is like them in type, because understanding comes more easily between similar types. At times, every student needs the challenge of being with a teacher of a different type. Finding the right balance between support and challenge for students is an important task for those who plan instruction. If a faculty has a mixture of types, and knows about the characteristic strengths of different types, then students can be better served.

(Lawrence, 1993, p. 20)
The Effective Teacher

The best teachers, effective teachers, are needed in our Canadian schools to successfully educate the minds of tomorrow. In an increasingly technological world, students must be educated for jobs that have not even been invented yet. The skills that students are learning today are easy to automate, digitize, and outsource (Asia Society, 2012).

Effective teachers are required in 21st century classrooms because they are open, flexible, and have the capability to adapt to the new and changing demands that are constantly being placed upon them (Rushton et al., 2007). In fact, several studies (e.g., Allington & Johnston, 2000; Barber & Mourshed, 2007; Sanders & Rivers, 1996; Stronge, 2010; Wright, Horn, & Sanders, 1997) demonstrate that effective teachers are the single most important factor associated with the variation in student learning and achievement.

The need for effective teachers cannot be overstated. These teachers create learning environments where students achieve academic gains beyond their expected level of growth. So, what do effective teachers do?

- They know and take a personal interest in their students.
- They set high expectations for themselves and for their students.
- They make content and learning meaningful and relevant to the future and respect students’ choices.
- They have a clear passion for their students, teaching, and for their content.
- They respect and trust their students.
KNOW AND TAKE A PERSONAL INTEREST IN YOUR STUDENTS

As teachers interact with their students, a connection is made that can provide students with four key feelings.

- Comfort
- Assurance
- Friendship
- Security

When students feel safe and cared for within their classrooms they are more inclined to take risks, speak their mind, and participate in new activities (Wentzel, 1997).

Spencer and Spencer (1993) explain that effective teachers take time to listen to their students in order to know their moods, feelings, background, interests, and needs. When teachers invest this time in getting to know their students, they are not only able to create lesson and unit plans that are more interesting and personally relevant to their students’ lives, but they are also able to demonstrate that they truly care about the feelings and well-being of their students.

Gentry, Steenbergen-Hu, and Choi (2011) found that students responded positively when teachers demonstrated a general liking towards them, which can increase on-task classroom behaviour.

There are many ways that effective teachers can show a genuine interest in developing relationships with their students. Being involved in the lives of students beyond the classroom will help any teacher be more in tune with his or her students’ needs. The following chart offers many suggestions for developing such relationships.
## Getting to Know Your Students

<table>
<thead>
<tr>
<th><strong>Talk With Your Students</strong></th>
<th>Simply talking with your students helps to make them feel more comfortable and relaxed. Their anxiety decreases, and they can think clearly on tasks. Conversations can be as simple as talking about what they did on the weekend or as serious as a death in the family.</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Observe Your Students</strong></td>
<td>As students work in groups, enter the classroom, have conversations with their peers, and complete tasks together, they can provide the teacher with invaluable information of their work habits and abilities, assertiveness, prior knowledge, social relationships, independence, etc.</td>
</tr>
<tr>
<td><strong>Know the Academic Skills of Your Students</strong></td>
<td>Take a look at your students’ Individual Education Plans (IEPs) and Ontario Student Reports (OSRs) from previous years in order to gain a better, well-rounded understanding of the strengths and weakness your students possess.</td>
</tr>
<tr>
<td><strong>Use a Survey for Both Parents and Students</strong></td>
<td>Providing both students and parents with a survey at the beginning of the school year or even when starting a new unit can provide the teacher with information he or she may not already know about students prior knowledge, interests, strengths, weaknesses, or medical concerns.</td>
</tr>
<tr>
<td><strong>Engage in Extracurricular Activities</strong></td>
<td>Engaging in activities such as coaching, academic competitions, attending student events, directing the musical, advising student council, competing in art fairs, and simply providing support for students’ extracurricular endeavors through conversations can enhance student–teacher relationships. (Gentry et al., 2011)</td>
</tr>
</tbody>
</table>
The following two resources provide examples of possible student and parent surveys that teachers can use in order gain information about their students. These resources were created by the author and are allowed to be copied for personal use.

The last resource in this section provides five icebreaker activities that an intermediate math teacher may use during the first 5 days of school in order to set the tone of the room for the remainder of the year, get to know students’ strengths and weaknesses, allow a chance for students to get to know each other, introduce mathematical concepts in an interesting way, create a seating plan, and decide which students would work best in cooperative group activities. These activities were written and created by Grade 8 teacher, Sandy Merz.
Student Information Questionnaire

Name: _______________________

Preferred Name or Nickname: _______________________

Birthday: _______________________

Pets (What kind, how many?):

Siblings (Male/Female, how many?):

Interests Outside of School or Extracurricular Involvement:

Rate how confident you feel with your math abilities:

Very Confident 5 4 3 2 1 I’m Scared of Math!

What part of math do you find easy?:

What part of math do you struggle with most?:

Do you prefer to work in groups or alone?:

Who do you think you work best with in this class? (2 or 3 names max.): 

What else is important that I should know about you?:

Parent Questionnaire – Getting to Know Your Child

Parent/Guardian Name(s): _______________________________

Name of Your Child: _______________________________

Does your child have any interests or extracurricular activities outside of school hours? If yes, please list them. If no, please skip this question and the next.

How much time does your child invest in these activities every week?

Does your child have easy access to the internet at home?

What else should I know about your child?

What questions or wonderings do you have for me?

What is the best way to contact you?

Cell: _______________________________
Home: _______________________________
Email: _______________________________
Get to Know Students through Seating Challenges

**Day 1:** Greet each student at the door and make sure he or she is in the right classroom. Next, before he or she has a chance to sit down, direct the student to follow the posted instructions: "Sit in birthday order so that the person with the birthday closest to January 1 sits in Seat 1. The year you were born doesn't matter. Don't skip seats. When everyone is seated, the student in Seat 5 will raise his or her hand and report that the class is ready to begin."

Observe the interactions: Look for organizers, active and passive participants, refusers, and disrupters. Be mindful that some students would rather be invisible and that the activity is probably something they haven't experienced before.

If anyone asks you what to do, redirect him or her to classmates and the posted instructions. Encourage students and remind them that you don't know the answer.

When Seat 5 reports in, do a couple of spot checks, show them where your birthday lies (just for fun!), and begin your lesson. By the end of this activity, every student will have interacted with other students and many will have reported to the whole class in a safe, nonthreatening way. (Ok, Seat 5 is under some pressure.)

**Day 2:** Ask the students to follow these instructions: "Line up in alphabetical order by the name you like to be called. Use last names and then middle names as tie-breakers. Then sit with an equal number of students at Tables 1-4. Remaining students sit at Table 5. When all are seated, the last student raises his or her hand and reports that the class is ready."

Adjusting to have equal numbers at each table produces a lot of interaction and some tension. Watch closely how students with different ideas negotiate. Don't intervene with the answer, but mediate if necessary. Have students quickly report their names. Treat alphabetizing mistakes kindly, of course.
Day 3 is different. Meet each student at the door with a paper that says, "Read this card completely. Do not enter the room until you understand the instructions. You may talk about the instructions before you enter the room. When you understand the instructions, give the card back to the teacher, enter the room, and begin." Here’s what the card says:

1) Complete this challenge in complete silence: Remain silent for the entire activity. Do not talk or whisper after you enter the room.

2) In the room, line up in order by height.

3) Then take your seats, with the shortest person in Seat 1.

4) Do not skip seats.

5) When the class is seated, the student in Seat 12 raises his or her hand and, when called on, reports that the class is ready.

Post the instructions in the room as well. Although the task is easy, the silent rule adds some stress, so observe which defense mechanisms students display. Note who is comfortable reading the cards and who avoids the task.

On Days 4 and 5, students sort themselves into groups and subgroups that may be lopsided. The instructions demand more judgment and decision-making from the students.

Day 4: The instructions read: "Sort yourselves into two groups: sneaker wearers and non-sneaker wearers. Next, each group forms two subgroups: students with curly hair and those with straight hair. You have curly or straight hair if you think you do. Each subgroup finds enough chairs and sits in order from the person with the shortest hair to the person with the longest hair."

A tree diagram showing the groups may help. Watch how they negotiate and decide where to sit.
**Day 5:** The instructions read: "Form two groups—students who prefer to spend free time indoors and those who prefer to spend it outdoors. You may like both, but choose just one. Within those groups, define your own subgroups based on the last thing you did when you spent free time the way you wanted to. Find a place to sit together and talk about your free time activity."

By the time you complete this series of seating challenges, you will have a good idea about how your classes will function and have a sense of the key players and personalities. The data can help guide you in your planning and execution throughout the course. And by training students to rely on each other and work together, you've demonstrated your norms rather than explaining them.

SET HIGH EXPECTATIONS FOR YOURSELF AND FOR YOUR STUDENTS

Many teachers model the type of behaviour they would like to see in their classrooms; so it is only suiting that teachers would have high expectations not only for their students, but also for themselves.

Effective teachers “challenge themselves and their students to find better and more interesting ways to think and learn.”

(Collinson, 1996, p. 12)

Teachers who understand that there is always something new to discover or more to learn constantly push themselves to be the most relevant and up-to-date with their teaching skills and their subject matter content (NCTM, 2000). Effective teachers use their need to stay relevant as a way to demonstrate to students the importance of being a lifelong learner.

Effective teachers not only find creative ways to educate their students, but they also try to make each day better than the last. Haberman (1995) states that effective, or ‘Star’ teachers, are continuously searching for the best way to help their students learn.

Not many would discredit the argument that teachers should set high expectations for themselves, but should they do the same for their students? According to Waldrip and
Fisher (2003), students were comfortable with a high level of strict behaviour and wanted their teachers to be somewhat stricter because they felt less strict teachers would negatively affect their learning. Similarly, Gentry et al. (2011) noted that students liked and trusted teachers who were demanding and not easy, because it showed that they cared.

Effective teachers do not just set high, unrealistic expectations; instead, they set high expectations and help their students to achieve them. These teachers challenge their students to continually achieve and support their students in these efforts. As a result of these efforts, students have demonstrated hard work, deep thought, risk-taking, and planning for their futures (Gentry et al., 2011).

Doug Lemov, a previous university, high school, and middle school teacher and now a teacher educator and school director, has written an internationally recognized book titled *Teach Like a Champion: 49 Techniques that Put Students on the Path to College*, (2010). In this book Lemov offers specific, effective teaching strategies that teachers can employ instantly within their classrooms to dramatically influence their learning environment and improve student achievement.

Lemov demonstrates throughout his book that effective teaching can be learned through the adoption of clearly explained teaching techniques. Within the first chapter, Lemov (2010) states that, “high expectations are the most reliable driver of high student achievement, even in students who do not have a history of successful achievement” (p. 27). Throughout the remainder of his first chapter,
Lemov describes his first five techniques: No Opt Out, Right is Right, Stretch It, Format Matters, and Without Apology. The following charts will provide a brief description of the five techniques and the intended actions by teachers and outcomes for students associated with each.

**Technique 1: No Opt Out**

A sequence that begins with a student unwilling or unable to answer a question ends with that student giving the right answer as often as possible even if they only repeat it.

**Teacher**

Provides answer and student repeats it; another student provides answer and first student repeats; provide cue and student uses it to find the answer; another student provides a cue and first student uses it to answer correctly.

**Student**

Is not able to avoid work or failure. Becomes increasingly familiar with success because he or she answers questions correctly more often.

**Technique 2: Right is Right**

Set and defend a high standard of correctness in your classroom.

**Teacher**

Do not accept partially or almost right answers; hold out for all the way; make students answer the question you asked and when you ask it (don’t let them get ahead of you); have students use technical vocabulary; tell students they are almost there or almost correct until they are 100% correct.

**Student**

Strives to provide precise answers to specific questions asked; Believes he or she is capable of getting answers right, like any other student anywhere else.
Technique 3: Stretch It
A sequence of learning does not end with the right answer; reward right answers with follow-up questions that extend knowledge and test for reliability (Differentiating Instruction).

**Teacher**
Respond to a right answer by asking a different/tougher question that builds and extends. Use questioning to make sure that a right answer is repeatable (ask how or why; ask for another way to answer; ask for a better word; ask for evidence; ask students to integrate a related skill; ask students to apply the same skill in a new setting).

**Student**
Knows how to get similar right answers again and again; Explains his or her thinking or applies knowledge in new ways; Pushed in a way that’s directly responsive to what they’ve shown they can already do.

Technique 4: Format Matters
It’s not just what students say that matters but how they communicate it. To succeed, students must take their knowledge and express it in the language of opportunity.

**Teacher**
Prepare students to succeed by requiring complete sentences and proficient grammar. Format Expectations: grammatical; complete sentence; audible; and unit measurements.

**Student**
Take knowledge and express it in a variety of clear and effective formats to fit the demands of the situation and of society.
**Technique 5: Without Apology**

The way we talk about expectations can inadvertently lower them. If we’re not on guard, we can unwittingly apologize for teaching worthy content. The skill of not apologizing to students is critical not only in the introduction and framing of material but also in reacting to students’ response to it.

**Teacher**

Refrain from apologizing for what we teach by assuming something will be boring; blaming it ("sorry, we have to learn it"); or not making it accessible (find a way to connect kids).

**Student**

Self-perception is raised because they know they can handle any content, no matter how difficult. They discover interest in content they might not have thought would be interesting.

All information included in charts obtained and adapted from:
http://dese.mo.gov/educator-quality/educator-effectiveness/educator-evaluation-system/teacher-evaluation
MAKE CONTENT AND LEARNING MEANINGFUL AND RELEVANT TO THE FUTURE AND RESPECT YOUR STUDENTS’ CHOICES (DIFFERENTIATION)

Effective teachers do not regurgitate a common script, but instead are responsive to the needs of their students (Allington, 2002). One of the best strategies that effective teachers use to make curriculum content meaningful is involving students in the learning process.

Teachers can actively involve students by giving them responsibility, allowing them to make suggestions, providing opportunities for students to teach each other, and giving students the chance to make their own choices (Collinson, 1996; Waldrip & Fisher, 2003; Waldrip, Reene, Fisher, & Dorman, 2008).

When students learn in a way that is more specified to their needs, they are able to better understand the content of the lesson and, in turn, the lesson becomes more meaningful to them. According to Alberta Education (2010), meaningful activities are:
Effective teachers continuously encourage dialogue within the classroom in order to find out what students understand and to push that knowledge further (Collinson, Killeavy, & Stephenson, 1998).

Within these conversations, students have the opportunity to voice their opinions, relate subject matter to real-life situations, learn about self-corrections and strategy refinement, practice respectful critiquing, and solve problems related to the real world and to their futures (Allington & Johnston, 2000; Collinson, 1996; Collinson et al., 1998; Gentry et al., 2011).

