The Finite Bandwidth Model for Spin Fluctuations in Pd

by

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Abstract

The frequency dependence of the electron-spin fluctuation spectrum, $P(\Omega)$, is calculated in the finite bandwidth model. We find that for Pd, which has a nearly full d-band, the magnitude, the range, and the peak frequency of $P(\Omega)$ are greatly reduced from those in the standard spin fluctuation theory. The electron self-energy due to spin fluctuations is calculated within the finite bandwidth model. Vertex corrections are examined, and we find that Migdal's theorem is valid for spin fluctuations in the nearly full band. The conductance of a normal metal-insulator-normal metal tunnel junction is examined when spin fluctuations are present in one electrode. We find that for the nearly full band, the momentum independent self-energy due to spin fluctuations enters the expression for the tunneling conductance with approximately the same weight as the self-energy due to phonons. The effect of spin fluctuations on the tunneling conductance is slight within the finite bandwidth model for Pd. The effect of spin fluctuations on the tunneling conductance of a metal with a less full d-band than Pd may be more pronounced. However, in this case the tunneling conductance is not simply proportional to the self-energy.

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I. Introduction

Palladium is remarkable for having a spin susceptibility which is greatly enhanced over the Pauli value,[1] and an absence of superconductivity down to 1.7 mK.[2] These characteristics have been attributed to spin fluctuations, or "paramagnons" which suppress the singlet pairing between electrons.[3-4] The interaction of electrons and spin-fluctuations leads to an enhancement of the effective electron mass, given by the mass renormalization parameter for spin fluctuations, λ_{sf}. This is analogous to the effective mass enhancement due to electron-phonon interaction, given by the electron-phonon mass renormalization parameter λ_{ep}. The total mass enhancement for Pd can be found by comparing the effective density of states at the fermi level from heat capacity measurements,[5] and from the de Haas-van Alphen effect[6] (2.20 states/eV-atom-spin) with the density of states from band structure calculations[7] (1.14-1.28 states/eV-atom-spin). The total mass enhancement parameter, $\lambda = (m^*/m - 1)$ is 0.58-0.77. The absence of superconductivity to 1.7 mK implies that the BCS coupling parameter, $(\lambda_{ep} - \lambda_{sf} - \mu^*) \lesssim 0$, where μ^*, the Coulomb pseudo-potential, is typically 0.13 for transition metals. Dumolin et al.[8] have interpreted the results of their proximity effect tunneling measurements as implying that $\lambda_{ep} \lesssim 0.2$, and $(\lambda_{ep} - \lambda_{sf} - \mu^*) = 0.00 \pm 0.05$, and have concluded that paramagnons are not responsible for the absence of superconductivity in Pd. This interpretation is inconsistent with a total mass enhancement $(\lambda_{ep} + \lambda_{sf})$ of 0.58-0.77, and with the calculated electron-phonon mass renormalization parameter $\lambda_{ep} = 0.41$.[9] It has been shown that the results of the proximity effect tunneling are not inconsistent with a larger electron-phonon interaction if

the characteristic spin fluctuation energy, Ω_{sf}, in the electron-paramagnon spectral function $P(\Omega)$ is comparable to the characteristic energy, Ω_{ep}, in the electron-phonon spectral function $\alpha^2F(\Omega)$.[10] Then, because $\alpha^2F(\Omega)$ and $P(\Omega)$ enter the gap equation with opposite signs, the effect of phonons may be masked by spin fluctuations in the tunneling density of states.[11] Stenzel and Winter,[12] in the calculation of the dynamic susceptibility for Pd (which is proportional to the spectral density for spin fluctuations), find that indeed the important frequency range is of the same order of magnitude as the phonon frequencies in Pd.

The enhancement of the static, long wavelength spin susceptibility $\chi(0,0)$ over the Pauli susceptibility χ_o is given by the Stoner factor $S = \chi(0,0)/\chi_o$. The experimentally measured susceptibility contains other contributions besides $\chi(0,0)$, e.g. core diamagnetic, which must be subtracted before comparing with χ_o. The Pauli susceptibility is $\chi_o = 2(\frac{1}{2}\mu_B)^2\, g_{eff}^2\, N(0)$, where $N(0)$ is the band structure density of states at the fermi level, μ_B is the Bohr magneton, and g_{eff} is the effective g factor averaged over the fermi surface.[7] Spin-orbit interaction can reduce g_{eff} from the spin value $g = 2$. The Stoner factor S for Pd has been taken by various authors to have values ranging from 6 to 16. In this work we will be taking $S = 14$, as suggested in Reference (13).

The usual model for the paramagnon theory is an electron gas with a contact repulsion between opposite spin electrons. Calculations based on this model with $S = 14$ produce a mass renormalization parameter $\lambda_{sf} = 5.3$, and a paramagnon spectral function which has a characteristic energy on the

order of the fermi energy, so $\Omega_{sf} >> \Omega_{ep}$ (see Numerical Results section). The contact interaction tends to overestimate λ_{sf}. When interatomic exchange is included,[4] λ_{sf} decreases somewhat, while Ω_{sf} does not change much.[14] MacDonald[13] suggested a model for Pd which takes into account the fact that the d-band is nearly full, and found that for the same Stoner factor, S = 14, the mass renormalization is greatly reduced to $\lambda_{sf} = 0.05$. We will see that this finite bandwidth model also produces a paramagnon spectral function with a characteristic energy Ω_{sf} that is much lower than that in the usual model.

The odd conductance of normal metal-insulator-normal metal junctions reflects self energy effects in the electrodes.[15] Normal state tunneling experiments have established the relationship between the odd conductance and the electron-phonon spectral function $\alpha^2F(\Omega)$.[16] We will derive the expression for the self-energy due to electron-spin fluctuation interaction, and find that it is the same as the expression for the self-energy due to electron-phonon interaction but with $\alpha^2F(\Omega)$ replaced by $P(\Omega)$. We will calculate the self-energy due to electron-phonon and electron-paramagnon interaction in Pd within the finite bandwidth model, and examine the possibility of using normal state tunneling as a probe for spin fluctuations in Pd. The Al-I-Pd normal state tunneling experiment of Rowell[17] will be examined in the context of these results.

II. Theory

The derivation of the expression for the self-energy due to electron-paramagnon interaction, Σ_{sf}, is given in detail in Section (A) for the case of no momentum cut-off. The changes in the theory which arise due to the finite width of the band are presented in Section (B). The first order vertex corrections for Σ_{sf} are examined in the limit in which Migdal's theorem is valid for phonons in Section (C). The momentum dependence of the self-energy is examined in Section (D). The results of a derivation of Σ_{ep} are in Section (E). The result relating the self-energy and the odd part of the tunneling conductance for normal metal-insulator-normal metal junctions is in Section (F). A method for inverting the tunneling data and an interpretation of the results of the inversion are in the last Section (G).

(A) Standard Paramagnon Theory

The standard paramagnon theory assumes spherical constant energy surfaces for the d electrons. The Hamiltonian for the non-interacting system of electrons in a parabolic band is

$$[1] \qquad \hat{H}_o - \mu\hat{N} = \sum_{\vec{p},\sigma} \epsilon_{\vec{p}} \hat{a}^{+}_{\vec{p}\sigma} \hat{a}_{\vec{p}\sigma} \ .$$

Here, $\hat{a}^{+}_{\vec{p}\sigma}$ and $\hat{a}_{\vec{p}\sigma}$ are creation and annihilation operators for electrons with momentum $\vec{p}$ and spin σ, and the energies, $\epsilon_{\vec{p}}$, are measured relative to the chemical potential μ. The single particle non-interacting temperature Green's function is

$$[2] \qquad G^{(o)}_{\sigma}(\vec{p}, i\omega_n) = \frac{1}{i\omega_n - \epsilon_{\vec{p}}} \ ,$$

where ω_n is the fermion Matsubara frequency. (We will use Roman subscripts to indicate fermion Matsubara frequencies, $\omega_n = (2n - 1)\pi/\beta$, and Greek subscripts to indicate boson Matsubara frequencies, $\omega_\nu = 2\nu\pi/\beta$, where $\beta = 1/k_B T$).

The interaction is taken to be a contact interaction between opposite spin electrons

$$[3] \qquad V_{\sigma\sigma'}(\vec{r} - \vec{r}') = U\delta(\vec{r} - \vec{r}')(1 - \delta_{\sigma\sigma'}) \ .$$

In second quantization, the interaction Hamiltonian is

$$[4] \qquad \hat{H}' = \tfrac{1}{2}\sum_{\vec{p},\vec{p}',\vec{q}} \sum_{\sigma\sigma'} I(1 - \delta_{\sigma\sigma'})\hat{a}^{+}_{\vec{p}+\vec{q},\sigma}\ \hat{a}^{+}_{\vec{p}'-\vec{q},\sigma'}\ \hat{a}_{\vec{p}',\sigma'}\ \hat{a}_{\vec{p},\sigma}$$

where $I = U/V$, with V the volume.

The correction to the non-interacting Green's function due to emission and absorption of transverse spin fluctuations is found by summing the series of diagrams in Figure (1a).

$$[5] \qquad [\delta G^{(o)}_{\sigma}(\vec{p},i\omega_n)]_t = G^{(o)}_{\sigma}(\vec{p},i\omega_n)\ \Sigma^{t}_{\sigma}(\vec{p},i\omega_n)\ G^{(o)}_{\sigma}(\vec{p},i\omega_n)$$

Here, Σ^{t}_{σ} is the self-energy resulting from the interaction between electrons and transverse spin fluctuations. For spin up electrons this self-energy is

$$[6] \qquad \Sigma^{t}_{\uparrow}(\vec{p},i\omega_n) = \frac{-1}{\beta}\int \frac{d\vec{p}'}{(2\pi)^3} \sum_{\omega_m} G^{(o)}_{\downarrow}(\vec{p}',i\omega_m)$$

$$x\ \{(-I)\sum_{k=1}^{\infty}[I(\tfrac{-1}{\beta})\int\frac{d\vec{p}''}{(2\pi)^3}\sum_{\omega_\ell} G^{(o)}_{\uparrow}(\vec{p}'',i\omega_\ell)$$

$$G^{(o)}_{\downarrow}(\vec{p}'' - (\vec{p} - \vec{p}'),i\omega_\ell - (i\omega_n - i\omega_m))]^{k}\}$$

The quantity in square brackets can be written in terms of the Lindhard function $u(\vec{q},i\omega_\upsilon)$ (see the Appendix).

$$[7] \qquad IN(o)u(\vec{q},i\omega_\upsilon) = I(\tfrac{-1}{\beta})\int\frac{d\vec{p}}{(2\pi)^3}\sum_{\omega_\ell} G^{(o)}(\vec{p},i\omega_\ell)\ G^{(o)}(\vec{p} - \vec{q},i\omega_\ell - i\omega_\upsilon)$$

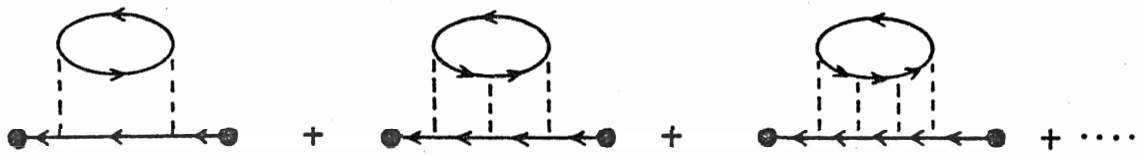
Figure 1

(a) Diagrams contributing to the electron Green's function due to transverse spin fluctuations.

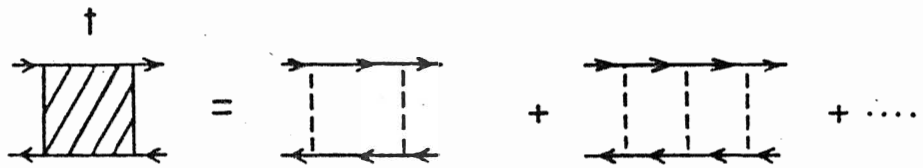
(b) The t-matrix for transverse spin fluctuations. The interaction (------) lines connect only electron (———) lines with opposite spin.

(c) Diagrams contributing to the electron Green's function due to longitudinal spin fluctuations. Only diagrams containing an odd number of bubbles contribute because the interaction (------) lines only connect electron (———) lines of opposite spin.

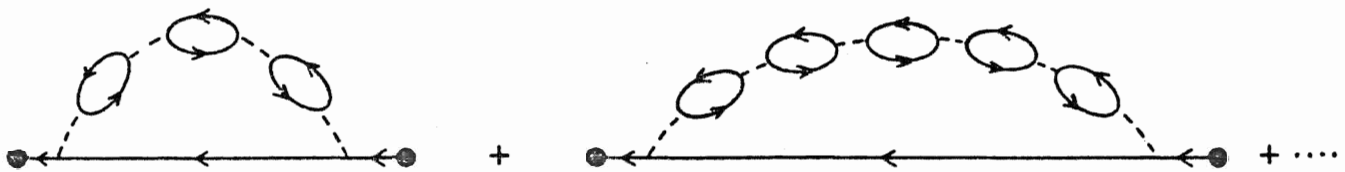
a)



b)



c)



Here, $i\omega_\nu$ is a boson Matsubara frequency, and N(o) is the single spin density of states. This self-energy can be expressed in terms of the t-matrix for transverse spin functuations (see Figure 1b),

$$[8] \quad t(\vec{q},i\omega_\nu) = -I \sum_{k=1}^{\infty} [IN(o)u(\vec{q},i\omega_\nu)]^k$$

$$= \frac{-I^2N(o)u(\vec{q},i\omega_\nu)}{1 - IN(o)u(\vec{q},i\omega_\nu)} \quad ,$$

as

$$[9] \quad \Sigma^t_\uparrow(\vec{p},i\omega_n) = \frac{-1}{\beta} \int \frac{d\vec{p}'}{(2\pi)^3} \sum_{\omega_m} G^{(o)}_\downarrow(\vec{p}',i\omega_m) t(\vec{p} - \vec{p}', i\omega_n - i\omega_m).$$

Another correction to $G^{(o)}_\sigma(\vec{p},i\omega_n)$, from longitudinal spin fluctuations, is found by summing the first diagram in Figure (1a) and the diagrams in Figure (1c).

$$[10] \quad [\delta G^{(o)}_\sigma(\vec{p},i\omega_n)]_\ell = G^{(o)}_\sigma(\vec{p},i\omega_n) \, \Sigma^\ell_\sigma(\vec{p},i\omega_n) \, G^{(o)}_\sigma(\vec{p},i\omega_n)$$

The self-energy due to electron-longitudinal spin fluctuation interaction for spin up electrons is

$$[11] \quad \Sigma_{\uparrow}^{\ell}(\vec{p},i\omega_n) = \frac{-1}{\beta}\int \frac{d\vec{p}'}{(2\pi)^3} \sum_{\omega_m} G_{\uparrow}^{(o)}(\vec{p}',i\omega_m)$$

$$x \{(-I)\sum_{k=1}^{\infty} [I(\frac{-1}{\beta})\int\frac{d\vec{p}''}{(2\pi)^3} \sum_{\omega_\ell} G^{(o)}(\vec{p}'',i\omega_\ell)$$

$$G^{(o)}(\vec{p}'' - (\vec{p} - \vec{p}'),i\omega_\ell - (i\omega_n - i\omega_m))]^{(2k-1)}\}$$

which can be written in terms of the Lindhard function $u(\vec{q},i\omega_\upsilon)$ as

$$[12] \quad \Sigma_{\uparrow}^{\ell}(\vec{p},i\omega_n) = \frac{-1}{\beta}\int\frac{d\vec{p}'}{(2\pi)^3} \sum_{\omega_m} G_{\uparrow}^{(o)}(\vec{p}',i\omega_m)$$

$$x \{(-I) \sum_{k=1}^{\infty} [IN(o)u(\vec{p} - \vec{p}',i\omega_n - i\omega_m)]^{(2k-1)}\}$$

$$= \frac{-1}{\beta}\int\frac{d\vec{p}'}{(2\pi)^3} \sum_{\omega_m} G_{\uparrow}^{(o)}(\vec{p}',i\omega_m)\frac{-I^2N(o)u(\vec{p} - \vec{p}',i\omega_n - i\omega_m)}{1 - [IN(o)u(\vec{p} - \vec{p}',i\omega_n - i\omega_m)]^2} .$$

The total self-energy due to the longitudinal and transverse spin fluctuations is

$$[13]\quad \Sigma_{\uparrow}(\vec{p},i\omega_n) = \frac{-1}{\beta}\int \frac{d\vec{p}'}{(2\pi)^3}\sum_{\omega_m} G^{(o)}(\vec{p}',i\omega_m)\{\frac{-I^2N(o)u(\vec{p}-\vec{p}',i\omega_n-i\omega_m)}{1-[IN(o)u(\vec{p}-\vec{p}',i\omega_n-i\omega_m)]^2}$$

$$- \frac{I^2N(o)u(\vec{p}-\vec{p}',i\omega_n- i\omega_m)}{1 - IN(o)u(\vec{p}- \vec{p}',i\omega_n- i\omega_m)} + I^2N(o)u(\vec{p}- \vec{p}',i\omega_n-i\omega_m)\}$$

where the contribution of the first diagram in Figure (1a), which occurs in the series for both Σ^{ℓ} and Σ^{t}, has been subtracted. The quantity in curly brackets in the last equation can be written in the form

$$\{\frac{3}{2}\frac{-I^2N(o)u}{1 - IN(o)u} + \frac{1}{2}\frac{I}{1 + IN(o)u} + I^2N(o)u - \frac{I}{2}\}$$

In the nearly ferromagnetic limit, $(1 - IN(o)) << 1$, the first term dominates, and the self-energy can be written in terms of the effective t-matrix for spin fluctuations,

$$[14]\quad t_{eff}(\vec{q},i\omega_\upsilon) = \frac{3}{2}\frac{-I^2N(o)u(\vec{q},i\omega_\upsilon)}{1 - IN(o)u(\vec{q},i\omega_\upsilon)} = \frac{3}{2}\, t(\vec{q},i\omega_\upsilon)\ ,$$

as

$$[15]\quad \Sigma_{\uparrow}(\vec{p},i\omega_n) = \frac{-1}{\beta}\int\frac{d\vec{p}'}{(2\pi)^3}\sum_{\omega_m} G^{(o)}(\vec{p}',i\omega_m)t_{eff}(\vec{p} - \vec{p}',i\omega_n- i\omega_m)\ .$$

The interacting Green's function is given by the Dyson equation

$$[16] \quad G_{\uparrow}(\vec{p},i\omega_n) = G^{(o)}_{\uparrow}(\vec{p},i\omega_n) + G^{(o)}_{\uparrow}(\vec{p},i\omega_n)\ \Sigma_{\uparrow}(\vec{p},i\omega_n)\ G_{\uparrow}(\vec{p},i\omega_n).$$

In the expression for the self-energy, $G^{(o)}(\vec{p}',i\omega_m)$ can be replaced by $G(\vec{p}',i\omega_m)$, which sums the contribution from multi-paramagnon processes. However in calculating the Lindhard function, the bare propagators, $G^{(o)}$, will be used. Schrieffer and Berk[18] have justified this for a nearly ferromagnetic system. Vertex corrections have not been included in [15], however there is no Migdal theorem for paramagnons which justifies this.[19] The characteristic paramagnon energy, Ω_{sf}, is not small compared to the fermi energy, and the first order corrections to the bare vertex are on the order of 1 (see Section (C)). There have been attempts to include the finite range of the interaction by introducing a momentum dependent I.[4] This reduces the peak height in the electron-paramagnon spectral function P (defined on page 16), without affecting the position much, and so will not be included here.

Next, to be able to do the sum over ω_m, we will want to write t_{eff} in its spectral representation. The spectral representation of $u(\vec{q},i\omega_\nu)$ is

$$[17] \quad u(\vec{q},i\omega_\nu) = \int_0^\infty d\Omega\ b(\vec{q},\Omega)\left[\frac{1}{i\omega_\nu - \Omega} - \frac{1}{i\omega_\nu + \Omega}\right] ,$$

where

$$[18] \quad b(\vec{q},\Omega) = \frac{-1}{\pi} \operatorname{Im} u(\vec{q},\Omega + i0^+)$$

is the spectral density. Then, using the dispersion relations for $u^n(\vec{q},\omega + i0^+)$, $n \geq 1$,

$$\operatorname{Re} u^n(\vec{q},\omega + i0^+) = \frac{-1}{\pi} P\int_{-\infty}^{\infty} d\Omega \frac{\operatorname{Im} u^n(\vec{q},\Omega + i0^+)}{\omega - \Omega}$$

$$[19] \quad \operatorname{Im} u^n(\vec{q},\omega + i0^+) = \frac{-1}{\pi} P\int_{-\infty}^{\infty} d\Omega \frac{\operatorname{Re} u^n(\vec{q},\Omega + i0^+)}{\omega - \Omega} \quad ,$$

t_{eff} can be written in its spectral representation

$$[20] \quad t_{eff}(\vec{q},i\omega_\nu) = \int_0^\infty d\Omega \; B(\vec{q},\Omega)\left[\frac{1}{i\omega_\nu - \Omega} - \frac{1}{i\omega_\nu + \Omega}\right]$$

where the spectral density is

$$[21] \quad B(\vec{q},\Omega) = \frac{-1}{\pi} \operatorname{Im} t_{eff}(\vec{q},\Omega + i0^+)$$

$$= \frac{-1}{\pi} \operatorname{Im}\left\{\frac{-3}{2} \frac{I}{1 - IN(o)u(\vec{q},\Omega + i0^+)}\right\} .$$

The spectral representation of t_{eff} can be used in summing over ω_m in the

expression for the self-energy [15]. The result is

$$[22] \quad \Sigma(\vec{p},i\omega_n) = \int_{-\infty}^{\infty} d\omega' \int \frac{d\vec{p}'}{(2\pi)^3} \left[\frac{-1}{\pi} \text{Im } G(\vec{p}',\omega' + i0^+)\right] \int_0^{\infty} d\Omega \; B(\vec{p} - \vec{p}',\Omega)$$

$$\times \left[\frac{f(-\omega')}{i\omega_n - \omega' - \Omega} + \frac{f(\omega')}{i\omega_n - \omega' + \Omega}\right]$$

$$+ \int_0^{\infty} \frac{d\Omega}{e^{\beta\Omega}-1} \int \frac{d\vec{p}'}{(2\pi)^3} B(\vec{p}-\vec{p}',\Omega)[G(\vec{p}',i\omega_n-\Omega) + G(\vec{p}',i\omega_n+\Omega)].$$

This can be analytically continued to just above the real frequency axis by setting $i\omega_n \to \omega + i0^+$. (See Reference (27) for a derivation which is similar to that leading to Equation [22].)

Now, since $\Sigma(\vec{p},\omega + i0^+)$ depends on $\vec{p}$ only through $|\vec{p}|$, the average over the constant energy surface $\epsilon = \epsilon_{\vec{p}}$ can be calculated using

$$F(\epsilon) = \frac{1}{N(\epsilon)} \int \frac{d\vec{p}}{(2\pi)^3} F(|\vec{p}|)\delta(\epsilon - \epsilon_{\vec{p}}) \; ,$$

for $\epsilon > -E_F$.

The result of the spherical averaging is

$$[23]\quad \sum(\epsilon,\omega + i0^+) = \int_{-\infty}^{\infty} d\omega' \int_{-\infty}^{\infty} d\epsilon' \frac{N(\epsilon')}{N(o)} \left[\frac{-1}{\pi} \operatorname{Im} G(\epsilon',\omega' + i0^+)\right]$$

$$\times \int_{o}^{\infty} d\Omega\, P(\epsilon,\epsilon';\Omega) \left[\frac{f(-\omega')}{\omega - \omega' - \Omega + i0^+} + \frac{f(\omega')}{\omega - \omega' + \Omega + i0^+}\right]$$

$$+ \int_{o}^{\infty} \frac{d\Omega}{e^{\beta\Omega} - 1} \int_{-\infty}^{\infty} d\epsilon' \frac{N(\epsilon')}{N(o)} P(\epsilon,\epsilon';\Omega) \left[G(\epsilon',\omega-\Omega+i0^+) + G(\epsilon',\omega+\Omega+i0^+)\right],$$

where the paramagnon spectral function is

$$[24]\quad P(\epsilon,\epsilon';\Omega) = \frac{N(O)}{N(\epsilon)N(\epsilon')} \int \frac{d\vec{p}}{(2\pi)^3} \int \frac{d\vec{p}'}{(2\pi)^3} B(\vec{p}-\vec{p}',\Omega)\ \delta(\epsilon-\epsilon_{\vec{p}})\ \delta(\epsilon'-\epsilon_{\vec{p}'})$$

for $\epsilon,\ \epsilon' > -E_F$. The integrals over spherical constant energy surfaces can be changed to an integral over $\vec{q} = \vec{p} - \vec{p}'$, with the result

$$[25]\quad P(\epsilon,\epsilon';\Omega) = \theta(\epsilon' + E_F)\ \theta(\epsilon + E_F) \int_{|p-p'|}^{(p+p')} \frac{dq\ q}{2pp'} B(\vec{q},\Omega)$$

where $p = p(\epsilon) = \sqrt{2m\epsilon + p_F^2}$ and $p' = p(\epsilon') = \sqrt{2m\epsilon' + p_F^2}$. Note that the ϵ-dependence of $\sum(\epsilon,\omega + i0^+)$ arises solely through the ϵ-dependence of $P(\epsilon,\epsilon';\Omega)$.