Students appreciate the fact that what they are learning in school can be applied to their lives. They become more invested in their education when it can help them outside the school or in other areas of study (Waldrip, Fisher, & Dorman, 2009). As teachers invite questions and critiques by their students, they are demonstrating respect for their students’ thinking and legitimate students’ thoughts and opinions (Collinson et al., 1998).

The following list offers suggestions for ensuring that classroom activities are meaningful, respectful, and engaging. Consider the following guidelines:

- Explicitly discuss the activity’s purpose and importance with students.
- Provide clear directions and expectations to reduce uncertainty, surprise, and disappointment.
- Challenge students of all ability levels to work to their full potential.
- Use learner profiles, pretesting, and other information to tailor activities and resources to student interests, backgrounds, and learning preferences.
• Anticipate any challenges or barriers to learning and build in supports to reduce these barriers.

• Share or develop assessment tools with students (e.g., criteria, rubrics, exemplars) to clarify expectations.

• Provide learning resources students will need for the activity that are accessible, engaging, and build understanding.

• Offer and model questioning strategies that address differing student abilities and readiness.

• Provide opportunities for students to apply what they have learned in contexts beyond the classroom setting.

• Provide problem-based learning contexts in which students actively solve problems in the same ways that professionals do in their jobs.

• Problem-based learning tasks can be structured individually, in small groups, or as a class.

• Offer tiered activities to have students work on the same concepts or skills, but with varying degrees of complexity, abstractness, and open-endedness.

• Model the use of graphic organizers to present information. For example, use a mind map to show and discuss connections between different ideas or concepts. Use a bubble map to present alternatives. Use a flowchart to discuss and work with sequence.

• Identify opportunities throughout the year for each student to take on the role of expert. The sharing of expertise builds both individual self-confidence and classroom community.

• At the start a new unit, find out about relevant student interests, then follow up on these throughout the unit.

All information obtained and adapted from Alberta Education (2010).
The inherent theme within this list of suggestions is differentiation. Successful differentiation involves respecting individual differences while at the same time maintaining the expectation that each student will do his or her best (Allington & Johnston, 2000; Collinson et al., 1998).

“In student choice is often thought of as the most obvious and straightforward aspect of differentiated instruction.”

(Alberta Education, 2010, p. 81)

In order to respect your students’ choices, you must first and foremost provide them with opportunities for choice. When students are given the opportunity to make their own choices, they enjoy class more and become more interested with the course material (Waldrip et al., 2009; Waldrip et al., 2008).

Students must learn how to make appropriate choices and develop many other skills that are beyond the basic subject matter in order to be successful in life beyond the classroom (Collinson, 1996). According to Alberta Education (2010), each and every student must learn how to consider alternatives and select options to become an independent learner. Within the classroom, students should be offered the opportunity to make choices about not only the way they learn (process), but also the way in which they will demonstrate their learning (product).

Some effective strategies for providing choice are listed in the chart Helpful Practices for Incorporating Choice into the Classroom below. Teachers are encouraged to make copies for classroom use.
### Helpful Practices for Incorporating Choice into the Classroom

<table>
<thead>
<tr>
<th>Best Practices</th>
<th>What This Can Look Like</th>
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</table>
| • Provide options for students to create products based on their learning interests, preferences, and readiness.  
• Products should be related to real problems, concerns, and audiences, and they should synthesize rather than summarize information. | Product choices could include:  
• a written report  
• an oral presentation  
• a group discussion on key concepts  
• a short book in which the key concepts are explained and described  
• a game |
| • Combine choice with flexible grouping to let students explore concepts in a variety of ways.  
• Students also can be provided with choices regarding the context in which they create the product; e.g., as individuals, with partners, or in small groups. | • Grouping strategies such as self-selected learning centres can accommodate student choice.  
• Plan more independent activities for groups that are capable of working without close guidance; provide more structure and supervision to those groups that need it. |
| • Structure choices within varying contexts, such as computation, explanation, application, and problem solving. | • Open-ended activities with multiple entry points. For example “The answer is 12, what is the question?”  
• Maria has \{12, 60, 121\} pine cones. She gave Evan \{5, 15, 46\} pine cones. How many pine cones does Maria have now? |
| • Encourage students to use their choice of different tools to perform the same task. | Students could use:  
• paper and pencil  
• manipulatives  
• computer  
• their own bodies  
• SMART Board |
In order to help students make informed decisions about their learning, it may be useful to have them determine their preferred learning style and/or perform a multiple intelligence test.

An online site students can access in order to complete a multiple intelligence test can be found at the following link: http://www.literacynet.org/mi/assessment/findyourstrengths.html. Teachers should ensure their students write their preferred intelligence on a cue card so that this information can be accessed by the teacher at a later date for planning and assessment purposes.

The following resource was provided and adapted from a colleague in the Halton District School Board. It not only provides an opportunity for students to learn about their preferred learning style (visual, auditory, or kinesthetic), but the content and outcome of the survey are directly related to the subject of math. At the end of the questionnaire, students are even encouraged to create a bar graph out of their results in order to display which learning styles they prefer most and least. Teachers are encouraged to copy and use the assessment within their own classrooms.
Math Learning Style

Circle the number that closest matches how well the sentence describes you. 1 (least) to 4 (most)

1. Reading a math problem out loud helps me learn better when I am studying. 1 2 3 4

2. I learn math better when I can talk about it. 1 2 3 4

3. Drawing a picture of a word problem helps me understand how to do it on a test. 1 2 3 4

4. Using math manipulatives helps me learn new concepts better. 1 2 3 4

5. Moving around while learning helps me concentrate more. 1 2 3 4

6. I enjoy making things with my hands for a hobby. 1 2 3 4

7. Watching someone complete a math problem helps me understand more. 1 2 3 4

8. Talking about a math problem while learning in class helps me understand it better. 1 2 3 4

9. When I solve a problem on a test, I talk it through in my head. 1 2 3 4

10. I learn math better when I watch someone do it. 1 2 3 4

11. I understand instructions better when someone tells me what they are. 1 2 3 4

12. Pictures and charts help me see how the parts of a problem go together. 1 2 3 4
13. When I solve math problems on a test, I picture my notes in my head to help me.

14. Making study aids with my hands helps me learn when I study.

15. I enjoy putting things together.

To Score

**Step One:** Fill in each answer score next to the appropriate question number.

Add the totals.

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To Score

**Step Two:** Graph It!

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HAVE A PASSION FOR YOUR STUDENTS, TEACHING, AND FOR CURRICULUM CONTENT

Most teachers enter the teaching profession because they want to make a difference in the lives of their students. According to Collinson et al. (1998), the ultimate goal for an effective teacher is to bring out the best in his or her students. To many teachers, students are the most important aspect of their career.

How experts describe effective teachers
(Gentry et al., 2011; Thompson, Warren, Foy, & Dickerson, 2008)

• Teachers ensure their students' needs are being met by knowing them as individuals with specific skills and interests
• They constantly monitor their students' progress
• They help students understand concepts which are particularly challenging to them

How students describe effective teachers
(Waldrip & Fisher, 2003)

• Their teachers were willing to help
• They knew whether their students understood the content
• They listened to their students
• They did not become angry quickly
• They took extra time to explain the material with which their students struggled

In order for teachers to differentiate their instruction and help students with their individual needs, these teachers must know their students extremely well. Effective teachers learn about their students' interests, hobbies, strengths, and weaknesses in order to better meet the needs within their classroom.
Students feel as though their teacher genuinely cares about them when their teacher knows each student as an individual and not just as part of a class (Stronge & Hindman, 2003). According to Collinson et al. (1998), effective teachers are honest and open with their students but are careful not to cross that professional line.

These teachers know that students must be put first. Anecdotes about the family can help create a bond with students and provide examples of life lessons for the class; but personal problems that exist outside of the classroom are inappropriate discussion material and must be set aside (Collinson et al., 1998).

When teachers are passionate about their profession and teaching curriculum content, this enthusiasm is passed on to their students. These students are excited to learn about subject matter because their teacher makes it interesting and engaging.

According to a number of different studies (Bishop, 1968; Sparks & Lipka, 1992; Waldrip et al., 2009), teachers who were deemed “the best” and “most successful” by their students:

- embraced student-centered education
- presented their subject matter in meaningful ways
• communicated content to their students in an effective manner
• used stimulating, motivating, and inspirational techniques to initiate or increase student interest in a particular subject area

The more interested students become in their learning, the easier it is for teachers to step out of the role of lecturer and into the position of advisor, observer, and collaborator.

Effective teachers are “skilful at encouraging complex functioning, learning, creativity, and flexibility to create intellectual, interactive classroom cultures.”

(Collinson, 1996, p.10)

Effective teachers understand that students must be interested and excited about what they are learning in order to be on task and working hard within the classroom. When students are busy working they are less likely to be disruptive and take time away from learning.

Beyond simply presenting information in meaningful ways, teachers can get students more invested in what they are learning by welcoming their input, giving them choices, allowing them to make decisions, and giving them opportunities to teach each other (Allington & Johnston, 2000; Collinson, 1996; Gentry et al., 2011; Waldrip et al., 2008).

**Teachers who are truly passionate about teaching realize that the best way to teach is to relinquish at least some control of the classroom to their students.**
Every teacher must be knowledgeable about the core material that they are teaching. Teachers who possess subject-matter competence are respected and deemed successful by their students (Bishop, 1968; Cruickshank & Haefele, 2001).

Good teachers connect with students and use this connection to introduce students to new and interesting subjects (Palmer, 1997). Effective teachers not only know their subject matter, but they also make it engaging and fun to learn. Gentry et al. (2011) found that effective teachers helped their students learn content by:

- creating a welcoming classroom climate
- using humour within the classroom
- having fun with their students

Effective teachers must commit to being lifelong learners and develop a keen sense of intrapersonal knowledge (Collinson, 1996). Students described teachers who made the greatest difference in their educational careers as those who had an obvious desire to constantly update their personal knowledge and understanding of curricular content (Bishop, 1968).

In an effort to expand and challenge their own perspectives, effective teachers engage in team teaching and collaboration with other adults (teachers or parent volunteers); (Collinson, 1996). According to Howey and Collinson (1995), these efforts to work in partnership with other adults are extremely beneficial because they allow teachers to develop different but effective ways to teach and learn, make more balanced decisions, and receive constructive criticism that encourages them to take risks within their classrooms. The handout on the following page can be used to aid in such processes.
Reflective Practice Sheet

The following model is based on Borton’s Developmental Framework for Reflection. It is based on the need for practitioners to operate in the real world of practice – the need to identify, make sense of, and respond to real-life situations. It asks the person to reflect using three basic starting points to the following questions:

<table>
<thead>
<tr>
<th><strong>What?</strong></th>
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<tr>
<td>e.g., What happened; what was I doing; what were others doing?</td>
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<td>This combines the reflective processes of identifying the experience and describing the detail.</td>
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<th><strong>So What?</strong></th>
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<tr>
<td>e.g., So what more do I need to know in order to understand the situation; so what could I have done that was different?</td>
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<td>This part breaks down the situation and tries to make sense of it by analysis and evaluation, drawing on previous experiences and knowledge.</td>
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<th><strong>Now what?</strong></th>
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<tr>
<td>e.g., Now what do I do to make things better? Now what will I do? Now what might be the consequences of this action?</td>
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<tr>
<td>This stage combines the processes of exploring alternatives and planning action that will be put into practice in order to develop or change practice.</td>
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HAVE RESPECT AND TRUST WITH YOUR STUDENTS

When teachers maintain respect for their students and expect the same in return, they are building a foundation of mutual respect within their classrooms and ultimately demonstrating to their students that they care (Collinson et al., 1998). According to researchers (Bishop 1968; Collinson et al. 1998; Stronge & Hindman, 2003; Waldrip et al., 2009), effective teachers exhibit respect for their students by:

- Requesting information and criticism from students
- Treating students fairly
- Knowing students on an individual basis
- Altering instruction for intellectual differences
- Communicating that teachers are learners too
- Keeping student information confidential
- Sharing personal stories from which students learn life lessons

Respect for Students includes...
Teachers must work at developing effective communication techniques and strong interpersonal skills in order to build positive relationships with their students (Wubbels, Levy, & Brekelmans, 1997). These relationships are essential for creating a classroom climate where students:

- engage in respectful interaction
- feel comfortable sharing their ideas
- know they will be considered valuable and important

(Deutsch, 2005; Stronge & Hindman, 2003)

Eliciting trust within the classroom is similar to developing respect because the teacher must still work at knowing his or her students on a more personal level. Many researchers (Collinson et al., 1998; Gentry et al., 2011; Haberman, 1995; Spencer & Spencer, 1993; Wubbels et al., 1997) have found trust to develop from:

- Genuinely listening and responding to students in a positive manner
- Creating learning environments where students feel successful
- Honestly admitting to mistakes, limitations, and fallibility
- Sharing appropriate anecdotes about life and life lessons
- Encouraging questioning in the learning process
- Providing students with classroom responsibilities and opportunities to make decisions
- Creating learning environments where students feel successful
Students who expressed feelings of trust with their teachers were in classes where the teachers viewed relationship-building with their students as important (Gregory & Ripski, 2008). The more trust teachers invest in their students, the more their students will trust them in return.

Although it takes a lot of time and effort to build student–teacher relationships within the classroom, Gregory and Ripski (2008) claim that trust will develop over time and students in turn become:

- more cooperative
- more engaged with course activities
- more willing to accept the rules and norms of the classroom
- better able to respond to requests from the teacher

Interestingly, teachers who work at building relationships within their classrooms are more likely to earn the trust of students who feel distance from their teachers because of such social determinants as race and social class (Gregory & Ripski, 2008).

The five themes discussed in this section of the handbook can be applied to any teacher looking to improve his or her practice. However, the following section of this handbook will describe teaching strategies that are specifically applicable and useful to intermediate math teachers.
Mathematics Strategies

There is a wide variety of effective math strategies teachers can utilize within their classrooms on a daily basis in order to improve the performance of their students.

The strategies within this resource are based on those that have been continuously discussed and presented in the literature (Chrisler, 2013; Henke, Chen, & Goldman, 1999; Jensen, 2013; Rapp, 2009, Reynolds & Muijs, 1999; Tuttle, 2007). These strategies include:

- the use of manipulatives
- using the SMART Board and other virtual manipulatives
- incorporating physical movement into lessons
- allowing students to play or create math games
- offering opportunities to work on the computer
- incorporating math into other subjects
- allowing students to complete their math without writing
- altering the pace of instruction
- ensuring math problems are related to the real world
- allowing students to work in various flexible groupings
- whole-class math discussions
- making use of the three-part lesson plan

These strategies are unlikely to be required all at the same time or for every student in the classroom; instead, an assortment of these practices should be selected and utilized by the teacher throughout the school year.

Teachers must attempt to cater to the needs and various learning styles of their students in order to ensure that each of them is engaged, motivated, and interested in learning math (Loong & Herbert, 2012).
One of the most common and widely discussed strategies for enhancing the mathematics classroom is the use of manipulatives. The Ontario Ministry of Education (2006) bases the valuable nature of manipulatives on their ability to provide useful insights into student thinking.