Next, as is usual, we will assume that the ϵ,ϵ'-dependence of $N(\epsilon')P(\epsilon,\epsilon';\Omega)$ can be ignored in evaluating $\sum(\epsilon,\omega + i0^+)$ for $|\epsilon|$, $|\omega| \lesssim \Omega_{sf}$. (This amounts to assuming that $\Omega_{sf} << E_F$, since for $|\epsilon|$, $|\omega| \lesssim \Omega_{sf}$, the important range in the ϵ' integration is for ϵ' on the order of Ω_{sf}. This assumption cannot be justified in this model, since the characteristic energy in the paramagnon spectral function is on the order of E_F.) After $N(\epsilon')P(\epsilon,\epsilon';\Omega)$ is replaced by $N(0)P(0,0;\Omega)$ in the expression for self-energy [23], the ϵ' integration can be performed by noting that in the important range of the ϵ' integration $\sum(\epsilon',\omega' + i0^+) = \sum(0,\omega' + i0^+)$. The result for $\sum(\omega + i0^+) \equiv \sum(0,\omega + i0^+)$ at $T = 0$ is

$$[26] \quad \sum(\omega + i0^+) = \int_{-\infty}^{\infty} d\omega' \int_0^{\infty} d\Omega\, P(\Omega)\left[\frac{\theta(\omega')}{\omega - \omega' - \Omega + i0^+} + \frac{\theta(-\omega')}{\omega - \omega' + \Omega + i0^+}\right]$$

where

$$[27] \quad P(\Omega) \equiv P(0,0;\Omega) = \int_0^2 \frac{\bar{q}d\bar{q}}{2}\left\{\frac{1}{\pi}\,\mathrm{Im}\left[\frac{3}{2}\,\frac{\bar{I}}{1 - \bar{I}u(\bar{q},\Omega + i0^+)}\right]\right\}.$$

Here, $\bar{I} = IN(0)$ and $\bar{q} = q/p_F$.

In this model, the enhancement of the uniform, static susceptibility over the Pauli susceptibility, $\chi_o = 2\mu_B^2 N(0)$, is given by the Stoner factor, S.

$$[28] \quad \chi(0,0) = S\chi_o = \frac{\chi_o}{1 - \bar{I}}\,.$$

(B) Paramagnons in the Finite Bandwidth Model

The finite width of the band is described by the momentum cut-off, p_c.[13] The zero order Hamiltonian is

$$[29] \quad \hat{H}_o - \mu\hat{N} = \sum_{\vec{p},\sigma} \epsilon_{\vec{p}} \hat{a}^+_{\vec{p}\sigma} \hat{a}_{\vec{p}\sigma} \theta(p_c - |\vec{p}|),$$

and the non-interacting single particle temperature Green's function is

$$[30] \quad G^{(o)}_\sigma(\vec{p},i\omega_n) = \frac{\theta(p_c - |\vec{p}|)}{i\omega_n - \epsilon_{\vec{p}}} \quad .$$

We can expect the momentum cut-off to have a drastic effect on the Lindhard function when $\bar{p}_c - 1 << 1$, where $\bar{p}_c = p_c/p_F$ (see the Appendix). The self-energy [15] can be calculated using the same approximations as in the standard paramagnon theory, but in the finite bandwidth model, the lowest order vertex corrections are expected to be much smaller than the bare vertex because of the greatly reduced phase space volume available for electron scattering when $\bar{p}_c - 1 << 1$ (see Section (C)). The paramagnon spectral function $P(\epsilon,\epsilon';\Omega)$ is calculated the same as in the standard paramagnon theory for ϵ_c $(= \frac{p_c^2}{2m} - \frac{p_F^2}{2m}) > \epsilon,\epsilon' > -E_F$, and is zero otherwise.

From the formulae for calculating $u(\vec{q},i\omega_\upsilon)$ given in the Appendix, we can see that $P(\Omega)$ peaks at $\Omega_p \simeq (\bar{p}_c^2 - 1)E_F = \epsilon_c$ and is non-zero up to

$\Omega_{max} = \bar{p}_c^2 E_F$ for $\bar{p}_c - 1 \ll 1$. Thus in the important range of ϵ' integration, for ϵ' within several Ω_{sf} of the fermi level, the energy dependence of the density of states cannot be neglected. To account for the finite width of the band we can take as the density of states

$$[31] \quad N(\epsilon') = N(o)\theta(\epsilon_c - E)\theta(E + E_F)$$

where $E = \epsilon' + \mathrm{Re}\Sigma(i0^+)$. $\mathrm{Re}\Sigma(i0^+)$ is the shift in the chemical potential due to interactions. Then, assuming that the ϵ,ϵ'-dependence of $P(\epsilon,\epsilon';\Omega)$ can be ignored for ϵ,ϵ' within several Ω_{sf} of the fermi level, we get for the self-energy at $T = 0$

$$[32] \quad \Sigma_{sf}(\omega + i0^+) = \int_{-\infty}^{\infty} d\omega' \tilde{N}_{sf}(\omega') \int_o^{\infty} d\Omega\, P(\Omega) \left[\frac{\theta(\omega')}{\omega - \omega' - \Omega + i0^+} + \frac{\theta(-\omega')}{\omega - \omega' + \Omega + i0^+}\right]$$

where

$$[33] \quad \tilde{N}_{sf}(\omega') = \int_{-\infty}^{\infty} d\epsilon' \frac{N(\epsilon')}{N(o)} \left[\frac{-1}{\pi}\, \mathrm{Im} \frac{1}{\omega' - \epsilon' - \Sigma_{sf}(\omega' + i0^+) + i0^+}\right] .$$

The ϵ'-dependence of the self-energy is neglected in evaluating $\tilde{N}_{sf}(\omega')$ since, as we have already argued, in the important range of the ϵ' integration, $\Sigma(\epsilon',\omega' + i0^+) = \Sigma(\omega' + i0^+)$. As in the standard paramagnon theory, the Stoner enhancement factor is given by $S = (1 - \bar{I})^{-1}$.

(C) Vertex Corrections in the Finite Bandwidth Model

The standard paramagnon theory, outlined in Section (A), implicitly assumes a Migdal theorem[20] by not considering any vertex corrections. Hertz, Levin and Béal-Monod[19] have examined the first order vertex corrections and have concluded that they are of the same order of magnitude as the bare vertex. In the standard paramagnon theory with S = 14, $P(\Omega)$ peaks at $\sim .2\ E_F$ and is non-zero up to $\Omega_{max} = 8\ E_F$, so the characteristic spin fluctuation energy Ω_{sf} is not small compared to E_F, and we cannot make the phase space argument that the vertex corrections are negligible and of order Ω_{sf}/E_F.[21] Hertz et al.[19] have also argued that because $\bar{I} = IN(o) \simeq 1$, the vertex corrections are comparable in magnitude to the bare vertex.

In the finite bandwidth model, with $\bar{p}_c - 1 \ll 1$, the peak in $P(\Omega)$ occurs at $\Omega_p \simeq (\bar{p}_c^2 - 1)E_F \ll E_F$, and $P(\Omega)$ is non-zero up to $\Omega_{max} = \bar{p}_c^2 E_F$, so the typical spin fluctuation energy is much smaller than in the case $\bar{p}_c \to \infty$. Moreover the phase space available for second order scattering is reduced by the finite momentum cut-off. These effects on the first order vertex corrections will be examined.

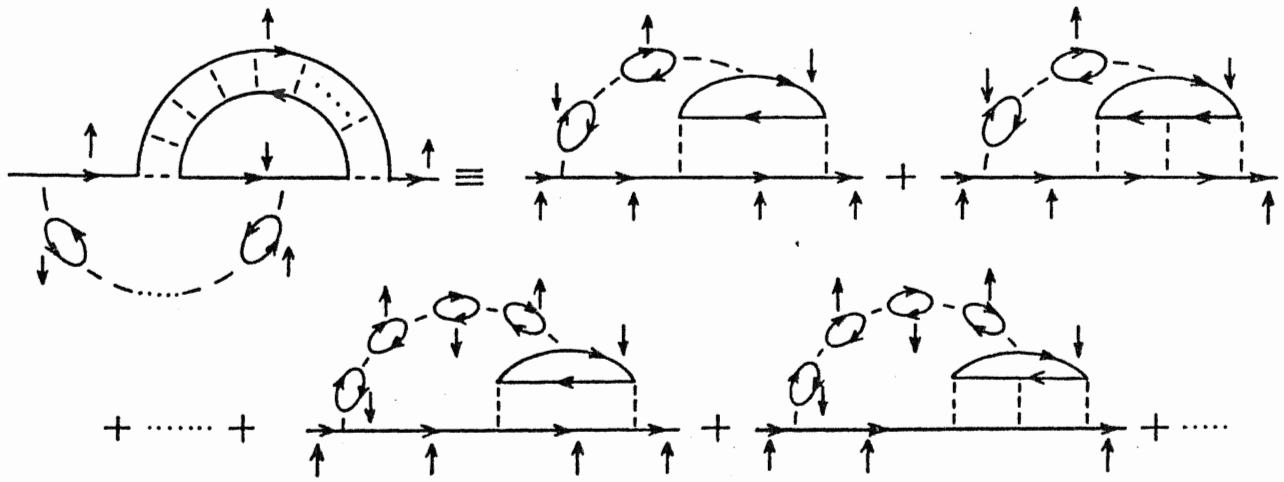
The lowest order vertex correction to the self-energy due to electron-longitudinal paramagnon interaction is found by summing diagrams of the type shown in Figure (2b). This sum can be written as

Figure 2

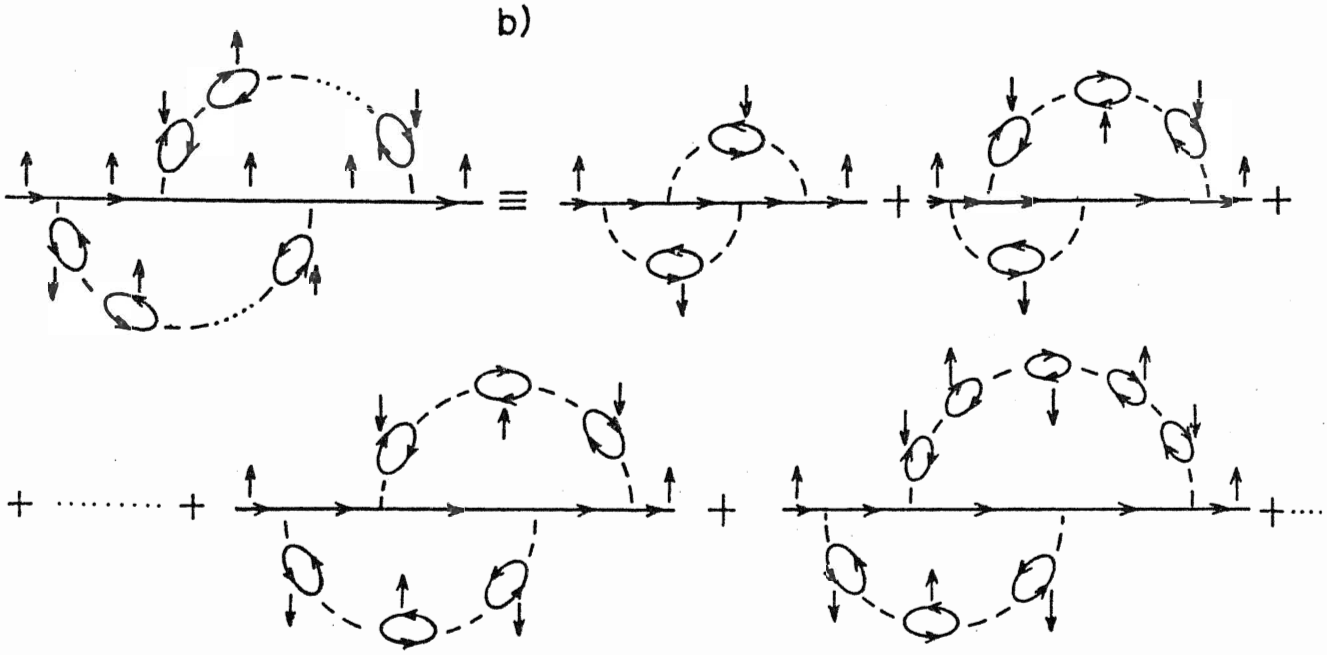
(a) Lowest order vertex correction to the electron self-energy due to electron-transverse spin fluctuation interaction. The strings contain an even number of bubbles.