According to researchers (Lim & Kor, 2012; NCTM, 1989; Rapp, 2009), teachers should provide students with as many different manipulatives as possible because they:

- encourage students to become actively engaged with their learning
- accommodate a variety of learning styles
- provide every student with an equal opportunity to participate
- help students develop a better understanding of numbers, patterns, geometry, and measurement

Typically, manipulatives refer to “concrete” or “hands-on” material like pattern blocks, unifix cubes, legos, drinking straws, paperclips, buttons, rulers, balances, geoboards and rubber bands, peg boards, beads and strings, checkers, coins, and tiles of various geometric shapes (Henke et al., 1999; NCTM, 1989; Rapp, 2009).

The charts presented on the following pages include an extensive list of manipulatives that can be used within the intermediate math classroom and also describe how these manipulatives connect to various mathematical concepts.
<table>
<thead>
<tr>
<th>Manipulative</th>
<th>Connections to Mathematical Concepts and Skills</th>
</tr>
</thead>
<tbody>
<tr>
<td>Abacus</td>
<td>problem-solving/thinking skills, algebra/patterns, counting, equality/equivalence, relationships/connections, reasoning, place value, number concepts/operations, estimation</td>
</tr>
<tr>
<td>Attribute blocks</td>
<td>classification, symmetry, reasoning, patterns, data collection/management/graphing/interpretation, number concepts/operations, problem-solving/thinking skills, similarity/congruence, angles</td>
</tr>
<tr>
<td>Balance and Weights</td>
<td>measurement/scale, money, reasoning, decimals, estimation, data collection/management/graphing/interpretation, number concepts/number systems/whole numbers, classification, weight/mass, equality/equivalence</td>
</tr>
<tr>
<td>Base 10 materials, with transparent overhead set</td>
<td>place value, money, measurement/scale, fractions, decimals, ratio/proportion/percent, patterns, area, similarity/congruence, classification, number concepts/operations, perimeter, relationships/connections, problem-solving/thinking skills, equality/equivalence</td>
</tr>
<tr>
<td>Calculators with two-line display and overhead calculator</td>
<td>problem-solving/thinking skills, algebra/patterns, counting, equality/equivalence, measurement/scale, reasoning, decimals, fractions, money, number concepts/operations, estimation, mental math, place value</td>
</tr>
<tr>
<td>Connecting plastic shapes to build 2-D shapes and 3-D nets (e.g., Polydron sets)</td>
<td>classification, perimeter, angles, reasoning, data collection/management/graphing/interpretation, similarity/congruence, area, problem-solving/thinking skills, spatial visualization, tessellations/tiling, fractions, transformational geometry, measurement/scale</td>
</tr>
<tr>
<td>Colour tiles</td>
<td>patterns, estimation, number concepts/operations, reasoning, equality/equivalence, place value, classification/sorting, fractions, problem-solving/thinking skills, probability/chance, measurement/scale, area, perimeter, data collection/management/graphing/interpretation, spatial visualization, similarity/congruence, relationships/connections, ratio/proportion/percent</td>
</tr>
<tr>
<td>Cuisenaire rods (coloured relational rods)</td>
<td>classification/sorting, number concepts/operations, similarity/congruence, fractions, symmetry, ratio/proportion/percent, place value, patterns, reasoning, estimation, problem-solving/thinking skills, relationships/connections</td>
</tr>
<tr>
<td>Counters (transparent)</td>
<td>measurement/scale, patterns, estimation, similarity/congruence, relationships/connections, place value, counting, estimation, problem-solving/thinking skills, volume, fractions, number concepts/operations, equality/equivalence, ratio/proportion/percent, reasoning</td>
</tr>
</tbody>
</table>


<table>
<thead>
<tr>
<th><strong>Dice/Number cubes</strong></th>
<th>number concepts/operations, mental math, fractions, decimals, probability/chance, problem-solving/thinking skills, reasoning, data collection/management/graphing/interpretation, equality/equivalence</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Fraction kit</strong></td>
<td>fractions, classification, spatial visualization, similarity/congruence, mental math, number concepts/operations, perimeter, area, ratio/proportion/percent, relationships/connections, problem-solving/thinking skills, equality/equivalence, estimation, reasoning</td>
</tr>
<tr>
<td><strong>Geoboards</strong></td>
<td>classification, area, perimeter, symmetry, fractions, coordinate geometry, angles, estimation, similarity/congruence, rotations, reflections, translations, polygons, spatial visualization, reasoning</td>
</tr>
<tr>
<td><strong>Geoboards</strong></td>
<td>classification, area, perimeter, symmetry, fractions, coordinate geometry, angles, estimation, similarity/congruence, rotations, reflections, translations, polygons, spatial visualization, reasoning</td>
</tr>
<tr>
<td><strong>Graduated beakers</strong></td>
<td>measurement/scale, volume, estimation, problem-solving/thinking skills, spatial visualization, data collection/management/graphing/interpretation, reasoning</td>
</tr>
<tr>
<td><strong>Hundreds chart, hundreds board, ninety-nine chart</strong></td>
<td>place value, counting, estimation, patterns, number concepts/operations, fractions, probability/chance, spatial visualization, ratio/proportion/percent, mental math, decimals, money, problem-solving, relationships/connections, reasoning</td>
</tr>
<tr>
<td><strong>Interlocking cubes</strong></td>
<td>number concepts/operations, place value, patterns, reasoning, ratio/proportion/percent, equality/equivalence, symmetry, weight/mass, spatial visualization, probability/chance, area, perimeter, volume, quantity, transformational geometry, fractions, estimation, mental math, problem-solving/thinking skills, money, measurement/scale, relationships/connections</td>
</tr>
<tr>
<td><strong>Measuring cups</strong></td>
<td>measurement/scale, estimation, number concepts/operations, data collection/management/graphing/interpretation</td>
</tr>
<tr>
<td><strong>Measuring spoons</strong></td>
<td>measurement/scale, estimation, number concepts/operations, data collection/management/graphing/interpretation</td>
</tr>
<tr>
<td><strong>Measuring tapes</strong></td>
<td>measurement/scale, area, perimeter, constructions, fractions, estimation, number concepts/operations, data collection/management/graphing/interpretation</td>
</tr>
<tr>
<td><strong>Mitre stick</strong></td>
<td>measurement/scale, area, perimeter, constructions, fractions, estimation, number concepts/operations, data collection/management/graphing/interpretation</td>
</tr>
<tr>
<td><strong>Miras (red transparent plastic tools)</strong></td>
<td>symmetry, similarity/congruence, transformational geometry, angles, mental math, problem-solving/thinking skills, spatial visualization</td>
</tr>
<tr>
<td>Item</td>
<td>Topics</td>
</tr>
<tr>
<td>-----------------------------</td>
<td>----------------------------------------------------------------------</td>
</tr>
<tr>
<td>Mirror symmetry</td>
<td>symmetry, similarity/congruence, transformational geometry, angles,</td>
</tr>
<tr>
<td></td>
<td>mental math, problem-solving/thinking skills, spatial visualization</td>
</tr>
<tr>
<td>Money</td>
<td>money, ratio/proportion/percent, counting, fractions, probability/chance, problem solving/ thinking skills, estimation, mental math, place value, relationships/connections, data collection/management/graphing/interpretation, reasoning, measurement/scale, decimals, number concepts/operations</td>
</tr>
<tr>
<td>Number lines</td>
<td>place value, counting, estimation, patterns, number concepts/operations, fractions, probability/chance, spatial visualization, mental math, decimals, money, measurement/scale, problem solving, similarity/congruence</td>
</tr>
<tr>
<td>Pattern blocks, with transparent overhead Set</td>
<td>patterns, angles, classification, ratio/proportion/percent, tessellations/tiling, symmetry, similarity/congruence, equality/equivalence, area, perimeter, transformational geometry, problem-solving/thinking skills, reasoning, fractions, decimals, spatial visualization, data collection/management/graphing/interpretation, measurement/scale, number concepts/operations</td>
</tr>
<tr>
<td>Pentominoes</td>
<td>geometry, spatial visualization, problem-solving/thinking skills, patterns, reasoning, fractions, similarity/congruence, perimeter, angles, classification, symmetry, transformational geometry, number concepts/operations, area, tessellations/tiling</td>
</tr>
<tr>
<td>Plastic polygons (wide variety of triangles and regular and irregular quadrilaterals)</td>
<td>classification, ratio/proportion/percent, number concepts/operations, perimeter, angles, reasoning, data collection/management/graphing/interpretation, similarity/congruence, area, problem-solving/thinking skills, spatial visualization, tessellations/tiling, fractions, transformational geometry, measurement/scale</td>
</tr>
<tr>
<td>Playing cards</td>
<td>estimation, number concepts/operations, mental math, problem-solving/thinking skills</td>
</tr>
<tr>
<td>Protractors, including full-circle protractors</td>
<td>constructions, angles, measurement/scale</td>
</tr>
<tr>
<td>Safety compass</td>
<td>constructions, angles, measurement/scale</td>
</tr>
<tr>
<td>Scales</td>
<td>measurement/scale, reasoning, decimals, estimation, data collection/management/graphing/interpretation, number concepts/number systems/whole numbers, classification, weight/mass, equality/equivalence</td>
</tr>
</tbody>
</table>
### Manipulatives

**Spinners (number, colour)**
- number concepts/operations, decimals, fractions, probability/chance, mental math, reasoning, problem-solving/thinking skills

**Standard masses**
- Measurement/scale, reasoning, decimals, estimation, data collection/management/graphing/interpretation, number concepts/number systems/whole numbers, classification, weight/mass, equality/equivalence

**Stopwatch**
- fractions, measurement/scale, number concepts/operations, relationships/connections

**Tangrams, with transparent overhead set**
- spatial visualization, problem-solving/thinking skills, patterns, reasoning, fractions, similarity/congruence, perimeter, ratio/proportion/percent, angles, classification, symmetry, transformational geometry, constructions, relationships/connections, number concepts/operations, area, tessellations/tiling

**Thermometers**
- measurement/scale, estimation, number concepts/operations, data collection/management/graphing/interpretation

**3-D geometric Solids**
- area, volume, classification, angles, reasoning, measurement/scale, symmetry, fractions, spatial visualization, perimeter, similarity/congruence, area, problem solving/thinking skills, weight/mass, relationships/connections, transformational geometry, tessellations/tiling

**Two-colour Counters**
- measurement/scale, patterns, estimation, similarity/congruence, place value, counting, estimation, problem-solving/thinking skills, fractions, number concepts/operations, classification, spatial visualization

**Trundle wheel**
- measurement/scale, estimation, number concepts/operations, data collection/management/graphing/interpretation

All information obtained from Ontario Ministry of Education (2006).

Manipulatives can also be represented in various technological formats. The novelty of SMART Boards, graphing calculators, and virtual manipulatives captivate students of all ages and ability levels.

Besides concrete manipulatives, there are many other resources and strategies that teachers can and should use within their classrooms. **Manipulatives should be thought of as just ONE resource, in a repertoire of many.** Additional techniques are described in detail on the following pages of this handbook.
THE SMART BOARD AND VIRTUAL MANIPULATIVES

SMART Boards have an endless number of programs that students can use to manipulate shapes, objects, numbers, graphs, and pictures, helping them to understand abstract concepts and make learning fun (Mueller, 2012).

The following SMART Board lesson on Pythagorean Theorem provides an example of a lesson that was designed on the SMART Board and incorporates independent, partner, and whole-class math activities.

Pythagorean Theorem

YOU MAY BE RIGHT, PYTHAGORAS, BUT EVERYBODY'S GOING TO LAUGH IF YOU CALL IT A "HYPOTENUSE."

Where did Pythagorean's Theorem Originate?

In Egypt, 5000 BC - 3000 BC, early civilizations used right triangles to divide land areas, estimate taxes, and to construct buildings (like the pyramids). However, the Theorem was not developed until 569 BC - 500 BC in Greece by the mathematician Pythagorus.
Pythagorean Theorem

A right triangle contains a right angle (90°).

right angles are identified by a square in the triangle corner.

The hypotenuse of a right triangle is the side opposite the right angle. It is the longest side of a right triangle.

The legs of the right triangle are the two sides that form the right angle.

Pythagorean Theorem ONLY applies to RIGHT angle triangles. Can you spot them?

Right angle triangles are always 90° and usually have a small square to show their location. Where do you think the hypotenuse is on this triangle?
The hypotenuse is always the longest side of the triangle and is always across from the 90° angle. Find the hypotenuse in each triangle...

REVIEW

Area of a square = side²
Area of a triangle = 1/2 base × height

A Right Triangle is a triangle in which one of it's interior angles is a right angle.

The Hypotenuse is the longest side of a right triangle, it is opposite the right angle.

What does the theorem mean? Let's explore together!

1) Draw a right triangle 4 units high and with a base of 3 units
2) Construct 3 squares off of each side, looking something like this...
3) Determine the area of each square.

What did you find?
To find the length of the hypotenuse, I can rotate the shapes

Find the area of each square.

What do you notice?
(Hint: Add the two small squares together)

Yes! The bigger square has the exact same area as the smaller two squares added together!

\[
\begin{align*}
9 + 16 &= 25
\end{align*}
\]
**PYTHAGOREAN THEOREM:**

The sum of the squares of the lengths of the legs of a right triangle is equal to the square of the length of the hypotenuse.

\[ a^2 + b^2 = c^2 \]

\[ 9 + 16 = 25 \]

What did we learn?

- In a right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.
- This can be written as \( a^2 + b^2 = c^2 \)
- Let's see if the equation works with our right triangle

\[ c^2 = a^2 + b^2 \]

\[ 5^2 = 3^2 + 4^2 \]

\[ 25 = 9 + 16 \]

\[ 25 = 25 \]

It works!

We can use the equation to find a side length we do not know!

Solution:

\[ c^2 = a^2 + b^2 \]

\[ c^2 = 5^2 + 12^2 \]

\[ c^2 = 25 + 144 \]

\[ c^2 = 169 \]

\[ c = \sqrt{169} \]

\[ c = 13 \]
The SMART Board is preferred by students over the same concrete manipulatives; however, only a few students can access the material at once (Mueller, 2012). Some teachers may prefer to use virtual manipulatives or graphing calculators so each student can participate in the lesson at the same time.

Students can use spreadsheet programs on school computers to create and manipulate graphs, or they can use interactive sites like NCTM’s Illuminations, the National Library of Virtual Manipulatives (NLVM), and Shodor’s Interactivate to access virtual manipulatives (Chrisler, 2013; Johnson, Campet, Gaber, & Zuidema, 2012).
Virtual manipulatives provide instant feedback to students and integrate pictures, sounds, verbal cues, and symbolic notations in order to demonstrate connections between various forms of representation (Johnson et al., 2012; Moyer-Packenham, Salkind, & Bolyard, 2008). These types of resources go beyond the basic capabilities of physical manipulatives and connect children to their 21st century world.
PHYSICAL MOVEMENT

When teachers introduce a new unit or concept, students are typically required to remain seated at their desks. Although this method of teaching may be necessary, it should not be the constant. Students learn and work best when they are able to move around and explore their learning environment (Beaudoin & Johnston, 2011).