(b) Diagrams giving the lowest order vertex correction to the electron self-energy due to electron-longitudinal spin fluctuation interaction. Each of the strings consists of an odd number of bubbles.

a)



b)



$$[34] \quad \delta\Sigma^{\ell}_{\uparrow}(\vec{k},i\omega_m) = \frac{-1}{\beta}\sum_{\omega_\nu}\int\frac{d\vec{q}}{(2\pi)^3}\, G^{(o)}_{\uparrow}(\vec{k}+\vec{q},i\omega_m+i\omega_\nu)\ \chi_{\ell}(\vec{q},i\omega_\nu)$$

$$\Gamma^{(1)}_{\ell}(\vec{k},\vec{k}+\vec{q};i\omega_m,i\omega_m+i\omega_\nu)$$

where

$$[35] \quad \chi_{\ell}(\vec{q},i\omega_\nu) = \frac{-I^2N(o)u(\vec{q},i\omega_\nu)}{1-[IN(o)u(\vec{q},i\omega_\nu)]^2}$$

is the propagator for longitudinal spin fluctuations, and the lowest order vertex correction for longitudinal spin fluctuations is

$$[36] \quad \Gamma^{(1)}_{\ell}(\vec{k},\vec{k}+\vec{q};i\omega_m,i\omega_m+i\omega_\nu) = \frac{-1}{\beta}\sum_{\omega_n}\int\frac{d\vec{p}}{(2\pi)^3}\, G^{(o)}_{\uparrow}(\vec{p},i\omega_n)\ G^{(o)}_{\uparrow}(\vec{p}+\vec{q},i\omega_n+i\omega_\nu)$$

$$\times\ \tfrac{1}{2}\{\frac{-I^2N(o)u(\vec{k}-\vec{p},i\omega_m-i\omega_n)}{1-[IN(o)u(\vec{k}-\vec{p},i\omega_m-i\omega_n)]^2}$$

$$\times\ [1+\frac{[1-(IN(o)u(\vec{k}-\vec{p},i\omega_m-i\omega_n))^2][1-(IN(o)u(\vec{q},i\omega_\nu))^2]}{1-[IN(o)u(\vec{k}-\vec{p},i\omega_m-i\omega_n)IN(o)u(\vec{q},i\omega_\nu)]^2}]\}\ .$$

In the nearly ferromagnetic limit, $1 - IN(o) \ll 1$, we can take

$\chi_{\ell}(\vec{q},i\omega_{\nu}) \simeq \frac{1}{2}t(\vec{q},i\omega_{\nu})$ where $t(\vec{q},i\omega_{\nu})$ is the t-matrix for transverse spin fluctuations (Equation [8]), and replace the quantity in curly brackets in Equation [36] by $\frac{1}{2}t(\vec{k} - \vec{p},i\omega_{m} - i\omega_{n})$. Then $\delta\sum_{\uparrow}^{\ell}(\vec{k},i\omega_{m})$ can be written in terms of

$$[37] \quad \Gamma^{(1)}(\vec{k},\vec{k} + \vec{q};i\omega_{m},i\omega_{m} + i\omega_{\nu})$$

$$\frac{-1}{\beta} \sum_{\omega_{n}} \int \frac{d\vec{p}}{(2\pi)^{3}} G_{\uparrow}^{(o)}(\vec{p},i\omega_{n})\ G_{\uparrow}^{(o)}(\vec{p}+\vec{q},i\omega_{n}+i\omega_{\nu})\ t(\vec{k}-\vec{p},i\omega_{m}-i\omega_{n})$$

and the propagator for transverse spin fluctuations as

$$[38] \quad \delta\sum_{\uparrow}^{\ell}(\vec{k},i\omega_{m}) = \frac{1}{8}\ \{\ \frac{-1}{\beta} \sum_{\omega_{\nu}} \int \frac{d\vec{q}}{(2\pi)^{3}} G_{\uparrow}^{(o)}(\vec{k} + \vec{q},i\omega_{m} + i\omega_{\nu})\ t(\vec{q},i\omega_{\nu})$$

$$\Gamma^{(1)}(\vec{k},\vec{k} + \vec{q};i\omega_{m},i\omega_{m} + i\omega_{\nu})\}\ .$$

The lowest order vertex correction for transverse spin fluctuations is found by summing diagrams of the type in Figure (2a). The sum of these diagrams is

$$[39] \quad \delta\Sigma_{\uparrow}^{t}(\vec{k},i\omega_m) = \frac{-1}{\beta}\sum_{\omega_\nu}\int\frac{d\vec{q}}{(2\pi)^3}\, G_{\downarrow}^{(o)}(\vec{k}+\vec{q},i\omega_m+i\omega_\nu)\; t(\vec{q},i\omega_\nu)$$

$$\Gamma_t^{(1)}(\vec{k},\vec{k}+\vec{q};i\omega_m,i\omega_m+i\omega_\nu)$$

where

$$[40] \quad \Gamma_t^{(1)}(\vec{k},\vec{k}+\vec{q};i\omega_m,i\omega_m+i\omega_\nu) = \frac{-1}{\beta}\sum_{\omega_n}\int\frac{d\vec{p}}{(2\pi)^3}\, G_{\uparrow}^{(o)}(\vec{p},i\omega_n)$$

$$G_{\downarrow}^{(o)}(\vec{p}+\vec{q},i\omega_n+i\omega_\nu)\left[\frac{I(IN(o)u(\vec{k}-\vec{p},i\omega_m-i\omega_n))^2}{1-(IN(o)u(\vec{k}-\vec{p},i\omega_m-i\omega_n))^2}\right]$$

is the first order vertex correction for transverse spin fluctuations. For $\bar{I}N(o) \simeq 1$,

$$[41] \quad \Gamma_t^{(1)}(\vec{k},\vec{k}+\vec{q};i\omega_m,i\omega_m+i\omega_\nu) \simeq -\tfrac{1}{2}\Gamma^{(1)}(\vec{k},\vec{k}+\vec{q};i\omega_m,i\omega_m+i\omega_\nu)\ .$$

The total first order vertex correction to the self-energy is

$$[42] \quad \delta\Sigma_{\uparrow}(\vec{k},i\omega_m) = \frac{-3}{8}\left\{\frac{-1}{\beta}\sum_{\omega_\nu}\int\frac{d\vec{q}}{(2\pi)^3}\, G^{(o)}(\vec{k}+\vec{q},i\omega_m+i\omega_\nu)\; t(\vec{q},i\omega_\nu)\right.$$

$$\left.\Gamma^{(1)}(\vec{k},\vec{k}+\vec{q};i\omega_m,i\omega_m+i\omega_\nu)\right\}\ .$$

We want to examine

$$\lim_{\vec{q}\to 0}\ \lim_{i\omega_\nu \to 0} \Gamma^{(1)}(\vec{k},\vec{k}+\vec{q};i\omega_m, i\omega_m + i\omega_\nu)$$

which is the limit in which Migdal's theorem is valid. The analysis is the same as in reference,[19] but with $G^{(o)}(\vec{p},i\omega_n)$ given by [30]. The result, for $\vec{k} = k_F\hat{k}$ and $\omega = 0$ is

$$[43] \quad \Gamma^{(1)} \equiv \lim_{\vec{q}\to 0}\ \lim_{\nu\to 0} \Gamma^{(1)}(k_F\hat{k},\ k_F\hat{k} + \vec{q};\ 0,\nu)$$

$$= \int \frac{d\vec{p}}{(2\pi)^3}\ \theta(p_c - |k_F\hat{k} + \vec{p}|) \int_0^\infty d\Omega\ B_t(\vec{p},\Omega)$$

$$\times \left[\frac{f(\epsilon_{\vec{k}+\vec{p}})}{(\epsilon_{\vec{k}+\vec{p}} - \Omega)^2} + \frac{1 - f(\epsilon_{\vec{k}+\vec{p}})}{(\epsilon_{\vec{k}+\vec{p}} + \Omega)^2} - \frac{2\delta(\epsilon_{\vec{k}+\vec{p}})}{\Omega}\right]_{\vec{k}=k_F\hat{k}}$$

where

$$[44] \quad B_t(\vec{p},\Omega) = \frac{-1}{\pi}\ \mathrm{Im}\ t(\vec{p},\Omega + i0^+)\ .$$

The results of the angular integrations are, in the case of a finite momentum cut-off

$$[45]\quad \Gamma^{(1)} = -\int_0^2 \frac{\bar{p}d\bar{p}}{2}\int_0^\infty d\bar{\Omega}\,\frac{N(o)B_t(\bar{p},\bar{\Omega})}{2\bar{p}+\bar{\Omega}-\bar{p}^2} - \int_0^{\bar{p}_c-1}\frac{\bar{p}d\bar{p}}{2}\int_0^\infty d\Omega\,\frac{N(o)B_t(\bar{p},\bar{\Omega})}{\bar{p}^2+\bar{\Omega}+2\bar{p}}$$

$$-\int_{\bar{p}_c-1}^{\bar{p}_c+1}\frac{\bar{p}d\bar{p}}{2}\int_0^\infty d\bar{\Omega}\,\frac{N(o)B_t(\bar{p},\bar{\Omega})}{\bar{p}_c^2-1+\bar{\Omega}} + \int_2^{\bar{p}_c+1}\frac{\bar{p}d\bar{p}}{2}\int_0^\infty d\bar{\Omega}\,\frac{N(o)B_t(\bar{p},\bar{\Omega})}{\bar{p}^2+\bar{\Omega}-2\bar{p}}$$

and for $\bar{p}_c \to \infty$

$$[46]\quad \Gamma^{(1)} = -\int_0^2 \bar{p}d\bar{p}\int_0^\infty d\bar{\Omega}\,N(o)B_t(\bar{p},\bar{\Omega})\,\frac{\bar{\Omega}+2\bar{p}}{(\bar{\Omega}+2\bar{p})^2-\bar{p}^4}$$

$$+\int_2^\infty \bar{p}d\bar{p}\int_0^\infty d\bar{\Omega}\,N(o)B_t(\bar{p},\bar{\Omega})\,\frac{2\bar{p}}{(\bar{p}^2+\bar{\Omega})^2-(2\bar{p})^2}\,,$$

where $\bar{p} = p/p_F$ and $\bar{\Omega} = \Omega/E_F$.

The first term in $\Gamma^{(1)}$ for the case $\bar{p}_c \to \infty$ is identical with the result of Hertz et al.[19] They combined their result with the expansion for $B_t(\bar{p},\bar{\Omega})$ valid for $\bar{\Omega} < 2\bar{p} << 1$ to obtain their estimate that $\Gamma^{(1)}$ is on the order of 1.

(D) Momentum Dependence of the Self-Energy

The self-energy due to transverse spin fluctuations is

$$[47] \quad \Sigma^t(\vec{k},i\omega_m) = \frac{-1}{\beta} \sum_{\omega_\nu} \int \frac{d\vec{q}}{(2\pi)^3} G^{(o)}(\vec{k}+\vec{q},i\omega_m+i\omega_\nu) t(\vec{q},i\omega_\nu)$$

$$\Gamma_t(\vec{k},\vec{k}+\vec{q};i\omega_m,i\omega_m+i\omega_\nu)$$

where the vertex function is

$$\Gamma_t(\vec{k},\vec{k}+\vec{q};i\omega_m,i\omega_m+i\omega_\nu) = 1 + \Gamma_t^{(1)}(\vec{k},\vec{k}+\vec{q};i\omega_m,i\omega_m+i\omega_\nu)$$
$$+ \text{higher order vertex corrections.}$$

To estimate the momentum dependence of $\Sigma^t(\vec{k},i\omega_m)$ we will replace the vertex function by 1 in Equation [47]. Then at T = 0,

$$[48] \quad \Sigma^t(\vec{k},\omega+i0^+) = \int_{-\infty}^{\infty} d\omega' \int \frac{d\vec{q}}{(2\pi)^3} \left[\frac{-1}{\pi} \operatorname{Im} G^{(o)}(\vec{k}+\vec{q},\omega'+i0^+)\right]$$

$$\times \int_0^{\infty} d\Omega\, B_t(\vec{q},\Omega) \left[\frac{\theta(\omega')}{\omega-\omega'-\Omega+i0^+} + \frac{\theta(-\omega')}{\omega-\omega'+\Omega+i0^+}\right],$$

where $B_t(\vec{q},\Omega)$ is the spectral density for transverse spin fluctuations. Then the result for the real part of the self-energy at $\omega = 0$ is

$$[49] \quad \mathrm{Re}\Sigma^t(\vec{k},i0^+) = -\int \frac{d\vec{q}}{(2\pi)^3}\,\theta(p_c - |\vec{k}+\vec{q}|)$$

$$\times \int_0^\infty d\Omega\, B_t(\vec{q},\Omega)\left[\frac{\theta(\epsilon_{\vec{k}+\vec{q}})}{\epsilon_{\vec{k}+\vec{q}} + \Omega} + \frac{\theta(-\epsilon_{\vec{k}+\vec{q}})}{\epsilon_{\vec{k}+\vec{q}} - \Omega}\right] .$$

The result of differentiating with respect to $\epsilon_{\vec{k}}$ is

$$[50] \quad \frac{\partial \mathrm{Re}\Sigma^t(\vec{k},i0^+)}{\partial \epsilon_{\vec{k}}} = \int \frac{d\vec{q}}{(2\pi)^3}\,\theta(p_c - |\vec{k}+\vec{q}| \int_0^\infty d\Omega\, B_t(\vec{q},\Omega)$$

$$\times \left[\frac{\theta(\epsilon_{\vec{k}+\vec{q}})}{(\epsilon_{\vec{k}+\vec{q}} + \Omega)^2} + \frac{\theta(-\epsilon_{\vec{k}+\vec{q}})}{(\epsilon_{\vec{k}+\vec{q}} - \Omega)^2} - \frac{2\delta(\epsilon_{\vec{k}+\vec{q}})}{\Omega}\right] \frac{\partial \epsilon_{\vec{k}+\vec{q}}}{\partial \epsilon_{\vec{k}}} ,$$

where the term from differentiating $\theta(p_c - |\vec{k}+\vec{q}|)$ has not been included. The result on the fermi surface is

$$[51] \quad \left.\frac{\partial \mathrm{Re}\,\Sigma^t(\vec{k},i0^+)}{\partial \epsilon_{\vec{k}}}\right|_{\vec{k}=k_F\hat{k}} = \int \frac{d\vec{p}}{(2\pi)^3}\,\theta(p_c - |k_F\hat{k}+\vec{q}|) \int_0^\infty d\Omega\, B_t(\vec{q},\Omega)$$

$$\times \left[\frac{f(\epsilon_{\vec{k}+\vec{q}})}{(\epsilon_{\vec{k}+\vec{q}} - \Omega)^2} + \frac{1 - f(\epsilon_{\vec{k}+\vec{q}})}{(\epsilon_{\vec{k}+\vec{q}} + \Omega)^2} - \frac{2\delta(\epsilon_{\vec{k}+\vec{q}})}{\Omega}\right]_{\vec{k}=k_F\hat{k}} \left(1 + \frac{\hat{k}\cdot\vec{q}}{k_F}\right).$$