Beaudoin and Johnston (2011) discuss two types of movements that teachers can incorporate into their mathematics classrooms:

- **Unrelated Movement**
  - This type of movement does not necessarily connect to the content of the lesson, but it can still help students engage in the learning process (Beaudoin & Johnston, 2011). For instance, students may participate in their learning by:
    - dancing
    - exercising
    - creating videos about math concepts
    - taking turns using the SMART Board
    - participating in silent math discussions on chart paper
    - exploring the school in a math scavenger hunt

Silent math discussions, often referred to as “Graffiti” activities, involve each student or group of students writing all they know about a particular topic on chart paper in complete silence. Typically, students rotate to all of the various pieces of chart paper, which incorporates the movement into the activity.

Chart paper can either be posted on walls around the room or at table/desk groups. In addition, the purpose of the activity can vary from reviewing concepts before a unit test to doing a diagnostic assessment of prior knowledge before a unit begins.
Purposeful Movement

Activities concentrating on purposeful movement would directly relate to mathematical content. For example, students could:

- walk around the edge of a desk to understand perimeter
- create numbers from body positioning
- divide themselves into sets in order to demonstrate a mathematical process like multiplication
- position themselves along a number line or coordinate plane
- display function transformations with the movement of a cut-out parabola

(Beaudoin & Johnston, 2011; Rapp, 2009)

The following picture demonstrates a “Human Grid” or, in other words, where students use their bodies as points on a co-ordinate plane made out of tape.

![Human Grid Picture](http://www.mathfilefoldergames.com/human-sized-coordinate-graph-game/)

Both unrelated and purposeful movement can be beneficial; however, when movement is purposeful and specifically related to the content being taught, students are said to demonstrate better academic gains and develop a more positive attitude towards the content of the lesson (Beaudoin, & Johnston, 2011).
MATH GAMES

The Ontario Ministry of Education (2006) recommends that games be included in mathematical materials within the classroom because they help to consolidate mathematical concepts by having students think about relationships between numbers and using numbers to solve problems.

“Students of all ages love to play games, and games, when chosen correctly, can reinforce mathematical concepts and provide engaging ways for students to practise basic facts or operations.”

(Ontario Ministry of Education, 2006, p. 18)

According to Kate Hammer (2011), education reporter for The Globe and Mail, play is extremely important because it teaches children about multiple viewpoints and self-regulation, qualities that can lead to a more fulfilling life in the future.

Board games and card games appropriate for the intermediate math classroom, which incorporate logical-thinking, problem-solving, and reasoning skills, include:

- Backgammon

- Monopoly
  ![Monopoly](http://abagond.wordpress.com/2010/12/07/if-america-were-a-game-of-monopoly/)

- Uno
  ![Uno](http://thingsifinished.blogspot.ca/2010/04/uno.html)

- Battleship
  ![Battleship](http://researchaccess.com/2014/04/crowdsourcing-throughout-the-research-process/battleship-board-game/)

- Go
  ![Go](http://entertainment.howstuffworks.com/leisure/brain-games/go.htm)

- Mancala
  ![Mancala](http://www.p-wholesale.com/cn-pro/11/472101/wooden-mancala-board-game-446227.html)

- Othello/Reversi
  ![Othello/Reversi](http://gamescrafters.berkeley.edu/games.php?game=othello)
Besides playing games provided by the teacher, students can create their own math games, giving them a sense of ownership. Both commercial and student-created games can help to build understanding and computational fluency for various math concepts (Legnard & Austin, 2012; Ontario Ministry of Education, 2006; Rapp, 2009, Silverman, 2002).

Teachers may also wish to create their own math games which directly link to the curriculum, by using the SMART Board or materials found within their classrooms (McKeny & Foley, 2012/2013; Mueller, 2012).

The graphic on the following page provides an example of a matching game developed on the SMART Board by the author. This game requires knowledge and understanding of equivalent fractions, improper fractions, and mixed numbers at a grade 7 and 8 instructional level.
No matter the content or what type of game is chosen to play, students are often willing and eager to participate. Dividing students into teams also adds an element of excitement to the game. Teachers are encouraged to reproduce similar games within their own classrooms and may use the game presented above as a guide.
Computer literacy, especially in mathematics, is essential for students growing up today because computer use is prevalent in all aspects of life. In addition, students solidify their problem-solving strategies and mathematical understandings by exploring concepts independently of the teacher.

Using the computer to complete a math activity is a rather commonplace occurrence in most schools. However, there are many applications that computers can be used for besides simple drill-and-practice activities. In fact, the Ontario Ministry of Education (2006) clearly states that math games used to fill time have little instructional value at all.

“Software chosen for use in Ontario classrooms should promote mathematical communication, understanding, and reasoning; encourage collaboration and social interaction; provide for the varying needs of each individual student; and be linked with the Ontario curriculum.”

(Ontario Ministry of Education, 2006, p. 18)

Teachers using computers to assist students in mastering math concepts must understand that students will not benefit from the technology if it is not used appropriately (Henke et al., 1999). Students, especially younger students, often need a prelesson or
scaffolding before attempting to master a concept or lesson on the computer (Chrisler, 2013; Kay, 2012).

In order to ensure a wide variety of students’ needs are being met, teachers should attempt to use the computer in several different ways. For instance, students can:

- utilize computer software (e.g., The Geometer’s Sketchpad) or websites such as coolmath4kids.com to complete individual or group activities which further explore the concepts introduced in the classroom (Rapp, 2009).

- create and use blogs, wikis, podcasts, videoconferencing, and discussion forums to communicate with others around the world about their mathematical learning (Henke et al., 1999; Loong & Herbert, 2012; Tuttle, 2007).

- enhance their knowledge by doing virtual math and science experiments (Henke et al., 1999).

- use spreadsheet software (e.g., Microsoft Excel) to generate and manipulate a variety of graphs (Chrisler, 2013; Jensen, 2013).

- explore the libraries of information on the internet to write a report about a mathematical theory or theorist (Henke et al., 1999).

- work collaboratively online via Prezi or Google Docs in order to write and edit mathematics presentations in real-time (Settle, Abrams, & Baker, 2011; Yang, 2010).

- pose questions to certified math teachers using the Government of Ontario’s funded Homework Help website or to mathematicians using the Ask NRICH Website managed by the Centre for Mathematical Sciences at the University of Cambridge (Loong & Herbert, 2012).
The following pictures demonstrate some examples of the types of graphs that can be created using spreadsheet software like Microsoft Excel. The graphs below depict a bar graph (A), circle graph (B), broken-line graph (C), scatter plot (D), and double bar graph (E).

All pictures from: http://www.edugains.ca/newsite/math/curriculum/curriculum7_8.html
Teachers can incorporate spreadsheet software into the design of collaborative group projects. Depending on the goals of these projects, they could involve data collection, graphing, problem-solving, student choice, and real-world applications. The graphs and reports produced by students from these activities can also provide excellent assessment opportunities for teachers.

As a caution, Johnson et al. (2012) explain that teachers should consider “the extent to which the online tool addresses the target math content, the ways in which the tool takes advantage of technology, and how to elicit responses that would yield meaningful insights about student understanding” (p. 202) before selecting a tool for assessment purposes.

When online tools are carefully selected and utilized appropriately, students can benefit from:

- visual scaffolding
- decreased cognitive load
- increased motivation and focus
- control over the learning process

(Kay, 2012)

These benefits contribute to more productive students and a better understanding of mathematical concepts overall.
SUBJECT INTEGRATION

Creating math lessons that are motivating, interesting, and applicable to various learning styles can be a difficult task. In order to pique the interests of every student in the classroom and ensure all their needs are being met, teachers should make an effort to integrate some of their math lessons with other subjects.

Math can be incorporated into art by studying patterns in quilts (Kurz, 2013); finding shapes in flags (Schad, Georgeson, & Bunten, 2009); and posing mathematical questions about pictures (Orr & Suh, 2013). Some example activities using pictures to investigate math concepts are shown below.

![Figure 1](image1.png)
This photograph engaged older students in an interesting math talk about partial units and how to decompose shapes.

![Figure 2](image2.png)
One Math Moment activity has students look at a picture of a 100-calorie snack and list all the math questions that come to mind.
Literacy can be interwoven into math in many different ways as well. For instance, students can read books, write poetry, and create short stories inspired by mathematical concepts (Cohen, 2013). Students can be provided with questions to solve which relate to the mathematical concepts of a book read in class, or they can create their own (Ontario Ministry of Education, 2006). In a more traditional sense, literacy can be used to describe numbers, number sense, and measurement (McKeny & Foley, 2012/2013).

According to the Ontario Ministry of Education (2006), using literature in mathematics:

- provides students with an opportunity to make connections with their own lives
- provides students with a context in which they can practice mathematics
- enriches students’ view of the world of mathematics
One of the easiest subjects to combine with math is science. Essentially almost every experiment in science involves a mathematical measurement, calculation, or description. A simple experiment about circuits can incorporate probability, counting, tables, charts, symbols, fractions, units of measurement, and math language (Hall, 2009). The following activity provides one such example:

**Light it up!**

Mr. Meester was a bit disorganized for teaching his science lesson on simple circuits. He pulled out the box of materials that contained copper wires, light bulbs, batteries, and battery holders. The last time he had used the materials for class, a number of batteries were dead, and some light bulbs were burned out. He did not have time to test and remove the defective materials, but he thought it would be a good task for his students.

On each table, he placed a battery holder, a light bulb holder, two wires, six light bulbs, and four batteries. He asked groups of students to create simple circuits and find all the light bulbs and batteries that were still working and place the nonworking ones aside for disposal.

After a few minutes of testing, Peyton said, “The bulbs only light about half the time.” Solana answered, “That’s because half the light bulbs are burned out.” Is her statement true? Justify your answer.

Activity and picture obtained from Hall (2009).

Similar to the previous task, teachers can also create large group activities that combine multiple subjects or various disciplines in order for students to use their mathematical knowledge in a variety of different ways on the same task (Jensen, 2013).

An activity combining art, language, science, and math would suit the needs of many students within the classroom and address a number of curriculum expectations at the same time. These activities would also help students experience math concepts in a manner that is more recognizable and meaningful to their lives.
NON-WRITING ACTIVITIES

Often teachers take for granted the literacy skills involved in understanding and correctly answering math questions. Students who are visual–spatial learners, struggle with language, have difficulty using fine motor skills, or simply are unmotivated by words may feel unsuccessful in math because they are unable to easily produce written output.

Students can demonstrate what they know in a variety of ways that do not involve writing. For instance, students can:

- draw pictures
- record their explanations using audiotape or the SMART Board
- take pictures using a digital camera and narrate a photo story
- dictate their answers or reasoning to a scribe
- create graphs with spreadsheet programs
- use computer presentation software like PowerPoint and Prezi to convey a math principle


Teachers can support these students by creating math assignments where solutions are found with hands-on activities and concrete objects like paper and string, toothpicks and marshmallows, pattern blocks, beads, or quilts (Holbert & Barlow, 2012/2013; Kurz, 2013).

Pictures provided by the author showing students using paper, string, and a ruler to determine the circumference of a circle and then the surface area of a cylinder.
The following activity involves students using marshmallows and toothpicks to create various 2-D shapes and investigate their areas. The handout that is provided with the activity can either be filled out by a scribe or perhaps the teacher can have students work in groups where a specific student (“the recorder”) is designated to fill in the required information.


Activity obtained from http://www.p12.nysed.gov/
While students describe concepts or ideas, teachers can record their explanations and use them for marking purposes or professional development exercises (Schifter, 2005).

The SMART Board is a great tool for recording student explanations because it can capture what the student is saying and the math work that is being performed by the student on the screen at the same time (Lapko, 2013). Oral communication is an important language skill and should be encouraged in the mathematics classroom.

This student is demonstrating how she uses the associative property of multiplication to solve problems with two or more digits. **Notice the recorder control panel in the upper right-hand corner.**

While an individual student demonstrated understanding of number sense, multiplication, facts, place value, and addition without regrouping, others used calculators to validate the solution.

All pictures and information obtained from Lapko (2013).
ALTERING THE PACE OF INSTRUCTION

When teachers provide instruction, introduce a new task, or observe students working in groups, they should constantly be monitoring students for success.

Some students will struggle with the material, while others will complete it at a brisk pace. Rapp (2009) suggests providing struggling students with more time to complete tasks so they may process their thinking and demonstrate their mathematical skills.

Extra time is especially important when questions are more cognitively demanding or require translation for such students as English Language Learners (Miller & Koesling, 2009; Reynolds & Muijs, 1999).

According to the Ontario Ministry of Education (2006), students require sufficient time, free from distractions and interruptions, to learn, reflect properly, and share their ideas with each other.

Increased time to think about math concepts + Increased time to reflect and share ideas about math = Effectively processing and understanding math concepts

Providing students with more time may not always be as simple as giving them “1 more minute.” Lemov (2010) explains that teachers should provide students with narrated wait times so that they know
what to do while the teacher is waiting. There are a number of phrases that teachers can use to further direct their students. For example:

1. “I’m waiting for more hands”
2. “I’d like to see at least 15 hands before we hear an answer”
3. “I’m waiting for someone who can connect this problem to another problem we did yesterday, ideally about surface area of a cylinder”
4. “I’m going to give everyone lots of time because this question is tricky. Your first answer may not be the best”
5. “I’m seeing people thinking deeply and jotting down thoughts. I’ll give everyone a few more seconds to do that”
6. “I’m seeing people going back to the chapter to see whether they can find an example. That seems like a great idea”
7. “I’m looking for someone who’s pointing to the place in the text where you can find the answer”
8. “I’ll start taking answers in 10 seconds”
9. “I’m starting to see more hands now. Four, five, seven. Great. People are really starting to get comfortable taking a risk here”

Adapted from Lemov (2010).

Alternatively, Stepanek (1999) recommends using a strategy for high-ability students in the mathematics classroom called “Most Difficult First.” Students are given the opportunity to complete the five most difficult problems of their assignment first, and, if answered correctly, they are allowed to skip the rest of the assignment and work on an alternative activity or enjoy free time (Stepanek, 1999).

The pace of instruction is optimal when each student is actively engaged with his or her work and able to respond to questions accurately, not when every student is working at the same pace.
REAL-WORLD PROBLEM SOLVING

Using mathematics in real-world contexts is imperative to student learning while at school and throughout their daily lives (Ontario Ministry of Education, 2006).

“As students make connections between mathematical concepts and ideas and make connections within and across strands, their understanding is enhanced and learning mathematics becomes more meaningful.”

(Ontario Ministry of Education, 2006, p. 51)

According to researchers (Chrisler, 2013; Jensen, 2013; Orr & Suh, 2013; Rapp, 2009; Schifter, 2005), math problems addressing real-world issues are necessary in the mathematics curriculum and helpful to students because they help to:

- make more sense of the surrounding world
- construct a larger conceptual framework for solving problems
- make connections between different types of problems
- develop skills like critical thinking, reasoning, and creativity

Activities that incorporate real-world ideas do not have to be overcomplicated; they can range from simple to complex. For instance, teachers can discuss fractions and division in terms of sharing food, such as pizza or cake (Schifter, 2005).
The following SMART Board graphics, created by the author, provide several examples of how to incorporate real-world ideas into fraction lessons in the intermediate math classroom.

- How many total cards are in a deck of cards?

- Write the following as a fraction:
  1) The number of aces in a deck of cards
  2) The number of RED cards in a deck of cards
  3) The number of diamond cards in a deck of cards

- Out of all the chairs in the room, how many are **green**? **blue**? **pink**? (write as fractions)

- How can you divide this money in half?
Other lesson ideas for incorporating real-world activities could involve students using grocery items to explore mathematical ideas and create real-world problems based on their findings (Orr & Suh, 2013; Tuttle, 2007).

Often students find it helpful to apply the Pythagorean Theorem to real-world activities like a ladder standing up against a wall or finding the distance between landmarks on a map.

Students can even use spreadsheet software to take on the role of mathematics consultants and find algebraic functions that appropriately model data from real-life scenarios (Chrisler, 2013; Jensen, 2013).

\[ \text{How long must a ladder be to reach the roof?} \]

\[ \begin{align*}
? & \quad 2.5 \text{ ft.} \\
8 \text{ ft} & \quad \downarrow 
\end{align*} \]

**Scenario: Shipping Costs**

The Civil Aeronautics Board wants to analyze shipping costs. Students are given data regarding the cost of shipping freight a certain distance. When they graph the data, they discover that a linear function most closely models this information. Once they determine the equation, they make predictions regarding the cost to ship crates varying distances. In addition, on the basis of the derived equation, students make recommendations to their client regarding the necessary budget for shipping.

Obtained and adapted from Jensen (2013).

Lessons and projects linking mathematical concepts to real-world ideas can take on many different forms; however, teachers should not feel overwhelmed. These activities should be a fun change of pace and chosen on the basis of student need and interest.
FLEXIBLE GROUPINGS AND MATH TALK

Students are best served when they are provided with a variety of learning situations (Ontario Ministry of Education, 2006). Students should be exposed to math concepts in the following ways:

- Independently
- In partners
- In a small group (3 or 4 students)
- In a large group (6 or 7 students)
- With the entire class (whole-class instruction)

When students learn from each other, not just from their teachers, they are motivated to learn and feel a sense of accomplishment when they complete a task (NCTM, 2000; Slavin, 1996).

According to researchers (Henke et al., 1999; Jensen, 2013; Lapko, 2013; Reynolds & Muijs, 1999; Stepanek, 1999), students benefit from working in groups because they:

- Learn to be flexible
- Provide and receive help through mathematical explanations
- Respect the opinions of others
- Reflect on their own thinking
- Overcome insecurities
- Consider multiple viewpoints at once
- Develop new problem-solving strategies
- Conisder multiple viewpoints at once

Students who work in groups...
Although group work has many benefits, it must be orchestrated with a great amount of thought and care in order to be successful. Students can become passive and off-task if expectations are not clear (Reynolds & Muijs, 1999).

Teachers must ensure that each person in a group is accountable and participating in some way (Lim & Kor, 2012; Reynolds & Muijs, 1999). For example, the teacher may assign roles for each group member (i.e., leader, note taker, etc.) and award points based on group participation (Lim & Kor, 2012).

A computer program like Google Docs can also help to increase individual accountability because the contribution made by each group member is highlighted in a different colour and can easily be tracked by the teacher in the revision history (Wood, 2011).

Remaining “flexible” about groupings ensures that students are working in new and different experiences on a constant basis while targeting their various learning needs (Alberta Education, 2010).

“At different times, groups may be based on friendship, a mix of abilities, concepts needing practice, interest, or random selection. Regardless of the type of group formed, students must have time to reflect and to share their discoveries, either in a small group, with a partner, or with the whole class.”

(Ontario Ministry of Education, 2006, p. 41)
In essence, groups vary based on who is selecting them and for what purpose. Students may be asked to choose their own groups or they could be teacher selected. Teachers may choose to create homogenous (students with like abilities) or heterogeneous (students with differing abilities) groups (Ontario Ministry of Education, n.d.).

There are numerous possibilities when it comes to designing and implementing a group activity or project. Teachers should not be overwhelmed by this fact, but instead feel confident in knowing that all group structures have merit (Alberta Education, 2010). If students succeed or struggle within a group, both are opportunities to learn.

Math Talk

To consolidate the learning from independent and group work, the entire class can participate in a discussion or activity (whole-class math discussions can also be used to introduce new concepts at the beginning of a lesson). From these discussions, students learn to:

- listen to one another
- share ideas
- learn new strategies for solving problems
- make connections between similar ways of thinking
- give teachers the opportunity to observe and assess their comprehension

(Cengiz, 2013; Reynolds & Muijs, 1999; Schifter, 2005; Sliman, 2013).
Through whole-class discussions, students are able to share ideas, strategies, and solutions, ultimately adding more problem-solving strategies to their repertoire (Ontario Ministry of Education, 2006).

Teachers should anticipate responses and solutions from students and be ready with questions to push the conversation forward (Cengiz, 2013). Students (especially in upper grades) should be required to justify their answers and be challenged to find the most efficient ways to solve their problems (Cengiz, 2013).

Discussions should not end once a solution is found. Teachers must continue to challenge their students, ask questions, and encourage students to extend their thinking beyond a simple answer.

Teachers can use the following questions as a way to reflect on their own grouping practices.

- Do I include a variety of flexible groupings in my lessons throughout the week?
- What opportunities were there for students to work in groups recently?
- What is preventing me from using groupings more often?
- Is my grouping strategy effective? Did it contribute to developing a Math-Talk Learning Community?
- If I were to teach the same lesson again, what grouping strategy could I use?

Adapted from Ontario Ministry of Education (n.d.).
The charts on the following pages provide examples of independent, paired, small group, and whole-class activities that teachers can use in their classrooms along with best practices that will enhance the learning experience for their students.

**Individual Learning Experiences**

<table>
<thead>
<tr>
<th>Best Practices</th>
<th>What This Can Look Like</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Provide structured opportunities for metacognition and self-reflection.</td>
<td>• “Learning logs” and “journals” are ways for students to track and reflect on their learning. Learning logs focus on more objective details of learning and encourage students to make connections and practise skills such as predicting, organizing, and evaluating. Journals, such as an opinions journal, encourage students to articulate their beliefs, attitudes, and perceptions about what and how they are learning.</td>
</tr>
<tr>
<td>• Individual learning experiences can be structured within other types of groups (e.g., small group, large group and whole class).</td>
<td>• “Interactive notebooks” use a specific notebook format to scaffold student interactions with information and text. On the right page of the notebook, students record notes during lessons, discussions, reading, viewing, group work, and research activities. On the left page, they record their individual interactions with the information, including reflecting, processing, making connections, doodling ideas, or posing questions. Teachers may provide suggestions but do not direct the content of the left page.</td>
</tr>
<tr>
<td>• Provide clear directions to encourage independence.</td>
<td>• “Agendas” are personalized lists of tasks that students must complete in a specific period of time. Tasks can be assigned, negotiated with students, or selected independently. Agendas should include both the task and the directions for completing it.</td>
</tr>
<tr>
<td>• Combine with larger group structures to emphasize collaboration and cooperation.</td>
<td>• “Think pad brainstorming” is a strategy for combining independent and group work. Students brainstorm individually on paper before sharing their ideas with a partner, group, or whole class.</td>
</tr>
</tbody>
</table>

All information obtained and adapted from Alberta Education (2010).
<table>
<thead>
<tr>
<th>Best Practices</th>
<th>What This Can Look Like</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Provide opportunities to develop and practise communication skills, make comparisons, and share learning in a nonthreatening context.</td>
<td>• “Study buddies” or “learning partners” can be established as a regular part of classroom routines.</td>
</tr>
<tr>
<td>• Assign partners based on learning needs, interests, and preferences.</td>
<td>• “Discussion breaks” are an opportunity for students to discuss ideas, questions, and information. Schedule 3 to 5 minutes into daily activities. Encourage students to record discussion points in their notebooks to help them stay on task.</td>
</tr>
<tr>
<td>• Consider when students should work with the same partner or switch partners for different learning contexts.</td>
<td>• “Exit cards” can provide opportunities to have students self-assess and reflect on their partnering activities and discussions by writing on an index card and handing it to the teacher as they leave the classroom.</td>
</tr>
<tr>
<td>• Encourage students to reflect and self-assess as a regular part of paired learning experiences.</td>
<td>• “Mind maps” or “bubble maps” are a good way for partners to synthesize their understandings. Each student records ideas on sticky notes. Then together partners make connections, identify similarities and differences, and represent content by creating pictures and symbols.</td>
</tr>
<tr>
<td>• Use paired grouping strategies to encourage students to challenge and support each other. Provide students with opportunities for peer teaching &amp; mentoring.</td>
<td>• A “resident expert” is a student who has received extra instruction in a topic or skill (or has expert knowledge and interest in a topic). Reteaching peers in a partner context can deepen the “resident expert” student’s own knowledge and skills.</td>
</tr>
<tr>
<td>• Use paired grouping structures to introduce, teach, and extend use of graphic organizers.</td>
<td>• In “team–pair–solo,” students complete problems first as a team, then with a partner, and finally on their own.</td>
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<tr>
<td></td>
<td>• In “say and switch,” partners take turns responding to topics at signaled but unpredictable times. The person listening must pick up from his or her partner’s train of thought before adding new ideas.</td>
</tr>
</tbody>
</table>

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<table>
<thead>
<tr>
<th>Best Practices</th>
<th>What This Can Look Like</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Make intentional decisions about when and how to organize small groups.</td>
<td>• Organize students into small groups for specific and focused instruction on concepts or skills with which they are experiencing difficulty. The other students in the classroom can work on independent or paired groupings appropriate to their understanding or skill development. These groupings are fluid and change as student needs change.</td>
</tr>
<tr>
<td>• Structure focused tasks based on content or skill development, learning interests, or preferences and readiness levels.</td>
<td>• In the “jigsaw strategy,” students are organized into groups of four or five. Each student in the group is assigned unique material to learn and then teach to his or her group members. Students working on the same material get together to decide what is important and how to teach it to their small group. After practising in these “expert” groups, the original groups re-form and students teach material to each other. By assigning the same material to those students who are challenged with concepts or skills, you can spend additional, focused time with this group.</td>
</tr>
<tr>
<td>• Consider grouping students based on both similarities and differences. Students benefit from opportunities to work with individuals whose interests and/or learning preferences differ from their own.</td>
<td>• “Learning centres” or “stations” allow students to work on specific tasks designed to target concept or skill development. Every small group does not necessarily have to complete all tasks at each learning centre. Time spent, tasks completed, and degree of choice can vary for each group of students.</td>
</tr>
<tr>
<td>• Use cooperative mixed-ability groupings to provide students with opportunities to rehearse information, learn from one another, build individual accountability in a group, engage in a high degree of activity, and receive support.</td>
<td>• In Spencer Kagan’s “numbered heads together” strategy, each group member is given a number. The teacher poses a problem, and all group members discuss it. Each group member is accountable for ensuring that every group member can complete the task. The teacher then calls a number, and that student is responsible for sharing the group’s solution to the problem.</td>
</tr>
</tbody>
</table>
• Build in strategies to ensure that every group member is involved in tasks, including answering questions, solving problems, or completing an activity.

• In “round robin brainstorming,” the class is divided into small groups with one person appointed as the recorder. An open-ended question is posed and students are given time to think about answers individually. Then members of the group share responses with one another, going around the circle, one after another, and the recorder writes down the answers of the group members.

• In “pass a problem,” the teacher creates problems for teams to solve and writes or attaches them to envelopes. Teams read the problems, place their solution in the envelope, and then exchange with another team to check their solutions and to determine whether they solved the problems in different ways.

• In “send a problem,” one student writes a problem on a card and asks group members to solve the problem. Group members solve the problem, and the question writer determines whether they have come up with a good solution.

• A “gallery walk” encourages students to learn from each other in small group settings. Groups record their work on a piece of chart paper. Each group appoints a docent to stay with their work (in some cases), while the remaining members rotate around, examining other groups’ ideas and asking questions of the docents. Another option is to have all group members rotate around the room and record their comments, questions, and/or suggestions for improvement using sticky notes. After 3 to 5 minutes, the groups rotate to the next solution. Rotation continues until all solutions are analyzed and responded to by all groups. As comments accumulate for each solution, the groups also review what previous groups have written and add only new comments, questions, and/or suggestions for improvement. When members return to their questions, they discuss the comments and add to their information.
- In “three stay, one stray,” three group members work together to solve a problem, while one group member “strays” to another group to compare and discuss their ideas.

- In a “visible quiz,” the teacher poses questions with multiple-choice responses and students discuss the responses in a group. At a signal, each group displays its answer written on a large card. A group also can be called upon to explain the group’s reasoning to the rest of the class.

- A “Mini-Congress” is used to prepare for the whole-class Math Congress. Students share their work with one another, check answers and strategies, and ask questions to provoke clarification and/or elaboration. During these small-group discussions, all students share their strategies, listen to the ideas of others, question what they do not understand, and defend their thinking. One student in each group facilitates this discussion, making sure that all strategies are shared and that everyone in the group has asked and/or responded to a question.

All information obtained and adapted from Alberta Education (2010) and Literacy and Numeracy Secretariat (2010).
## Whole-Class Learning Experiences

### What This Can Look Like

<table>
<thead>
<tr>
<th>Best Practices</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>• Provide students with opportunities to work collaboratively as a whole class. Encourage students to build on each other's ideas and strengths. This builds a sense of community in the classroom as students learn that everyone has something to contribute.</td>
<td>• Use a cooperative learning strategy, such as Kagan's <strong>board share</strong>, to involve student groups in a whole-class activity. Each group brainstorms responses related to a question or task. One member of each group is the “runner and recorder” and writes the group’s ideas on the board. The class then discusses and reflects on the whole-class effort.</td>
<td></td>
</tr>
<tr>
<td>• Use cooperative learning strategies to involve smaller groups in a whole-class activity.</td>
<td>• <strong>“Think–pair–share”</strong> and <strong>“Think–pair–square”</strong> encourage students to share their learning and thinking processes with the whole class. In think–pair–share, individual students think silently about a question posed by the teacher. Students share thoughts with a partner, then partners share responses with the whole class. In think–pair–square, partners combine into a small group before the whole-class discussion.</td>
<td></td>
</tr>
<tr>
<td>• Model and teach skills through student interactions with each other.</td>
<td>• In <strong>carousel brainstorming</strong>, the teacher posts charts on the wall, with key questions or ideas at the top. Groups are formed, and one person scribes for the group and adds to the chart as they brainstorm. Groups then rotate to a new chart, read the other groups' responses, and then add to the chart.</td>
<td></td>
</tr>
<tr>
<td>• Introduce different questioning strategies to help students learn to ask meaningful questions and understand what effective responses can provide.</td>
<td>• An <strong>“inside/outside circle”</strong> strategy encourages interaction and conversation between all class members. Divide the class in half. One half forms a circle facing outward, the other half finds one person in the circle to stand opposite, so there are two circles of students facing each other. Students discuss a question or topic with the person facing them. On a signal, the outer circle moves one person and the conversation begins again.</td>
<td></td>
</tr>
</tbody>
</table>
• Use a “three-minute pause” to stop at any point during an activity and encourage students to review what has been discussed, ask clarifying questions, or reflect on their learning.