(We get a similar result for $[\partial \mathrm{Re}\, \Sigma^{\ell}(\vec{k},i0^+)/\partial\epsilon_{\vec{k}}]_{\vec{k}=k_F\hat{k}}$, except that $B_t(\vec{q},\Omega)$ is replaced by $B_\ell(\vec{q},\Omega)$, the spectral density for longitudinal spin fluctuations.) This is the same as the result for $\Gamma^{(1)}$ in Equation [43], except for the factor $[1 + (\hat{k}\cdot\vec{q})/k_F]$. Then we can expect that when the vertex corrections are unimportant, the momentum dependence of the self-energy may be small compared to 1. When the result for $\Gamma^{(1)}$ is not small, the result for $[\partial \mathrm{Re}\, \Sigma^{t}(\vec{k},i0^+)/\partial\epsilon_{\vec{k}}]_{\vec{k}=k_F\hat{k}}$ may be even larger because of the factor $[1 + (\hat{k}\cdot\vec{q})/k_F]$. In addition, when $\Gamma^{(1)}$ is not small compared to 1, we cannot replace the vertex function by 1 in Equation [47]. This suggests that when vertex corrections are important in the calculation of the self-energy, the ϵ-dependence of $\Sigma_{sf}(\epsilon,\omega + i0^+)$ may be dominated by the effect of vertex corrections.

An alternative viewpoint, when vertex corrections are negligible, is to consider the ϵ-dependence of the self-energy to result from the ϵ-dependence of $P(\epsilon,\epsilon';\Omega)$. From Equation [25] we have, for $\epsilon,\epsilon' << E_F$

$$[52] \quad P(\epsilon,\epsilon';\Omega) = \int_{\frac{1}{2}|\bar{\epsilon}-\bar{\epsilon}'|}^{2+\frac{1}{2}(\bar{\epsilon}+\bar{\epsilon}')} \frac{d\bar{q}\ \bar{q}}{2[1 + \frac{1}{2}(\bar{\epsilon} + \bar{\epsilon}')]} \frac{1}{\pi} \mathrm{Im}\{ \frac{3}{2} \frac{\bar{I}}{1 - \bar{I}u(q,\Omega + i0^+)} \}$$

where $\bar{\epsilon} = \epsilon/\epsilon_F$, $\bar{\epsilon}' = \epsilon'/E_F$ and $\bar{q} = q/p_F$, and

$$[53] \quad \frac{\partial P(\epsilon=0,\epsilon'=0;\Omega)}{\partial\epsilon} = \frac{-1}{2E_F} P(\Omega) + \frac{1}{2\pi E_F} \mathrm{Im}\{ \frac{3}{2} \frac{\bar{I}}{1 - \bar{I}u(2p_F,\Omega + i0^+)} \} .$$

When the second term in Equation [53] is negligible compared to the first, and if we can ignore the ϵ'-dependence of $P(\epsilon,\epsilon';\Omega)$ in the important range of the ϵ' integration, we have

$$[54] \qquad \frac{\partial \Sigma_{sf}(\epsilon = 0, \omega + i0^+)}{\partial \epsilon} \simeq \frac{-1}{2E_F} \Sigma_{sf}(\omega + i0^+)$$

where $\Sigma_{sf}(\omega + i0^+)$, given by Equation [32] is calculated using $P(\Omega) \equiv P(0,0;\Omega)$.

(E) Self-Energy Due to Electron-Phonon Interaction

The electron-phonon contribution to the self-energy is obtained in the usual way[22] as

$$[55] \quad \Sigma_{ep}(\epsilon,\omega + i0^+) = \int_{-\infty}^{\infty} d\omega' \int_{-\infty}^{\infty} d\epsilon' \frac{N(\epsilon')}{N(o)} \left[\frac{-1}{\pi} \text{Im } G(\epsilon',\omega' + i0^+)\right]$$

$$\times \int_{o}^{\infty} d\Omega \; \alpha^2F(\epsilon,\epsilon';\Omega) \left[\frac{f(-\omega')}{\omega - \omega' - \Omega + i0^+} + \frac{f(\omega')}{\omega - \omega' + \Omega + i0^+}\right]$$

$$+ \int_{o}^{\infty} \frac{d\Omega}{e^{\beta\Omega} - 1} \int_{-\infty}^{\infty} d\epsilon' \frac{N(\epsilon')}{N(o)} \; \alpha^2F(\epsilon,\epsilon';\Omega) [G(\epsilon',\omega-\Omega+i0^+) + G(\epsilon',\omega+\Omega+i0^+)]$$

where

$$[56] \quad \alpha^2F(\epsilon,\epsilon';\Omega) = \frac{N(o)}{N(\epsilon)N(\epsilon')} \sum_{k} \sum_{k'} \sum_{\lambda} |g_{kk'\lambda}| \; B_\lambda(k-k',\Omega)\delta(\epsilon-\epsilon_k)\delta(\epsilon'-\epsilon_{k'}),$$

$B_\lambda(q,\Omega)$ is the phonon spectral weight function for polarization λ, and $g_{kk'\lambda}$ is the electron-phonon coupling function. Assuming, as in the electron-paramagnon problem, that for ϵ and ϵ' within several Ω_{ep} of the fermi level ($\Omega_{ep} << E_F$) $\alpha^2F(\epsilon,\epsilon';\Omega)$ can be set equal to $\alpha^2F(\Omega) \equiv \alpha^2F(0,0;\Omega)$ but the energy dependence of $N(\epsilon')$ cannot be neglected, the result for Σ_{ep} at $T = 0$ is

$$[57] \quad \Sigma_{ep}(\omega + i0^+) = \int_{-\infty}^{\infty} d\omega' \tilde{N}_{ep}(\omega') \int_{0}^{\infty} d\Omega\ \alpha^2F(\Omega) \left[\frac{\theta(\omega')}{\omega-\omega'-\Omega+i0^+} + \frac{\theta(-\omega')}{\omega-\omega'+\Omega+i0^+}\right]$$

where $\tilde{N}_{ep}(\omega')$ is defined as in the electron-paramagnon problem [33] in terms of $\Sigma_{ep}(\omega' + i0^+)$.

The combined electron-paramagnon and electron-phonon self-energy is calculated as

$$[58] \quad \Sigma_{tot}(\omega + i0^+) = \int_{-\infty}^{\infty} d\omega' \tilde{N}_{tot}(\omega') \int_{0}^{\infty} d\Omega\ [P(\Omega) + \alpha^2F(\Omega)]$$

$$\left[\frac{\theta(\omega')}{\omega-\omega'-\Omega+i0^+} + \frac{\theta(-\omega')}{\omega-\omega'+\Omega+i0^+}\right]$$

where $\tilde{N}_{tot}(\omega')$ is defined in terms of $\Sigma_{tot}(\omega' + i0^+)$. However $\Omega_{sf}^{max} >> \Omega_{ep}^{max}$, so the result of calculating Σ_{sf} using Equations [32,33] and Σ_{ep} using Equation [57] and combining the results as

$$[59] \quad \Sigma_{tot}(\omega + i0^+) = \Sigma_{sf}(\omega + i0^+) + \Sigma_{ep}(\omega + i0^+)$$

is not expected to differ much from the result of equation [58].

(F) Normal Metal Tunneling

The transfer Hamiltonian result for elastic tunneling current from metal b to metal a at T = 0, with metal a biased positive is

$$[60] \quad I(V) = \frac{4\pi e}{\hbar} \int_{-eV}^{0} d\omega \int_{-\infty}^{\infty} d\epsilon_a N_a(\epsilon_a) \int_{-\infty}^{\infty} d\epsilon_b N_b(\epsilon_b) |T(\epsilon_a,\epsilon_b)|^2$$

$$\times \; A_a(\epsilon_a,\omega+eV)\; A_b(\epsilon_b,\omega) \;,$$

where V is the applied voltage. Here, $|T(\epsilon_a,\epsilon_b)|^2$ is the matrix element squared for transfer of an electron, and $A(\epsilon,\omega)$ is the electron spectral weight function

$$[61] \quad A(\epsilon,\omega) = \frac{1}{\pi} \frac{|\mathrm{Im}\Sigma(\epsilon,\omega)|}{\{\epsilon - \omega + \mathrm{Re}\Sigma(\epsilon,\omega)\}^2 + \{\mathrm{Im}\Sigma(\epsilon,\omega)\}^2} \;.$$

If we assume that the densities of states are constant, that $|T(\epsilon_a, \epsilon_b)|^2 = |T|^2$, and $\Sigma(\epsilon,\omega) = \Sigma(\omega)$ the expression for current reduces to $I(V) \propto V$. Hermann and Schmid[23] took for the matrix element

$$[62] \quad N_a(\epsilon_a)N_b(\epsilon_b)|T(\epsilon_a,\epsilon_b)|^2 = N_a(0)N_b(0)|T|^2\{1 + \alpha(\frac{\epsilon_a + \epsilon_b}{\mu})\}$$

and estimated that $\alpha \sim 30$, which justified neglecting the ϵ-dependence of $\Sigma(\epsilon,\omega)$. With these approximations they obtained for the odd part of the conductance ($\sigma = dI/dV$; $\sigma_{odd}(V) = \frac{1}{2}[\sigma(V) - \sigma(-V)]$),

$$[63] \quad \sigma_{odd}(V) = \sigma_o\{\frac{-\alpha}{\mu} \mathrm{Re}\Sigma_{odd,a}(eV) + \frac{\alpha}{\mu} \mathrm{Re}\Sigma_{odd,b}(eV)\} \ .$$

where $\sigma_o = 4\pi e^2 \ N_a(o)N_b(o) \ |T|^2/\hbar$.

Based on this result Svistunov et al.[16] derived an inversion formula (see Section (G)) to obtain the electron-phonon spectral function from normal state tunneling measurements. They were able to apply this to Bi, Pb, and Pb-Bi alloys to obtain phonon spectra which agreed well in shape with those obtained from superconductive tunneling, although they did not obtain absolute magnitudes for the spectra. However, it is found experimentally that $\alpha \lesssim 1$, which removes the justification for ignoring the ϵ-dependence of $\Sigma(\epsilon,\omega)$. Also, it has been argued by Appelbaum and Brinkman[24] that the electrons tunnel at their renormalized energies, $\epsilon + \mathrm{Re}\Sigma$, in which case $\sigma_{odd} = 0$. Leavens and Mitrović[25] have recovered Equation [63] for the odd conductance (with a different interpretation of the coefficients) by including the ϵ-dependence of the self-energy. They have considered the ϵ-dependence of $\Sigma(\epsilon,\omega + i0^+)$ which arises from the ϵ-dependence of $\alpha^2F(\epsilon,\epsilon';\Omega)$. They have assumed that for ϵ,ϵ' within several Ω_{max} of the fermi level,

$$[64] \quad \alpha^2F(\epsilon,\epsilon';\Omega) = \alpha^2F(\Omega)[1 + \gamma(\frac{\epsilon + \epsilon'}{\mu})]$$

where γ is on the order of 1. Then from Equation [55] they find, at $T = 0$,

$$[65] \quad \Sigma(\epsilon,\omega + i0^+) = \Sigma(\omega + i0^+) + \frac{\gamma\epsilon}{\mu}\Sigma(\omega + i0^+) + \xi(\omega + i0^+)$$

where $\sum(\omega + i0^+)$ is given by Equation [57], and $\xi(\omega + i0^+)$ is the small (on the order of $\sum(\omega + i0^+)/\mu$) change in the ϵ-independent part of $\sum(\epsilon,\omega + i0^+)$. They corrected the expression for the matrix element to include the renormalized energies as

$$|T|^2\{1 + \frac{\alpha_a}{\mu_a}(\epsilon_a + \text{Re}\sum_a(\epsilon_a,\omega)) + \frac{\alpha_b}{\mu_b}(\epsilon_b + \text{Re}\sum_b(\epsilon_b,\omega - eV))\}$$

for ϵ_a, ϵ_b within several Ω_{max} of the fermi level. They also considered the possibility that the ϵ-dependences of the densities of states are not completely cancelled by a factor $[N_a(\epsilon_a)N_b(\epsilon_b)]^{-1}$ in $|T(\epsilon_a,\epsilon_b)|^2$. To correct for this they have included a factor

$$(1 + \delta_a \frac{\epsilon_a}{\mu_a})(1 + \delta_b \frac{\epsilon_b}{\mu_b})$$

with δ_a, δ_b at most on the order of 1. Then, keeping in mind that these expansions are valid only for ϵ_a, ϵ_b within several Ω_{max} of the fermi level, they obtained for the odd conductance

$$[66] \quad \sigma_{odd}(V) = \sigma_o\{(\frac{\delta_a}{\mu_a} - \frac{\delta_b}{\mu_b})eV - (\frac{\delta_a + \gamma_a}{\mu_a})\text{Re}\sum_{odd,a}(eV)$$

$$- (\frac{\delta_b + \gamma_b}{\mu_b})\text{Re}\sum_{odd,b}(-eV)\} \ .$$

The first term would be subtracted together with the linear background due to barrier asymmetry.[16] Then the result is the same as that of Hermann and Schmidt, except that the coefficients of $\mathrm{Re}\Sigma_{odd}$ are on the order of 1.

We can do the same type of derivation when there is also a contribution to the self-energy from spin fluctuations by replacing $\alpha^2F(\epsilon,\epsilon';\Omega)$ by $P(\epsilon,\epsilon';\Omega)$. However, as we have seen in Section (D), the ϵ-dependence of $\Sigma_{sf}(\epsilon,\omega + i0^+)$ may be dominated by the effects of vertex corrections. Then the ϵ-dependence of $\Sigma_{sf}(\epsilon,\omega + i0^+)$ does not arise solely through the ϵ-dependence of $P(\epsilon,\epsilon';\Omega)$. In addition, the second term in Equation [53] may not be negligible compared to the first, and we may not be able to make an expansion of $P(\epsilon,\epsilon';\Omega)$ like that for $\alpha^2F(\epsilon,\epsilon';\Omega)$ in Equation [64]. We will assume that for $\epsilon << E_F$, we can write

$$[67] \quad \Sigma(\epsilon,\omega + i0^+) = \Sigma(0,\omega + i0^+) + \beta(\omega)\epsilon$$

where

$$[68] \quad \beta(\omega) = \frac{\partial\Sigma(\epsilon = 0,\omega + i0^+)}{\partial\epsilon} = \beta^{(1)}(\omega) + i\beta^{(2)}(\omega) \ ,$$

$\beta^{(1)}(\omega)$ and $\beta^{(2)}(\omega)$ are real. We will assume that $\beta^{(1)}(\omega),\beta^{(2)}(\omega) << 1$, so that in expanding the electron spectral weight functions we will retain only the terms up to first order in β's. As before, the parameter α from the

expansion of the matrix element does not enter the expression for $\sigma_{odd}(V)$. Then taking the matrix element and the densities of states as constant, the zero temperature conductance is

$$[69] \quad \sigma(V) = \sigma_o\{1 - \beta_a^{(1)}(\omega) - \beta_b^{(1)}(-\omega)\} \ .$$

The result for the odd conductance reduces to Equation [66] if $\partial\sum(\epsilon = 0, \omega + i0^+)/\partial\epsilon = \frac{\gamma}{\mu}\sum(\omega + i0^+)$.

(G) Inversion of Normal Metal Tunneling Data

Information about the electron-phonon and -paramagnon interactions is contained in the data from normal metal-insulator-normal metal junction tunnel current measurements. We will assume that $\sigma_{odd}(V)$ is given by Equation [66], that is that the ϵ-dependence of $\sum(\epsilon,\omega + i0^+)$ comes only through the ϵ-dependence of $G(\epsilon,\epsilon';\Omega)$ (G is α^2F for the electron-phonon interaction, P for the electron-spin fluctuation interaction.), and that we can expand $G(\epsilon,\epsilon';\Omega)$ as in Equation [64] for ϵ,ϵ' within several Ω_{max} of the fermi level. For simplicity, we will assume that $\alpha^2F_a(\Omega) >> \alpha^2F_b(\Omega)$ for $\Omega \leq \Omega_a^{max}$, that $(\delta_b + \gamma_b)/\mu_b \lessapprox (\delta_a + \gamma_a)/\mu_a$, and that the electron-paramagnon interaction is negligible compared to the electron-phonon interaction in both metals. In addition it is assumed that the linear term in σ_{odd} has been subtracted with the linear background due to barrier asymmetry. Then the odd conductance is

$$[70] \quad \sigma_{odd}(V) = -\sigma_o \frac{c_a}{\mu_a} \mathrm{Re}\, \sum\nolimits^{(o)}_{odd,a}(eV)$$

where c_a is less than or on the order of 1.

The inversion formula of Svistunov et al.[16] can be derived from the expression for the self-energy at $T = 0$, calculated when the energy dependence of $N(\epsilon')$ and $\alpha^2F(\epsilon,\epsilon';\Omega)$ can be ignored for ϵ and ϵ' within several Ω_{max} of the fermi level.

$$[71] \quad \sum\nolimits^{(o)}(\omega+i0^+) = \int_{-\infty}^{\infty} d\omega' \int_{o}^{\infty} d\Omega\, \alpha^2F(\Omega) \left[\frac{\theta(\omega')}{\omega - \omega' - \Omega + i0^+} + \frac{\theta(-\omega')}{\omega - \omega' + \Omega + i0^+}\right]$$

For this quantity, we have the following results:

$$[72] \quad \mathrm{Re}\Sigma^{(o)}(\omega) = -\mathrm{Re}\Sigma^{(o)}(-\omega)$$

$$[73] \quad \mathrm{Im}\Sigma^{(o)}(\omega) = -\pi\int_{o}^{|\omega|} d\Omega\ \alpha^2F(\Omega)$$

$$[74] \quad \mathrm{Im}[\Sigma^{(o)}(\omega) - \Sigma^{(o)}(\infty)] = \frac{2}{\pi}\int_{o}^{\infty} d\omega'\ \frac{\mathrm{Re}\Sigma^{(o)}(\omega')\omega'}{\omega^2 - \omega'^2} \quad .$$

The dispersion relation [74] makes use of Equation [72]. If the ϵ'-dependence of $N(\epsilon')$ is negligible in the important range of ϵ' values, i.e. if $\Sigma(\omega) = \Sigma^{(o)}(\omega)$, then Equation [70] can be combined with Equations [73] and [74] to obtain the inversion formula

$$[75] \quad \alpha^2F(\omega) = \frac{2\mu_a\omega}{c_a\sigma_o\pi^2}\int_{o}^{\infty} \frac{d\sigma_{odd}(\omega')}{d\omega'}\ \frac{d\omega'}{\omega^2 - \omega'^2} \quad .$$

When the ϵ'-dependence of $N(\epsilon')$ cannot be neglected in the important range of ϵ' values, we have

$$[76] \quad \mathrm{Im}\,\Sigma_{even}(\omega) = -\pi\int_{o}^{|\omega|} d\omega'\tilde{N}_{even}(\omega')\ \alpha^2F(|\omega| - \omega') \quad .$$

$\mathrm{Re}\Sigma(\omega)$, which is calculated neglecting only the energy dependence of

$\alpha^2F(\epsilon,\epsilon';\Omega)$, is even only if $N(\epsilon')$ is symmetric about $\epsilon' = 0$. If $Re\sum(\omega)$ is not even, [74] is replaced by the more general dispersion relation

$$[77] \quad Im[\sum(\omega) - \sum(\infty)] = \frac{1}{\pi} \int_{-\infty}^{\infty} d\omega' \frac{Re\sum(\omega')}{\omega - \omega'} .$$

The result of using the inversion formula [75] when the energy dependence of $N(\epsilon')$ is not negligible is an effective spectrum

$$[78] \quad \alpha^2F_{eff}(\Omega) = \frac{-1}{\pi} \frac{d}{d\Omega} Im \sum_{even}(\Omega) .$$

It has been found that when $N(\epsilon')$ has a peak in the density of states near the fermi level, $\alpha^2F_{eff}(\Omega)$ has a negative high frequency tail and a shift of weight to lower frequencies.[25]

III. Numerical Results

(A) Paramagnon Spectral Function

The electron-paramagnon spectral function $P(\Omega)$, calculated with $S = 14$, in the model with no momentum cut-off is shown in Figure (3) as a function of $\bar{\Omega} = \Omega/E_F$. It is necessary to fix the value of E_F to scale the horizontal frequency axis. As pointed out by Stenzel and Winter,[12] there is no agreement on the value of the parameter E_F; in the paramagnon theory, values ranging from 0.25 eV to 0.9 eV have been used in calculations. We will take $E_F = 0.7$ eV, which is the width of the peak in the density of states about the fermi level. With this choice of E_F, the peak in $P(\Omega)$ occurs at $\Omega_p = 0.14$ eV $>> \Omega_{ep}$, and $P(\Omega)$ is non-zero up to $\Omega_{max} = 5.6$ eV. The paramagnon mass renormalization parameter, which does not depend on E_F, but only on S, is $\lambda_{sf} = 5.3$. (To calculate the mass renormalization we have assumed that $[\partial\sum(\vec{k},i0^+)/\partial\epsilon_{\vec{k}}]_{\vec{k}=k_F\hat{k}}$ is small compared to 1, which is the usual assumption. This may not be a bad approximation for the nearly full band, however when $\bar{p}_c \to \infty$, $[\partial\sum(\vec{k},i0^+)/\partial\epsilon_{\vec{k}}]_{\vec{k}=k_F\hat{k}}$ may be large. See page 50.) It was to obtain a more reasonable value of λ_{sf} that Schrieffer introduced the q-dependent Stoner factors,[4] which act to decrease the peak height slightly in $P(\Omega)$, without affecting the position of the peak much. In the finite bandwidth model, with $\bar{p}_c = p_c/p_F = 1.02$ and $S = 14$, the magnitude, the range and the peak frequency of $P(\Omega)$ are drastically reduced (Fig. (3)). By examining the expressions for Im $u(\bar{q},\bar{\Omega} + i0^+)$ given in the Appendix, we can see that $P(\Omega)$ increases linearly up to the peak frequency $\Omega_p \simeq (\bar{p}_c^2 - 1)E_F$,

and is non-zero up to $\Omega_{max} = \bar{p}_c^2 E_F$, for $\bar{p}_c - 1 << 1$. Taking $E_F = 0.7$ eV, we get $\Omega_p = 28$ meV $\simeq \Omega_{ep}$ and $\Omega_{max} = 728$ meV. However, in this model, $P(\Omega) << \alpha^2F(\Omega)$ for frequencies less than the maximum phonon frequency (Fig. (4)), so this model does not predict a large cancellation of $\alpha^2F(\Omega)$ against $P(\Omega)$ in the proximity effect tunneling. There is a large reduction in the mass renormalization parameter, $\lambda_{sf} = 2\int_0^\infty d\Omega\ P(\Omega)/\Omega$, to 0.113. (The true mass renormalization is even smaller--see page 51.)

The spectral density $B(\bar{q},\bar{\Omega})$ is plotted in Figure (5) as a function of $\bar{q}$ for several values of $\bar{\Omega}$, for $\bar{p}_c = 1.02$ and in the inset, for $\bar{p}_c \to \infty$. The effect of the finite momentum cut-off is to shift the peak to a lower $\bar{q}$ value for fixed $\bar{\Omega}$, and to reduce the width of the peak from that in the case with no momentum cut-off. The peak heights also decrease sharply with increasing $\bar{\Omega}$. The spectral density $B(\bar{q},\bar{\Omega})$ is proportional to the dynamic susceptibility $\chi_s(q,q,\Omega)$. In a detailed calculation of $\chi_s(q,q,\Omega)$, Stenzel and Winter[12] have found that the important contribution to the dynamic susceptibility is from the frequency range of the same order of magnitude as Ω_{ep} for Pd. Therefore the inclusion of a simple momentum cut-off reproduces the main result of Stenzel and Winter's work.