• A “Socratic seminar” poses a thoughtful question to students to help them understand ideas, issues, and values in their text readings. Students develop questions for classmates in order to dig into a text they all have read. This strategy encourages involvement of all class members, develops critical and creative thinking, emphasizes respect for others and for differing viewpoints, encourages students to support their arguments with textual evidence, and reinforces effective communication skills.

• A “math congress” occurs over two lesson periods. After the mini-congress (refer to Small Group Learning Experiences Chart), the whole class gathers together to discuss solutions presented by two or three different pairs of students. These students defend and support their mathematical thinking as the teacher guides the whole-class discussion toward important mathematical ideas and strategies. In order to prompt students to reason and make mathematical generalizations, these sorts of questions are posed: “How is this strategy similar to and different from the first solution presented? Will this strategy always work? How do you know? When will it not work? Why not?”

• The purpose of “Bansho” is to organize and record mathematical thinking derived from and collectively produced by students on a large-size chalkboard or dry-erase board. Such board writing includes the use of mathematical expressions, figures, and diagrams of students’ solutions and strategies to a lesson problem. Because this written record enables simultaneous comparison of multiple-solution methods, there is the potential for students to construct new mathematical ideas and deepen their mathematical understanding. Because the chalkboard is a written record of the entire lesson, the students and teacher have a
whole view of the class’ mathematical discussion throughout the lesson. Also, by modelling effective organization, Bansho fosters student note-taking skills.

1. **Before – Getting started.** The teacher records the discussion prompt or activating problem. About 1/8 of board space is needed.

2. **During – Teaching/learning.** The teacher records the lesson problem and a list of the information that the class identifies to use. About 1/8 of board space.

3. **After (a) Consolidation.** Two to four different solutions are posted on the board as a visual for students to use to explain their solutions and for other students and the teacher to offer questions and comments. The teacher records these mathematical elaborations or mathematical annotations on and around the solutions, so that the mathematical thinking behind the solution is explicit to everyone. About 1/2 of board space.

   (b) **Highlights/summary.** Key mathematical concepts, algorithms, and/or strategies related to the lesson learning goal are summarized and recorded in a list so that the learning from the lesson is explicit to all students. About 1/8 of board space.

   (c) **Practice.** One of the practice problems and two student solutions to it are recorded on the board. About 1/8 of board space is needed.

All information obtained and adapted from Alberta Education (2010) and Literacy and Numeracy Secretariat (2010).
THE THREE-PART LESSON PLAN

Students are successful in mathematics classes when teachers actively present information in structured lessons (Reynolds & Muijs, 1999). The Government of Ontario has released several documents that encourage the implementation of the three-part lesson in mathematics (Literacy and Numeracy Secretariat, 2010, 2011a, 2011b; Ontario Ministry of Education, 2006).

The three parts describe the actions teachers should be taking before, during, and after a math lesson. In essence, these steps include:

- **Part 1 "Before"**
  - preparing students for the lesson by activating prior knowledge or introducing a new concept

- **Part 2 "During"**
  - allowing students time to work through the problem in pairs, small groups, or individually to find a solution

- **Part 3 "After"**
  - bringing students together to consolidate, summarize, and present their learning in creative ways


Using these steps as a guide, teachers can create unique lessons that help make concepts and information more memorable (Reynolds & Muijs, 1999). Examples that incorporate the three-part lesson into various math activities can be found in numerous articles (Chrisler, 2013; Legnard & Austin, 2012; Literacy and Numeracy Secretariat, 2010; Shimizu, 2009; Sliman, 2013).
The example presented below depicts a three-part lesson from an Integer unit that could be applied in a Grade 8 classroom. The lesson was obtained from an excellent resource site developed for teachers by the Ontario Ministry of Education. To get to this site, use the following link: http://www.edugains.ca/newsite/HOME/index.html

<table>
<thead>
<tr>
<th>Unit 1: Day 2: Living with Negatives</th>
<th>Grade 8</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Math Learning Goals</strong></td>
<td><strong>Materials</strong></td>
</tr>
<tr>
<td>• Students will solve a variety of application questions requiring the choosing of operations and the applying of skills (adding/subtracting) with integers.</td>
<td>• BLM 1.2.1, 1.2.2, 1.2.3</td>
</tr>
<tr>
<td><strong>PART 1</strong></td>
<td><strong>Coloured counters</strong></td>
</tr>
<tr>
<td><strong>Minds On…</strong></td>
<td><strong>Number line</strong></td>
</tr>
<tr>
<td><strong>Whole Class → Problem Solving</strong></td>
<td><strong>Thermometer</strong></td>
</tr>
<tr>
<td>Have a big problem on the board for when students enter the classroom. The problem should address concerns with notation (e.g., Owed money is represented using a negative sign) and allow for incorrect notations to be discussed (representing owing money with a positive amount).</td>
<td><strong>Calculator</strong></td>
</tr>
<tr>
<td>Example Problem: Emmanuelle owes her brother $20 for a CD that he bought for her and is getting $10 from her grandmother for mowing the lawn. If she started out with $25, how much money will she have now? Have students share solutions and discuss any discrepancies.</td>
<td></td>
</tr>
<tr>
<td><strong>PART 2</strong></td>
<td></td>
</tr>
<tr>
<td><strong>Small Group → Connecting</strong></td>
<td></td>
</tr>
<tr>
<td>Set up five stations around the classroom and break students into groups around each station. See BLM 1.2.1 for activities for each station.</td>
<td></td>
</tr>
<tr>
<td>Recommended manipulatives: Station A: algebra tiles</td>
<td></td>
</tr>
<tr>
<td>Station B: coloured counters/ two-colour discs</td>
<td></td>
</tr>
</tbody>
</table>
Station C: number line
Station D: thermometer
Station E: calculator

Give students BLM 1.2.2. Allow students sufficient time at each station to discuss the problem and record their work.

**Content Expectations/Observation/Mental Note:** Circulate to assess whether students are understanding and using the rules discussed on Day 1. The recognition and understanding of these rules is key to success in this unit.

<table>
<thead>
<tr>
<th>PART 3</th>
<th>Whole Class → Discuss</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>As a class, summarize and discuss their results from the “Action!” section. Have students put samples of their answers to each station on the board and discuss other possible representations. Discuss which manipulatives worked best for what situations.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Exploration Reflection</th>
<th>Home Activity or Further Classroom Consolidation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Students complete BLM 1.2.3</td>
</tr>
</tbody>
</table>

All of the black line masters mentioned in the lesson plan are available on the website. The activities from this site can also be modified by teachers themselves in order to accommodate their specific learning environments.

Teachers are encouraged to try the three-part lesson design in their own classrooms and decide whether the structured format is a good fit for them and their students.
You’ve got to have...Personality!

An effective teacher does not simply develop a collection of good teaching strategies in order to be successful. Teaching involves an understanding of self and knowing one’s strengths and weaknesses. Teachers do not motivate and inspire by what they teach, but how they teach it.

In order for teachers to make an informed decision about their daily practices within the classroom, they need to have a clear understanding of how they affect their students (Sanders & Horn, 1998).

The ways that teachers respond to situations arising within their classroom depend on personal traits like education, values, and previous classroom experiences. In particular, teachers’ personality types can predict:

- classroom management styles
- responses to teacher-team meetings and parent–teacher conferences
- reactions to students’ misbehaviours
- levels of student achievement
- the occurrence of burnout

WHY SHOULD YOU KNOW YOUR PERSONALITY TYPE?

In order for teachers to be effective, they should be aware of their own personality types (Rushton et al., 2007). They can use this information to improve their practice and better connect with the students in their classes.

Feedback about personality types can also help to affirm career choices, suggest alternative career paths, or help teachers choose an appropriate age level to teach (Lessen & Frankiewicz, 1992; Thornton et al., 2005).

Each type has its strengths and limitations; one type may suit the needs of a couple of students in the class but leave the others bored (Lawrence, 1993). Because a variety of personality types are represented by the student population, teachers must learn how to adapt their personality type in order to meet these needs.

According to Jane Kise (2007), past president of the Association for Psychological Type and self-proclaimed MBTI expert, teachers often raise objections about adjusting their teaching style to meet the needs of their students. The four common objections and Jane’s response to them are detailed below.

1) Shouldn’t students learn to follow each teacher’s rules? Doesn’t that mirror the real world?

Students aren’t mature. They aren’t yet skilled at using the preferences least natural for them. Whereas yes, students need to learn to adjust, the teacher needs to consider whose needs—emotional and academic—are greatest, the teacher’s or the student’s.
By developing an understanding of personality type, teachers can self-assess their interactions with students, parents, and staff in order to determine how best to teach, communicate, and collaborate with other personality types. In addition, students can learn about their type and use it to make decisions within the class.

By developing an understanding of personality type, teachers can self-assess their interactions with students, parents, and staff in order to determine how best to teach, communicate, and collaborate with other personality types. In addition, students can learn about their type and use it to make decisions within the class.

2) **Some of these suggestions are contrary to my natural style. I don’t want to use them!**

You always start with your own style and then adjust for your opposite. Which suggestions would be easiest for you? Which could you use at least once in a while to meet the students’ needs?

3) **If I’m meeting the needs of students with one preference, doesn’t that mean that the needs of students with the opposite preference go unmet?**

Remember, the goal isn’t to meet the needs of all students at all times. The goal is to help them appreciate differences and know how to adjust when schoolwork or activities require it. For teachers, another goal is to ensure that no students are consistently expected to operate outside of their preferences.

4) **What about students with ADHD or ODD (oppositional defiance disorder) or other definite problems?**

Whereas type is about normal differences among normal people, researchers are finding that many of these conditions represent exaggerated forms of the various personality preferences. Many of the strategies that research is showing are most successful in helping these students with academic achievement are similar to type-based strategies. Specific interventions or medications may still be needed, but using strategies grounded in type also helps these students achieve more academically.

Information obtained and adapted from Kise (2007).
THE MYERS-BRIGGS TYPE INDICATOR

The most widely used instrument for assessing personality characteristics is the Myers-Briggs Type Indicator (MBTI) assessment (Quenk, 2009; Rushton et al., 2007). This instrument is based on Carl Jung’s theory of psychological types (Thornton et al., 2005).

Jung (1976) discovered that contradictory types of attitude, extraversion (E) and introversion (I), could be characterized by four basic psychological functions: sensation (S), intuition (N), thinking (T) and feeling (F).

Throughout his book, Jung never explicitly states that judging and perceiving are a fourth pair of opposites; however, after Briggs and Myers spent years intensively reading his work, they concluded that this pairing was implied throughout his work and added the fourth pair to Jung’s system (Quenk, 2009).

Instead of using the judging and perceiving dimension to categorize the functions like Jung, the MBTI measures this attitude as a separate dimension (Cranton & Knoop, 1995).

The Jung/Myers theory maintains that all eight categories (extraversion, introversion, sensing, intuition, thinking, feeling, judging, perceiving) are used by every person at some point in time (Quenk, 2009).
The MBTI questionnaire, developed by Katharine Briggs and her daughter Isabel Briggs Myers, is used to identify the “consistent differences in the ways that normal people use their minds” (Quenk, 2009, p. 5), as described in the Jung/Myers theory of psychological types.

As of 1998, the standard paper-and-pencil questionnaire (Form M) has included 93 forced-choice items that measure the four personality dimensions (Bayne, 1995; Quenk, 2009). This question format is imperative for determining the natural preferences of an individual towards a specific mental function or attitude (Quenk, 2009).

Once individuals have completed the MBTI, they are provided with four letters that describe their personality type (Cranton & Knoop, 1995). According to Rushton et al. (2007), these letters can be described as:

- **Extroversion (E) and Introversion (I).** Extraverted individuals obtain information through an orientation toward the outer world of people, events, or things. They enjoy meeting new people, thinking aloud, and being active. Introversion types seek the introspection of ideas, thoughts, and concepts. They prefer to process their thoughts internally before speaking, have few close friends, and often seek conversations that tend to be deeper in nature.

- **Sensing (S) and Intuition (N)** relate to individuals’ preferences in how they receive and make sense of
information or data from the external world. Sensing types are more aware of their senses in relation to their environment, are often factually based, focus on practical concrete problems, and, generally, believe that if something works, it is best left alone. Individuals who have a tendency to understand the world through an intuitive process prefer to live in a world of possibilities and options, often looking toward the future. They also tend to focus on complicated abstract problems, seeing the big picture, sometimes at the expense of the details.

- **Thinking (T) and Feeling (F)** are considered the “rational processes” by which we come to certain conclusions and judgments regarding the information collected. Thinking types (T) prefer to focus on making decisions based on an impersonal objective position. Feeling types (F) have a tendency to respond well and easily to people’s values and are adept at assessing the human impact of decisions.

- **Judging (J) and Perceiving (P)** relate to how we “live our outward life.” Judging types prefer to live a structured, organized life. They also tend to be self-disciplined, enjoy making decisions, and thrive on order. Perceiving types prefer to live a lifestyle that is more flexible and adaptable. They tend to thrive on spontaneity, prefer to leave things open, require more information in order to make decisions, and often get things done at the last minute.

From these dichotomous pairs, 16 different letter combinations can be formed, representing 16 different personality profiles (Rushton et al., 2007). For example, one of these combinations could be expressed as ESTP, representing an extroverted, sensing, thinking, and perceiving personality type.
Teachers can learn their own types by completing the MBTI (with payment) by accessing the following site:

**MBTI Questionnaire Link**

Teachers can also take a free, unaccredited, Jung Typology test at:
http://www.humanmetrics.com/cgi-win/jtypes2.asp

Students and teachers can explore their type preferences together by completing the simple checklists from Jane Kise’s book *Differentiation through Personality Types: A Framework for Instruction, Assessment, and Classroom Management* (2009) provided below.