Figure 3

The electron-paramagnon spectral function $P(\bar{\Omega})$ for S = 14 in the usual theory (•-•-•- line), and in the finite bandwidth model for $\bar{p}_c = 1.02$ (——— line). ($\bar{\Omega} = \Omega/E_F$)

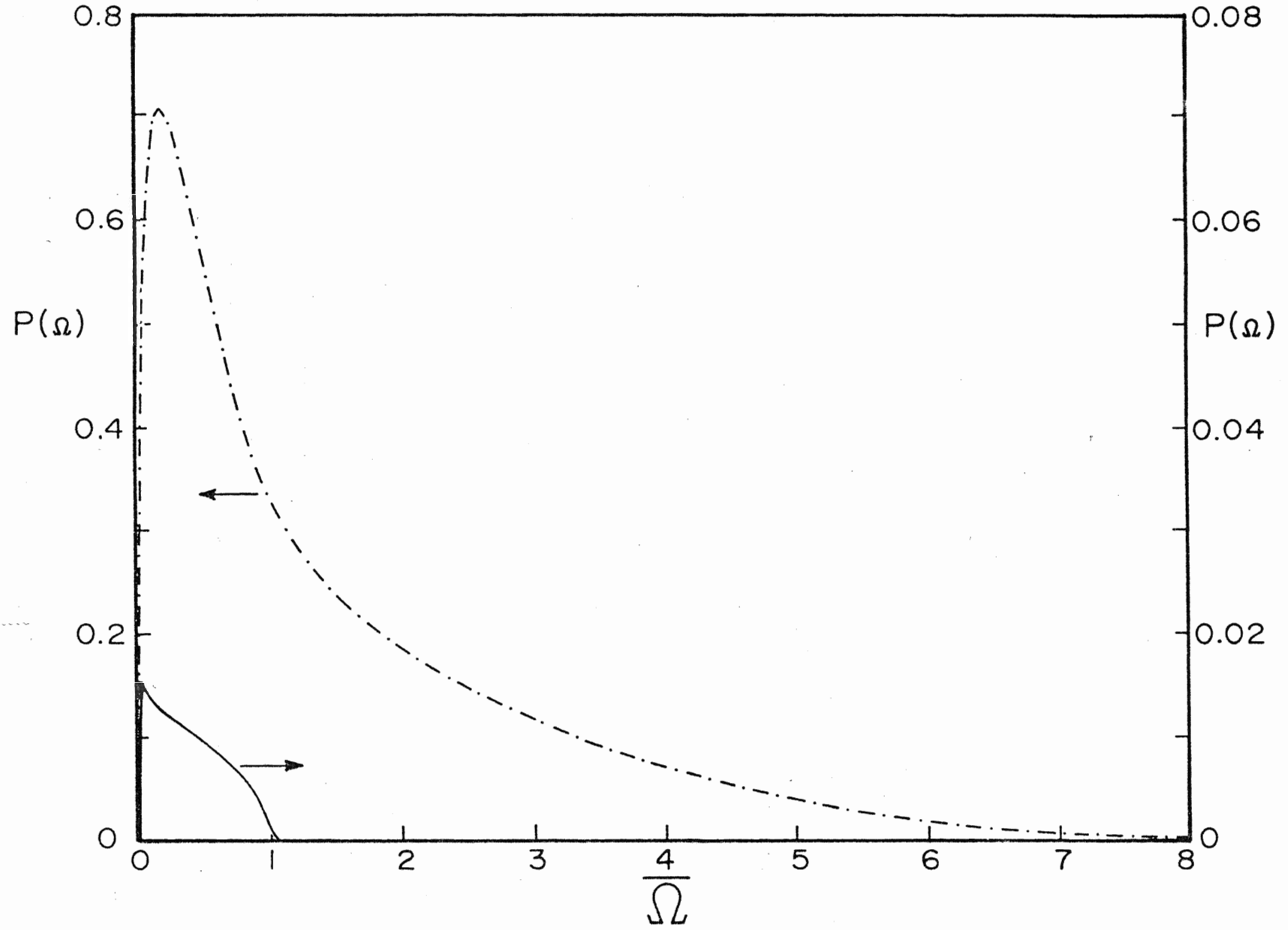

0.8
0.6
0.4
0.2
0
P(Ω)
0.08
0.06
0.04
0.02
0
P(Ω)
0
1
2
3
4
5
6
7
8
Ω

Figure 4

The electron-paramagnon spectral function $P(\Omega)$ calculated within the finite bandwidth model with $\bar{p}_c = 1.02$, $S = 14$, and $E_F = 0.7$ eV together with the calculated electron-phonon spectral function $\alpha^2F(\Omega)$ for Pd from Reference (9) (——— line) and the calculated $\alpha^2F(\Omega)$ for Cu from Reference (26) (----- line).

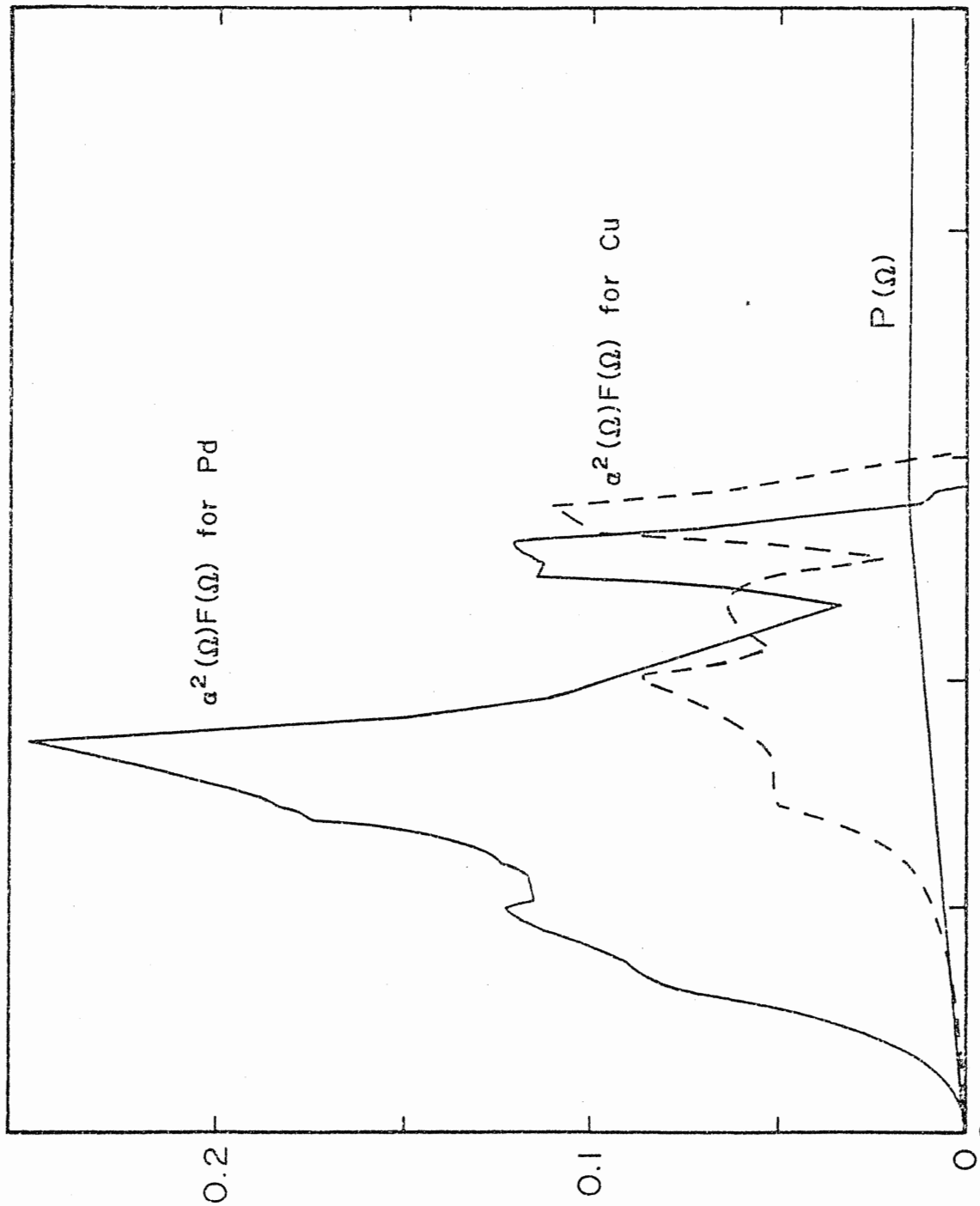
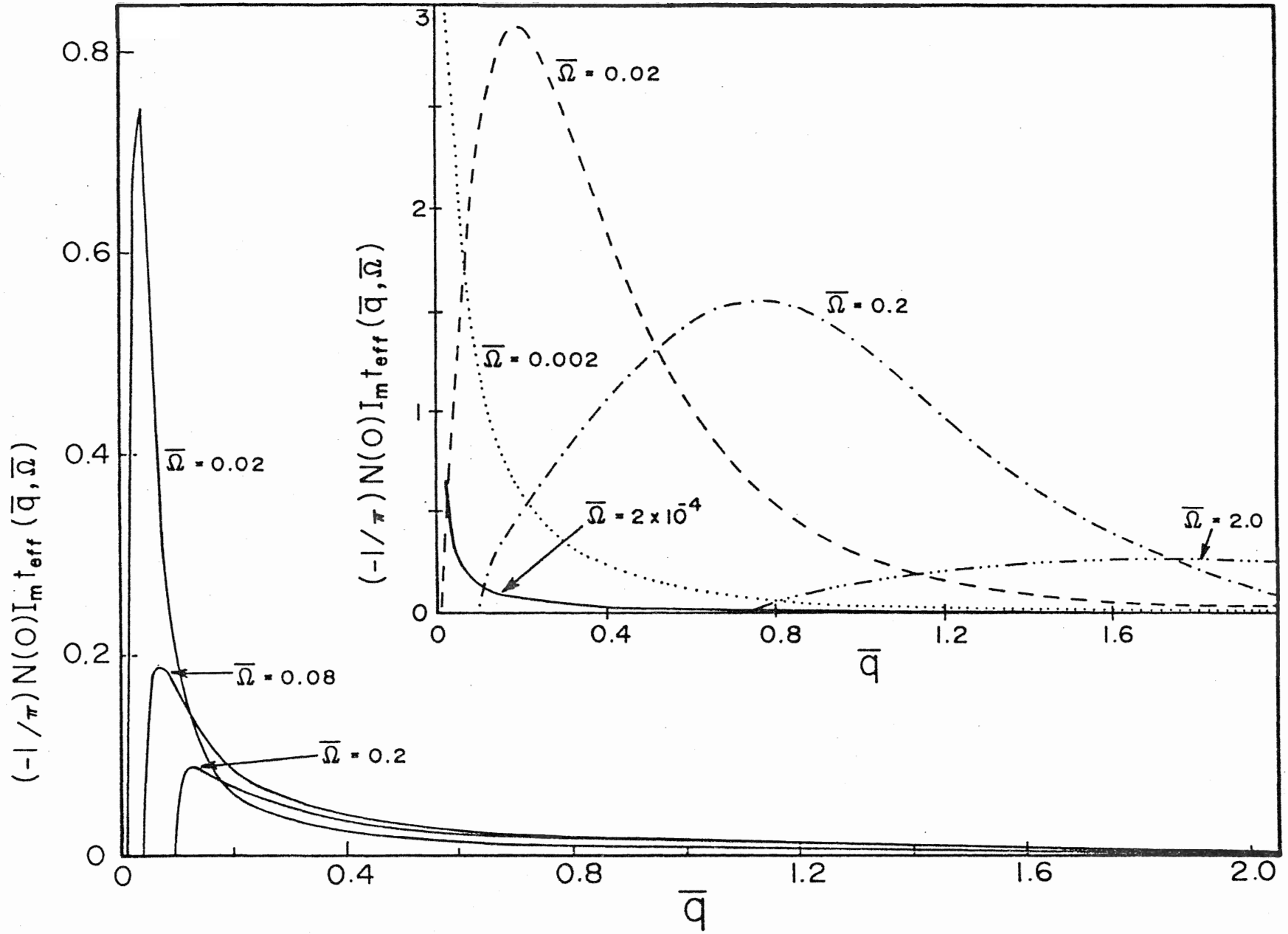

0.2
0.1
0
0
20
40
Ω (meV)
α²(Ω)F(Ω) for Pd
α²(Ω)F(Ω) for Cu
P(Ω)

Figure 5

The imaginary part of t_{eff} as a function of $\bar{q} = q/p_F$ for several values of $\bar{\Omega} = \Omega/E_F$ for $\bar{p}_c = 1.02$, and in the inset for the case with no momentum cut-off.


$(-1/\pi)\,N(0)\,I_m\,t_{eff}\,(\bar{q},\bar{\Omega})$
$\bar{q}$
$\bar{\Omega} = 0.02$
$\bar{\Omega} = 0.08$
$\bar{\Omega} = 0.2$
$\bar{\Omega} = 0.002$
$\bar{\Omega} = 2 \times 10^{-4}$
$\bar{\Omega} = 2.0$

(B) Vertex Corrections and the Momentum Dependence of Σ_{sf}

The expressions for $\Gamma^{(1)}$, Equations [45] and [46], were evaluated numerically in the case of $\bar{p}_c \to \infty$, and for $\bar{p}_c = 1.02$. In their evaluation of the lowest order vertex correction in the case with no momentum cut-off, Hertz et al.[19] considered an expression containing only the first term in Equation [46] for $\Gamma^{(1)}$. However, the results of the numerical calculations for $\bar{p}_c \to \infty$ are -0.61 for the first term and 0.20 for the second term in Equation (46), so the second term is not negligible compared to the first. For $\bar{p}_c \to \infty$, we have $\Gamma^{(1)} = -0.41$ which is not negligible compared to 1. In the finite bandwidth model, with $\bar{p}_c = 1.02$, $\Gamma^{(1)}$ is reduced to -0.029 which is negligible compared to the bare vertex and of the same order of magnitude as the first order vertex correction in the electron-phonon problem. Note that the first order vertex correction for the nearly full band is small even though $\bar{I} = IN(o) \simeq 1$. For the nearly full band ($\bar{p}_c - 1 \ll 1$), we have $\Omega_{sf} \ll E_F$ and $\epsilon_c \ll E_F$, so we expect that the higher order vertex corrections are also negligible.