**Extraversion or Introversion:**
Where do you get your Energy?

<table>
<thead>
<tr>
<th>EXTRAVERSION</th>
<th>INTROVERSION</th>
</tr>
</thead>
<tbody>
<tr>
<td>Thinks out loud (talks!)</td>
<td>Thinks inside (quiet!)</td>
</tr>
<tr>
<td>Likes to work in groups</td>
<td>Likes to work alone or with close friend</td>
</tr>
<tr>
<td>Likes noise</td>
<td>Dislikes noise</td>
</tr>
<tr>
<td>Prefers to speak</td>
<td>Prefers to read or write</td>
</tr>
<tr>
<td>Lots going on</td>
<td>One activity at a time</td>
</tr>
<tr>
<td>Says what they’re thinking</td>
<td>Keeps thoughts inside</td>
</tr>
</tbody>
</table>

Remember: Extraverts need some time alone. Introverts need time with people. The question is how much and for how long?

Circle which describes you best:

E (Extraversion) I (Introversion) U (Not Sure)
Sensing or Intuition:
What Information gets your attention?

SENSING
☐ Likes facts and concrete things
☐ Experience first
☐ Sees the trees—details
☐ Wants clear expectations
☐ Step-by-step learning
☐ Practical, common sense

INTUITION
☐ Likes ideas & imagination
☐ Explanation first
☐ Sees the forest—big ideas
☐ Wants room to roam
☐ Random learning
☐ New insights

Remember: Sensing types use facts to build to the big picture. Intuitive types start with the big picture and use facts to support it.

Circle which describes you best:
S (Sensing) N (INtuition) U (Not Sure)

Thinking or Feeling:
How Do You Make Decisions?

THINKING
☐ Decide with head
☐ Principles important
☐ Work first
☐ Quick to give advice
☐ Find the flaw
☐ Reasons—objective truth

FEELING
☐ Decide with heart
☐ Feelings important
☐ Friendship first
☐ Quick to give comfort
☐ Find the positive
☐ Values—personal choice

Remember: Thinkers have feelings. Feelers have thinking skills.

Circle which describes you best:
T (Thinking) F (Feeling) U (Not Sure)
Judging or Perceiving:  
How Do You Approach Life?

<table>
<thead>
<tr>
<th>JUDGING</th>
<th>PERCEIVING</th>
</tr>
</thead>
<tbody>
<tr>
<td>□ You plan your work—stick to it</td>
<td>□ You go with the flow—keep options open</td>
</tr>
<tr>
<td>□ Organized</td>
<td>□ Flexible</td>
</tr>
<tr>
<td>□ Work before play</td>
<td>□ Play and work together</td>
</tr>
<tr>
<td>□ Steady effort</td>
<td>□ Last minute effort</td>
</tr>
<tr>
<td>□ Schedules and lists</td>
<td>□ Spur of the moment</td>
</tr>
<tr>
<td>□ Enjoy finishing</td>
<td>□ Enjoy starting</td>
</tr>
</tbody>
</table>

Remember: Judging types ARE NOT JUDGEMENTAL. They like to come to “judgments.” Perceivers ARE NOT more PERCEPTIVE. They like to get more “perceptions” or information.

Circle which describes you best:
J (Judging)  P (Perceiving)  U (Not Sure)

Teachers should be careful to introduce and discuss the meaning of personality type before the checklists are distributed and after the checklists are completed. Excellent activities to supplement these checklists can be found at: http://www.janekise.com/resources/
SIXTEEN TYPOLOGICAL TYPES

The following chart displays the 16 different personality type combinations that are possible to attain from the MBTI questionnaire.

<table>
<thead>
<tr>
<th>Type</th>
<th>Dynamic Name</th>
<th>Tertiary</th>
<th>Inferior</th>
</tr>
</thead>
<tbody>
<tr>
<td>ISTJ</td>
<td>Introverted Sensing with Extraverted Thinking</td>
<td>Feeling</td>
<td>Extraverted Intuition</td>
</tr>
<tr>
<td>ISFJ</td>
<td>Introverted Sensing with Extraverted Feeling</td>
<td>Thinking</td>
<td>Extraverted Intuition</td>
</tr>
<tr>
<td>ESTP</td>
<td>Extraverted Sensing with Introverted Thinking</td>
<td>Feeling</td>
<td>Introverted Intuition</td>
</tr>
<tr>
<td>ESFP</td>
<td>Extraverted Sensing with Introverted Feeling</td>
<td>Thinking</td>
<td>Introverted Intuition</td>
</tr>
<tr>
<td>INTJ</td>
<td>Introverted Intuition with Extraverted Thinking</td>
<td>Feeling</td>
<td>Extraverted Sensing</td>
</tr>
<tr>
<td>INFJ</td>
<td>Introverted Intuition with Extraverted Feeling</td>
<td>Thinking</td>
<td>Extraverted Sensing</td>
</tr>
<tr>
<td>ENTP</td>
<td>Extraverted Intuition with Introverted Thinking</td>
<td>Feeling</td>
<td>Introverted Sensing</td>
</tr>
<tr>
<td>ENFP</td>
<td>Extraverted Intuition with Introverted Feeling</td>
<td>Thinking</td>
<td>Introverted Sensing</td>
</tr>
<tr>
<td>ISTP</td>
<td>Introverted Thinking with Extraverted Sensing</td>
<td>Intuition</td>
<td>Extraverted Feeling</td>
</tr>
<tr>
<td>INTP</td>
<td>Introverted Thinking with Extraverted Intuition</td>
<td>Sensing</td>
<td>Extraverted Feeling</td>
</tr>
<tr>
<td>ESTJ</td>
<td>Extraverted Thinking with Introverted Sensing</td>
<td>Intuition</td>
<td>Introverted Feeling</td>
</tr>
<tr>
<td>ENTJ</td>
<td>Extraverted Thinking with Introverted Sensing</td>
<td>Sensing</td>
<td>Introverted Feeling</td>
</tr>
<tr>
<td>ISFP</td>
<td>Introverted Feeling with Extraverted Sensing</td>
<td>Intuition</td>
<td>Extraverted Thinking</td>
</tr>
<tr>
<td>INFP</td>
<td>Introverted Feeling with Extraverted Sensing</td>
<td>Sensing</td>
<td>Extraverted Thinking</td>
</tr>
<tr>
<td>ESFJ</td>
<td>Extraverted Feeling with Introverted Sensing</td>
<td>Intuition</td>
<td>Introverted Thinking</td>
</tr>
<tr>
<td>ENFJ</td>
<td>Extraverted Feeling with Introverted Intuition</td>
<td>Sensing</td>
<td>Introverted Thinking</td>
</tr>
</tbody>
</table>

Obtained from Quenk (2009).
Once students have identified their types, teachers should provide them with information about their preferred learning styles and take this information into consideration when they are teaching or assessing. Kise (2011) identified four ways that students learn math in association with four specific personality preference combinations.

### Four Ways of Learning

- **"Let me master it!"** (Preferences: introversion and sensing). If these students aren't comfortable with mathematics, they want certainty before proceeding. They like direct instruction and practice work, and they dislike experimenting unless they can receive immediate feedback.

- **"Let me do something!"** (Preferences: extraversion and sensing). These students learn through movement, interaction, and hands-on manipulatives. For example, instead of using little paper fractions strips, they thrive on working in groups with large magnetized fractions strips at the whiteboard. They often use a purposeful trial-and-error method working with pictures or manipulatives to solve problems until they "see" that their answer matches the mathematics of a problem.

- **"Let me think!"** (Preferences: introversion and intuition). These students process ideas internally and pride themselves on unique or creative solutions. They are drawn to concepts, not procedures. They prefer working with numbers and may not always benefit from hands-on tools.

- **"Let me brainstorm!"** (Preferences: extraversion and intuition). These students process their ideas best out loud with partners or in groups, transfer new knowledge to new situations easily, and prefer a variety of challenging tasks as opposed to practice work.
WHAT THE RESEARCH SAYS

Several studies have found that the typical elementary and preservice teacher has an ESFJ or an ISFJ profile (Hinton & Stockburger, 1991; Lawrence, 1993; Macdaid, McCaulley, & Kainz, 1986; McCutcheon, Schmidt, & Bolden, 1991; Reid, 1999; Rushton et al., 2007; Sears, Kennedy, Kaye, & Gail, 1997; Thornton et al., 2005).

These profile types are quite fitting, especially for the primary grades, because they represent a patient, nurturing, loyal, and devoted personality (Rushton et al., 2007). However, introverts prefer a quiet and orderly learning environment, whereas extroverts are more likely to welcome noise and movement (Lawrence, 1993).

The teaching style of the ISFJ teacher involves “pencil-and-paper drills, workbook assignments, and quiet deskwork to teach their lessons. They often use course outlines and contracts with students. Their students are encouraged to memorize facts and will be exposed to audiovisuals as a way of teaching them about reality. ISFJ teachers also like short periods of teacher-led questions and answers, and brief lectures.”

(Fairhurst & Fairhurst, 1995, p. 61)

Most often ISFJ teachers are called “Stabilizers” or “Traditionalists” because they appreciate clearly defined rules, enjoy consistency, and have a strong desire to keep things under control (Rushton et al., 2007). These teachers do prefer to stay set in their ways; however, if they feel a change is necessary and are provided with a detailed explanation, they can change over time.
In a study conducted by Rushton et al. (2007), outstanding teachers were found to have a personality profile composed of Extroversion, Intuition, Feeling, and Perceiving (ENFP) types.

The nature of an ENFP type is generally enthusiastic, highly creative, adaptable, and able to rely on improvisation instead of preparation (Lawrence, 1993; C. Martin, 1997).

As teachers, ENFPs create novel, stimulating lectures; encourage class discussions, team-building, and cooperative learning; and are sensitive to the needs of their students (Fairhurst & Fairhurst, 1995; C. Martin, 1997; Rushton et al., 2007).

"ENFPs are energetic and enthusiastic teachers. They often stimulate students to seek out what is unknown and make it known. They promote imagination and creativity in their classrooms through many different kinds of activities. Their students usually feel that their ENFP teachers understand them and help them to deal with their personal problems."

(Fairhurst & Fairhurst, 1995, p. 61)

These teachers are viewed as “idealists” or “advocates” because they are flexible, open-minded, intolerant of routine, proud to be unique, innovative, problem-solvers, and they look forward to the future (Rushton et al., 2007).

According to Sears et al. (1997), the SFJ-type teachers will not be the future leaders in education. The risk-takers and visionaries who easily accept change and naturally adapt to new situations are more likely to be the leaders of tomorrow (Rushton et al., 2007).
Additional Resources

The following list can be utilized by teachers, parents, and other education administrators looking for resources to assist and further understand the complex nature of intermediate (grades 7 and 8) math learners.

A brief paragraph accompanies the sources listed below in order to provide the reader with a description of the useful material and learning activities that are available from each source. Connections will also be made as to how some of these sources were utilized in this research paper.

WEBSITES

http://education.alberta.ca/home.aspx

The Alberta Government has designed this site for student, parent, and teacher use. There are a variety of different resources that provide lesson and activity ideas for the intermediate math classroom. One resource from this site that was used throughout this research paper was *Making a difference: Meeting diverse learning needs with differentiated instruction*, which can be found at the following link: http://education.alberta.ca/teachers/resources/cross/making-a-difference.aspx. This page divides the document into easily accessible chapters that provide information and examples on effective teaching techniques such as differentiated instruction and flexible groupings. Chapter 5, entitled Differentiated Learning Experiences, was especially utilized in this research paper and can be found at http://education.alberta.ca/media/1233989/8_ch5%20learning.pdf.

http://dese.mo.gov/about

This website was created by the Missouri Department of Elementary & Secondary Education. Although this site is mostly directed to schools in the state of Missouri, the resources for teachers are excellent. One document on this site summarizes the behaviours of both teachers and students in relation to Doug Lemov’s book *Teach like a Champion: 49 Techniques that put students on the*
path to college. The PDF document containing this summary can be found at: http://dese.mo.gov/sites/default/files/11-Research-ProvenPracticesTLAC.pdf.

http://www.edugains.ca/newsite/math/tips.html

EduGains, a website created by the Ontario Ministry of Education, provides extremely useful math resources for teachers in Grades K–12; specifically, three-part lesson plans with accompanying black line master handout sheets. TIPS4RM (Targeted Implementation and Planning Supports for Revised Mathematics) lessons, designed for Grade 7 and 8 students in both English and French, are clearly linked to curriculum expectations and already structured in manageable units for teachers. Unit 1 in the English Grade 8 section of the site was utilized to demonstrate a three-part lesson in this research paper.


The New York State Education Department has designed this website specifically for K-12 educators. Resources can be found for a variety of subjects including mathematics. The “Mathematics Publications” page is especially useful and provides a variety of Key Idea Tasks to Enhance Success (KITES). This page can be located at http://www.p12.nysed.gov/ciai/pub/pubmath.html. These resources demonstrate how a single activity can be modified for use throughout grades K-12. Teachers can ensure that all of their students’ needs are being met because the same activity can apply to such a wide range of student abilities. The sample marshmallow activity for students in Grade 8 that was included in this research paper can be found at http://www.p12.nysed.gov/ciai/mst/pub/kite4.pdf.

http://www.dreambox.com/school

DreamBox is a site that offers math lessons to students in grades K–8. Teachers who have access to the site and set up student accounts can access their students’ learning assessments and can even decide at which level their students should begin their lessons on the site. DreamBox’s trademarked “adaptive learning technology” takes over the role of the teacher and automatically changes the difficulty of the lessons based on the progress of the student through each previous lesson. Students who are not granted access through a school board can have their parents help them start a free trial on the site so they may practice math concepts at home.
http://www.explorelearning.com/

This website is extremely unique in the online simulations called “Gizmos” that it provides for students and teachers. With an account, teachers can assign “Gizmos” and their associated questions for homework. Alternatively, teachers can use a projector or whiteboard to display “Gizmos” in class and create math conversations or activities from the content on the screen. Some school boards purchase access to the site for their teachers. As with other education websites, students can sign up for a free trial of the site; however, many great aspects of the site are offered for free without purchase. Teachers are encouraged to explore the site and test out some of the “Gizmos” for themselves.

http://nlvm.usu.edu/
http://illuminations.nctm.org/

Both websites offer interactive manipulatives for students to use at home or within the mathematics classrooms. Teachers can create activities that involve the interactive online manipulatives, or they can use them with an interactive whiteboard or projector to teach a lesson. These lessons can utilize the online manipulatives by demonstrating a concept or by posing a question and having the class participate in a whole-class discussion about the chosen manipulative. The first link provides the teacher or parent with additional information for each manipulative, whereas the second link provides actual lessons that teachers can use along with the manipulatives.

BOOKS


This book provides explicit instructions for teachers looking to differentiate their teaching based on Jungian typology. A section of the book specifically addresses differentiated math instruction and how it can be applied to benefit a variety of different personality preferences. Additionally, the book focuses on how to address classroom management concerns by explaining how to react to students with different type preferences. Some concerns that are addressed in the book include: students are talking when they should be listening, students blurt out
answers, questions or comments, students participate unequally in class
discussions, and students struggle to work well in groups. Each concern is
addressed individually and type concepts are applied in order to provide teachers
with specific strategies for tackling these common classroom problems.

Lemov, D. (2010). *Teach like a champion: 49 Techniques that put

In this book, Doug Lemov presents 49 teaching techniques that teachers can
apply to their classrooms in order to create learning environments that are
engaging and encourage academic success. The techniques are explained well
and can be applied to the classroom immediately. At the end of each chapter,
teachers have the opportunity to reflect on their own teaching practices and
consider how they relate to the technique being discussed. The book is also
accompanied by a disk that provides videos of the techniques being applied in
real classrooms by real teachers.

Toronto, Canada: Nelson Education.

Unlike the other two books, the content in this book is more closely linked to
Canadian classrooms and teachers. The chapter problems, math concepts, and
teaching strategies presented in this book are directly related to Canadian
curriculum standards. Marian Small presents mathematical learning in a way that
is interesting to students and also makes sense to them. The book includes
actual student work that teachers can refer to for examples as well.

**ONLINE VIDEOS**

http://tedxtalks.ted.com/video/TEDxEnola-Jane-Kise-Neuroscienc

In this video, Jane Kise, author of *Differentiation Through Personality Types: A
framework for instruction, assessment, and classroom management* (2007),
presents findings from her research on mathematics students and Jungian type.
She discusses how students with differing personality preferences approach and
solve math problems in different ways. In addition, she demonstrates how
teachers can assist students with various Jungian types to better understand and
complete math problems.
References


CHAPTER FIVE: SUMMARY, IMPLICATIONS, AND CONCLUSIONS

This chapter summarizes and discusses possible implications of the handbook *Using Effective Teaching Strategies and Personality Type to Enhance the Mathematics Classroom: A Handbook for Intermediate Math Teachers*. In addition, the chapter explores the formative feedback obtained from three teacher evaluations of the handbook. Both positive elements of the handbook and suggestions for improvement are addressed. The chapter concludes with a brief closing statement.

**Summary of the Handbook**

The handbook was developed specifically for intermediate math teachers looking to reflect on their professional practice. Teachers who fit this designation may include beginning teachers who are new to the profession, experienced math teachers who are looking for something new to inspire their teaching, and experienced teachers who are new to the field of mathematics and looking to prepare themselves for the upcoming school year. The content in the handbook is unique in that it does not simply focus on teaching strategies; it also encourages teachers to take an inward look at their own personal characteristics and personality traits and discover how these factors could affect their current teaching practices.

The beginning of the handbook lays the framework for being an effective teacher. Five key themes are used to explore the characteristics of effective teachers, while a number of different resources are presented that support these themes. The second section of the manual provides several practical math strategies that teachers can apply within their classrooms to enhance their current pedagogy. Next, the handbook explains how personality traits can affect both teaching and learning in mathematics. Within this
section of the handbook suggestions and resources are provided for teachers looking to determine their own personality type and also for teachers who are attempting to educate students about their personality types. The final section lists a variety of web-based and print resources that teachers can make use of to supplement the information they have learned and utilized within the handbook.

**Positive Elements of the Handbook From Formative Feedback**

Following the completion of the handbook, three teacher colleagues within a district school board in southern Ontario who taught intermediate math within the past year were asked to evaluate the content and structure of the handbook. Overall, the feedback provided from these professionals was extremely positive. Each evaluator agreed that she would use the handbook to reflect on and modify her own intermediate teaching practices. In addition, they all stated that other intermediate math teachers would be able to easily integrate the concepts from the handbook into their classrooms. One evaluator even went on to suggest that the handbook be professionally marketed.

In terms of content, the teacher evaluators provided the following comments: “The handbook is well-written and thought out and is in keeping with the present directions of numeracy,” “very readable–friendly language,” “I like the classroom applications, they were concise and to the point,” “you added ‘real’ classroom ideas,” “the language and strategies are easily accessible,” and “you are very up to date with your ideas.” Teachers were duly impressed with the wide variety of content and ideas presented in the handbook as well as the practicability of the strategies and examples.
Suggestions for Improvement from Formative Feedback

Although the teacher evaluators all agreed that the handbook was easy to read and well organized, the majority of their suggestions for improvement were related to the structure of the handbook. For instance, one teacher suggested that the order of the sections within the handbook be changed by beginning with personality traits, following with effective teaching characteristics, and ending with the math strategies. Although this order would leave the reader with a final focus on math strategies, this order was purposefully avoided for specific reasons. First, the author wanted to make a clear separation between the sections on personal characteristics and personality type to draw attention to the fact that these two attributes are significantly different. In addition, the author sought to introduce and end the handbook in a way that encouraged teachers to think of effective math teaching as more than just a collection of teaching strategies.

Another suggestion for improvement related more to the content within the handbook. One of the teacher evaluators suggested that strategies such as math congress, gallery walk, and bansho be included as ways for students to present their learning during the consolidation section of the three-part lesson plan. Although the “gallery walk” strategy was specifically addressed in the Small Group Learning Experiences chart, the other two were not. For the final copy of the handbook, both the “math congress” and “bansho” activities were included in the Whole Class Learning Experiences chart. These strategies were not specifically detailed within the Three-Part Lesson Plan section of the handbook in order for readers to avoid interpreting these strategies as the only options to use for successfully consolidating a lesson.
Many of the strategies, lessons, and information in the handbook can be easily adapted and applied to other grade levels. However, one teacher evaluator mentioned that primary and junior examples of activities would further encourage teachers of other grade levels to utilize the handbook. Because the handbook was intended to specifically target intermediate math classrooms, the activities and strategies were left unchanged. Either future editions of the original handbook could be used to expand into the younger grade levels, or additional handbooks with similar content and structure could be created to specifically address the other grade levels.

A final recommendation suggested that the surveys and activities within the handbook be organized into an appendix at the end of the document. Although this idea was logical, the original intent of the handbook was for teachers to not only collect a variety of effective math instruction strategies but also to develop an understanding of why these strategies are recommended and how they contribute to student achievement. In order for this connection to remain evident, it was decided to leave the strategies and activities in the main part of the handbook with their corresponding explanations.

**Practical Uses of the Handbook**

The handbook developed for this project has a variety of different uses. Although the handbook was developed for intermediate math teachers, teachers of other subjects and even parents of school-aged children struggling in mathematics could find this handbook useful. The information provided in the first and third sections of the handbook relates to personal characteristics and personality type. Although these topics were related to math teachers in the handbook, essentially any teacher could use these practices to improve their current classroom dynamics. Teachers can be more effective if they take a
personal interest in their students, set high expectations for themselves and for their 
students, make content and learning meaningful, have a passion for their students and 
teaching, and show respect and trust for their students (Gentry et al., 2011). In addition, 
as teachers become more familiar with their personality type and the effect it can have on 
their students’ learning, they will be able to better understand themselves and the way in 
which they teach.

Parents, tutors, and preservice education students could all benefit from the 
information presented in the handbook. These individuals are always working with 
children and looking to inspire them to learn. This handbook offers a variety of different 
ways to approach teaching mathematics while really focusing on the needs of the student. 
The suggestions presented in this guide are many and should provide a wide enough 
variety of activities that a number of different student needs can be met. Parents, tutors, 
and preservice education students are encouraged to try as many different approaches to 
teaching math from the handbook as they possibly can in order to determine what works 
best for their particular students. Although this handbook was written with teachers in 
mind, the language and format used throughout makes this manual accessible to a variety 
of different groups of people who are interested in introducing intermediate math 
concepts in a new light.

**Implications for Theory**

The literature review that framed the content within the handbook provides an in-
depth look at both teaching effectiveness research and personality type theory. A variety 
of different resources have attempted to define an effective teacher (Aaronson et al., 
2007; Collinson et al., 1998; Gentry et al., 2011; Waldrip et al., 2009); however, the
handbook developed for this paper provides a more unique outlook at the definition of effective teaching by combining both personal characteristics and teaching practices. This combination should be considered significant by other researchers looking to define effective teaching because it describes teachers in a more holistic manner. Simply focusing on the academic gains of students by the implementation of particular teaching practices completely overlooks the fact that personal characteristics play a huge role in student learning and teacher effectiveness (Ayme et al., 2009; Kokkinos, 2007; Lessen & Frankiewicz, 1992; N. K. Martin et al., 1998; Thornton et al., 2005).

Personality type assessments were developed from Carl Jung’s theory of psychological types. The handbook demonstrates how this theory connects to the teaching of mathematics. Both student comprehension and teacher instruction of mathematics will vary from person to person, depending on their personality types (Kise, 2007, 2011; Lawrence, 1993). The information presented in the handbook provides theorists, teachers, parents, and other individuals working with students with a resource that connects personality type, teaching, and mathematics. In addition, lesson ideas and activities are provided within the handbook to encourage adults to teach their students (or children) about personality type and how it can affect their learning.

**Implications for Practice**

One of the most significant goals of this handbook was to influence the current teaching practices in Canadian math classrooms. Since math scores have been declining across the country, a need for this handbook was determined (Alphonso, 2013; Canadian Press, 2013). The information and strategies presented in the handbook were specifically targeted to intermediate (grades 7 and 8) math teachers looking for new and fresh ideas to
inspire and teach their students. The content within the handbook is intended to enhance their current pedagogical practices while improving academic achievement for the students within their classrooms. Although this handbook addresses teaching mathematics through both personal and practical lenses, it is not all encompassing. Intermediate math teachers are encouraged to use this handbook in combination with a variety of other resources in order to gain an authentic, well-rounded perspective on teaching mathematics.

The section in the handbook that focuses on personality traits directly affects teachers’ classroom practices. As teachers learn what their personality types are and how they affect their students, they develop a better understanding of how to reach the needs of all their learners (Rushton et al., 2007; Sanders & Horn, 1998). Teachers may find it useful to alter their pedagogy or teaching style based on the results of their personality assessment. In addition, preservice teachers may use the handbook to educate themselves about personality type. According to researchers (Lessen & Frankiewicz, 1992; Thornton, et al., 2005), feedback about personality type could assist preservice teachers in affirming their career choices, selecting an alternative career path, or choosing an appropriate age level to teach.

**Future Research**

The information presented in this research paper, from both the literature review and handbook itself, may impact future research. Throughout the first section of the handbook, five themes are used from Gentry et al. (2011) to explore the definition of effective teaching. Future researchers should take a more in-depth look at this topic and attempt to concretely define the characteristics of an effective teacher. Even more, the
research conducted in their field could highlight lists of personal attributes and teaching strategies typically associated with effective teaching, or perhaps even exemplary teaching practices. These lists could be utilized by teachers for making their own personal comparisons or by school boards looking to hire effective teachers.

Within the literature review, personality traits were presented as a way to further reflect on and assess effective teaching. Many studies have already determined that the typical elementary and preservice teacher has an ESFJ or an ISFJ profile (Hinton & Stockburger, 1991; Lawrence, 1993; Macdaid et al., 1986; McCutcheon et al., 1991; Reid, 1999; Rushton et al., 2007; Sears et al., 1997; Thornton et al., 2005). Future research should take these studies one step further and determine the personality traits of teachers deemed to be extremely effective or exemplary. Patterns may evolve from this research that could suggest a single personality trait being associated with highly effective teachers; perhaps subject and grade level will demonstrate associations with particular types as well.

**Concluding Remarks**

Throughout the development of this research paper and embedded handbook, the focus remained on providing teachers with a resource that would assist them in developing outstanding mathematics classrooms at the intermediate level. The premise for this handbook developed out of personal desire and then translated into an obvious need in the field of mathematics. The holistic approach taken throughout this handbook to describe and encourage teacher effectiveness is significant. Personal characteristics, effective teaching strategies, and personality are all necessary factors in becoming an excellent teacher. Great teachers are not effective simply because of what they teach, but
more often by how they teach. Although the handbook includes a plethora of information and activities, it will not suffice as a stand-alone document. Instead, the handbook created for this project should be used in combination with a variety of other sources to maximize the potential for successful lessons and effective teaching.
References


Appendix

Intermediate Math Handbook Questionnaire

1. What are your overall impressions of the handbook?
   ____________________________________________________________
   ____________________________________________________________
   ____________________________________________________________
   ____________________________________________________________

2. What suggestions would you make to improve the overall effectiveness of the design of the handbook?
   ____________________________________________________________
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   ____________________________________________________________
   ____________________________________________________________

3. What would you add or delete from the content of the handbook to make it easier to implement?
   ____________________________________________________________
   ____________________________________________________________
   ____________________________________________________________
   ____________________________________________________________

4. Would intermediate math teachers be able to easily integrate the concepts from the handbook into their classrooms?
   ____________________________________________________________
   ____________________________________________________________
   ____________________________________________________________
   ____________________________________________________________

5. Which aspects of the handbook do you feel will be most effective in improving the teaching practices of intermediate math teachers?
   ____________________________________________________________
   ____________________________________________________________
   ____________________________________________________________
   ____________________________________________________________
6. Which aspect of the handbook do you believe would be hardest/least effective to apply to an intermediate math classroom? What suggestions would you make to change this?
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7. Can you suggest any effective intermediate math teaching strategies that have been left out of the handbook and should be included?
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8. Would you use this handbook to improve your own intermediate math teaching practices?
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9. Do you believe the content from this handbook can be easily applied to Grade 7 and 8 math classrooms? Please explain.
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10. Can you make any suggestions that would add to the versatility of the handbook for numerous grades and levels of students?
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___________________________________________________________________________
11. Please provide additional comments below

___________________________________________________________________________
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Thank you for taking time to complete this questionnaire. Your comments will add to the validity and improvement of the handbook for teachers and students!
## Intermediate Math Handbook Evaluation Survey

**SA** = Strongly Agree  **A** = Agree  **U** = Undecided  **D** = Disagree  **SD** = Strongly Disagree

Please use a checkmark in the appropriate column.

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<th><strong>SA</strong></th>
<th><strong>A</strong></th>
<th><strong>U</strong></th>
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<th><strong>SD</strong></th>
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</thead>
<tbody>
<tr>
<td>1</td>
<td>The purpose of the Handbook is clearly stated and understandable.</td>
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<td>2</td>
<td>The Handbook is easy to read.</td>
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<td>3</td>
<td>The Handbook is well organized.</td>
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<td>4</td>
<td>The Handbook will be useful for an intermediate math teacher.</td>
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<td>5</td>
<td>The Handbook provided readily usable classroom activities and lessons.</td>
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<td>6</td>
<td>The Handbook provided useful templates for an intermediate math program.</td>
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<td>7</td>
<td>The Handbook clearly explained a variety of effective math strategies for teaching in the intermediate math classroom.</td>
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<td>8</td>
<td>The Handbook clearly explained personality type and its effect on the intermediate math classroom.</td>
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<td>9</td>
<td>The terms used in the Handbook were clearly defined or explained.</td>
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<td>10</td>
<td>I learned new or useful information from the Handbook.</td>
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**In terms of usefulness, my overall opinion of the Handbook is:**

_______________________________________________________________________________
_______________________________________________________________________________

**Suggestions for improving the Handbook would be:**

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_______________________________________________________________________________
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Thank you for your willingness to read and evaluate the Handbook. Please use the remainder of this page if you require additional space.

**Additional Comments:**