From this we can expect that for $\bar{p}_c = 1.02$ the momentum dependence of the self-energy is small. The upper limit of the $\bar{q}$ integration in Equation [51] is 2.02, so the additional factor $[1 + (\hat{k}\cdot\vec{q})/k_F]$ will not make $\partial\Sigma(\vec{k} = k_F\hat{k}, i0^+)/\partial\epsilon_{\vec{k}}$ comparable to 1. By comparing Figures (4) and (5) we can see that the second term in Equation [53] is negligible compared to the first, and so we can make an expansion of the type in Equation [65] for

$\sum_{sf}(\epsilon,\omega + i0^+)$, with γ_{sf} on the order of 1. Then for the nearly full band, the odd conductance will contain $\sum_{sf,odd}(\omega + i0^+)$. For $\bar{p}_c \to \infty$, $\Gamma^{(1)}$ is not negligible compared to 1, and the upper limit of the $\bar{q}$ integration in Equation [51] is ∞. This may make the result of Equation [51] for $\partial\sum(\vec{k} = k_F\hat{k},i0^+)/\partial\epsilon_{\vec{k}}$ very large. (MacDonald[13] says that this derivative diverges when $\bar{p}_c \to \infty$). However Equation [51] is now a poor approximation for $\partial\sum(\vec{k} = k_F\hat{k},i0^+)/\partial\epsilon_{\vec{k}}$ because vertex corrections are important in the calculation of $\sum(\epsilon,\omega + i0^+)$ when $\bar{p}_c \to \infty$. We also note, by comparing Figures (4) and (5), that the second term of Equation [53] is not negligible compared to the first, and so we cannot make an expansion of the type in Equation [65] for $\sum_{sf}(\epsilon,\omega + i0^+)$ when $\bar{p}_c \to \infty$. The odd conductance then contains $\beta^{(1)}_{odd}(\omega) = [\partial Re\sum(\epsilon = 0,\omega + i0^+)/\partial\epsilon]_{odd}$.

(C) Electron-Spin Fluctuation Self-Energy

The self-energy due to electron-spin fluctuation interaction in the finite bandwidth model is shown in Figures (6) and (7), together with the self-energy calculated by taking $\tilde{N}_{sf}(\omega') = 1$ in Equation [32] for comparison. The self-energy was calculated with $S = 14$, $\bar{p}_c = 1.02$ and $E_F = 0.7$ eV. When the density of states is symmetric about the fermi level, $\mathrm{Re}\Sigma_{sf}(\omega + i0^+)$ is an odd function of ω, and $\mathrm{Im}\,\Sigma_{sf}(\omega + i0^+)$ is an even function of ω. In the finite bandwidth model, where the density of states (Equation [31]) is not symmetric about the fermi level, $\mathrm{Re}\Sigma_{sf}(\omega + i0^+)$ is not an odd function of ω, and $\mathrm{Im}\Sigma_{sf}(\omega + i0^+)$ is not an even function of ω. In addition, the asymmetry in the density of states causes a shift in the true interacting chemical potential[22] of $\delta\mu = \mathrm{Re}\Sigma_{sf}(0 + i0^+) = 8.5$ meV. With a non-constant density of states, we no longer have $(m^*/m - 1) = 2\int_0^\infty d\Omega\, P(\Omega)/\Omega$. The true effective mass enhancement is given by $m^*/m = [1 - \partial\Sigma_{sf}(0,0)/\partial\omega]/[1 + \partial\Sigma_{sf}(0,0)/\partial\epsilon]$. We have already argued that we can neglect the ϵ-dependence of $\Sigma_{sf}(\epsilon,\omega + i0^+)$ near $\epsilon = 0$ for the nearly full band. The true effective mass enhancement due to spin fluctuations is $(m^*/m) = [1 - \partial\Sigma_{sf}(0,0)/\partial\omega] = 1.07$, which is very close to the value obtained by MacDonald[13] of 1.05. Note that the result of choosing a different value for the fermi energy, E_F', is to rescale both the horizontal and vertical axes of Figures (6) and (7) by E_F'/E_F. So with a different choice for the fermi energy, the effective mass enhancement is unchanged, while $\delta\mu$ is rescaled by E_F'/E_F. Also note that using a different

functional form for $N(\epsilon)$, for example the parabolic density of states $N(\epsilon) = N(0)(1 + \epsilon/E_F)^{1/2}\theta(\epsilon_c - \epsilon)\theta(\epsilon + E_F)$, affects $\Sigma_{sf}(\omega + i0^+)$ only for large ω, and does not alter much the numerical value of m^*/m.

The effective paramagnon spectrum $P_{eff}(\Omega)$ in Figure (8) is the result of using the inversion method of Svistunov et al.,[16] which assumes a constant density of states, to invert the self-energy which was calculated in the finite bandwidth model. This is the effective spectrum which would yield the input $Re\Sigma_{odd}(\omega)$ (Fig. (9)), if used in Equation [26], which assumes a constant density of states. The effective spectrum has the negative high frequency tail and shift of weight to lower frequencies which are characteristic of a peak in the density of states near the fermi level.[25]

Figure 6

The real part of the self-energy due to electron-spin fluctuation interaction in the finite bandwidth model with $\bar{p}_c = 1.02$, $S = 14$ and the density of states given by Equation [31] with $E_F = 0.7$ eV (•-•-•- line). The solid line was obtained using the same $P(\Omega)$, but assuming a constant density of states.

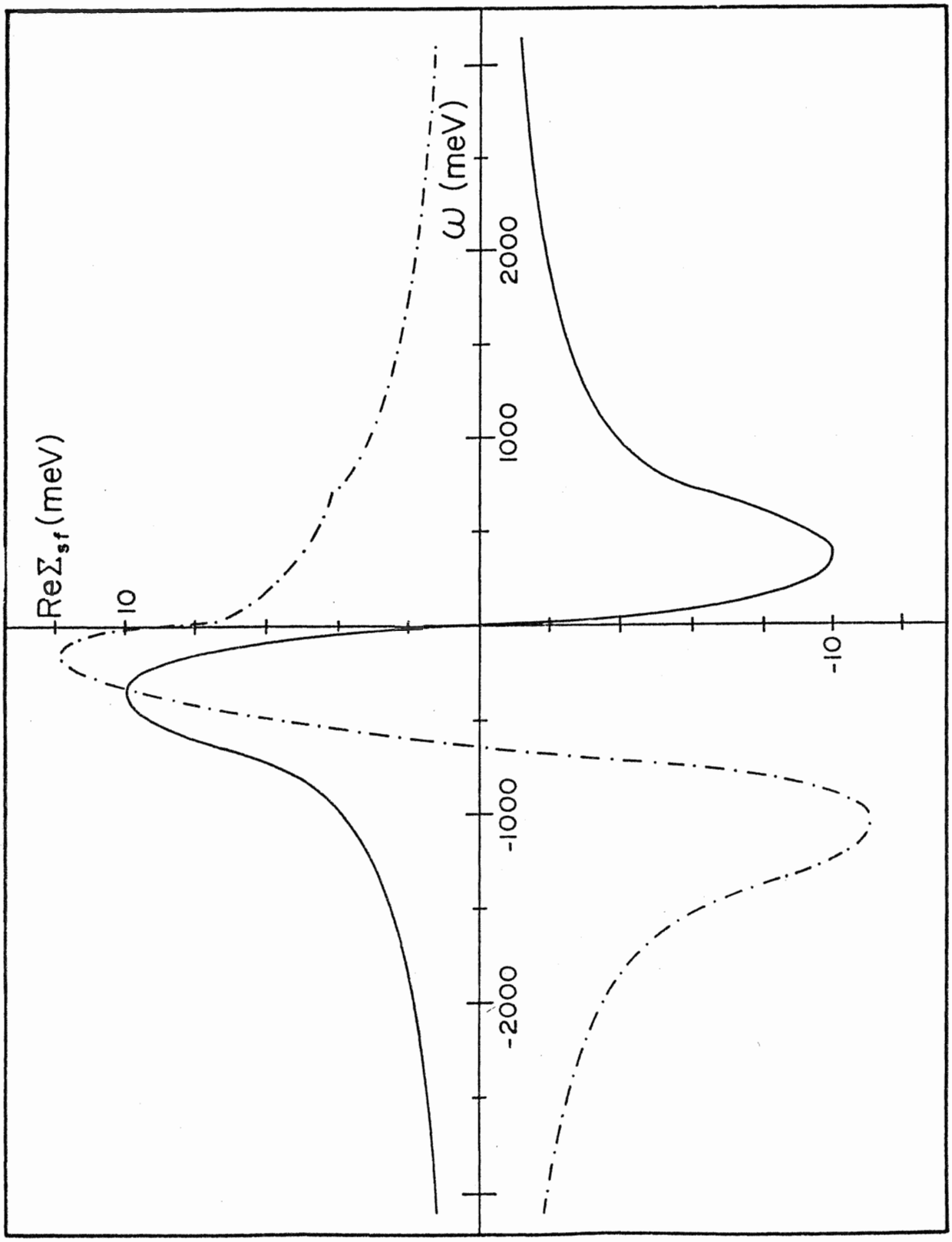

ReΣ_{sf} (meV)
10
ω (meV)
2000
1000
-10
-1000
-2000

Figure 7

The imaginary part of the electron self-energy due to spin fluctuations calculated for the same parameters as in Figure (6).

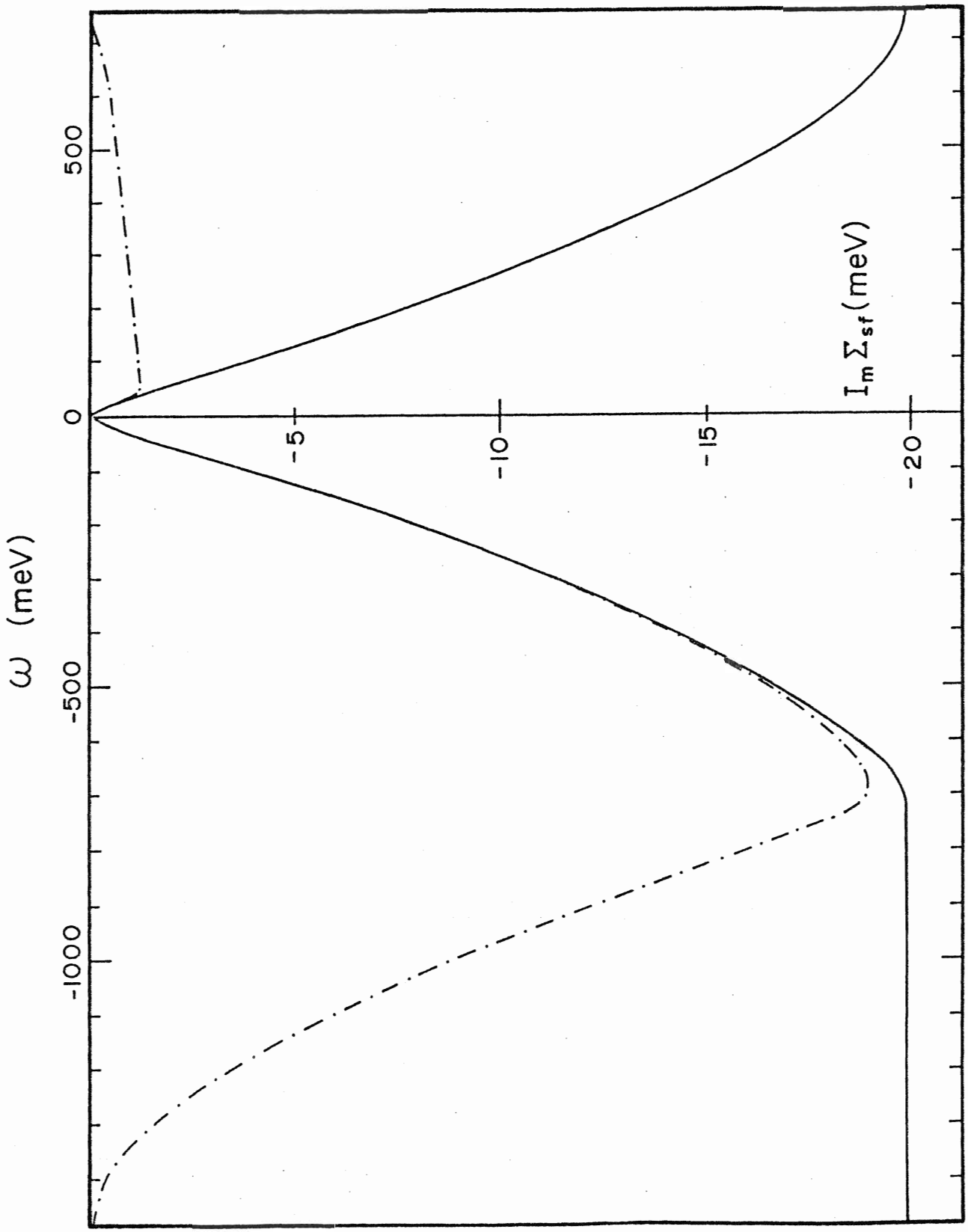

500
0
-500
-1000
ω (meV)
-5
-10
-15
-20
$I_m \Sigma_{sf}$ (meV)

Figure 8

The effective paramagnon spectrum obtained by using the inversion method of Svistunov et al.[16] to invert the self-energy calculated in the finite bandwidth model (•-•-•- line) together with $P(\Omega)$ (——— line).

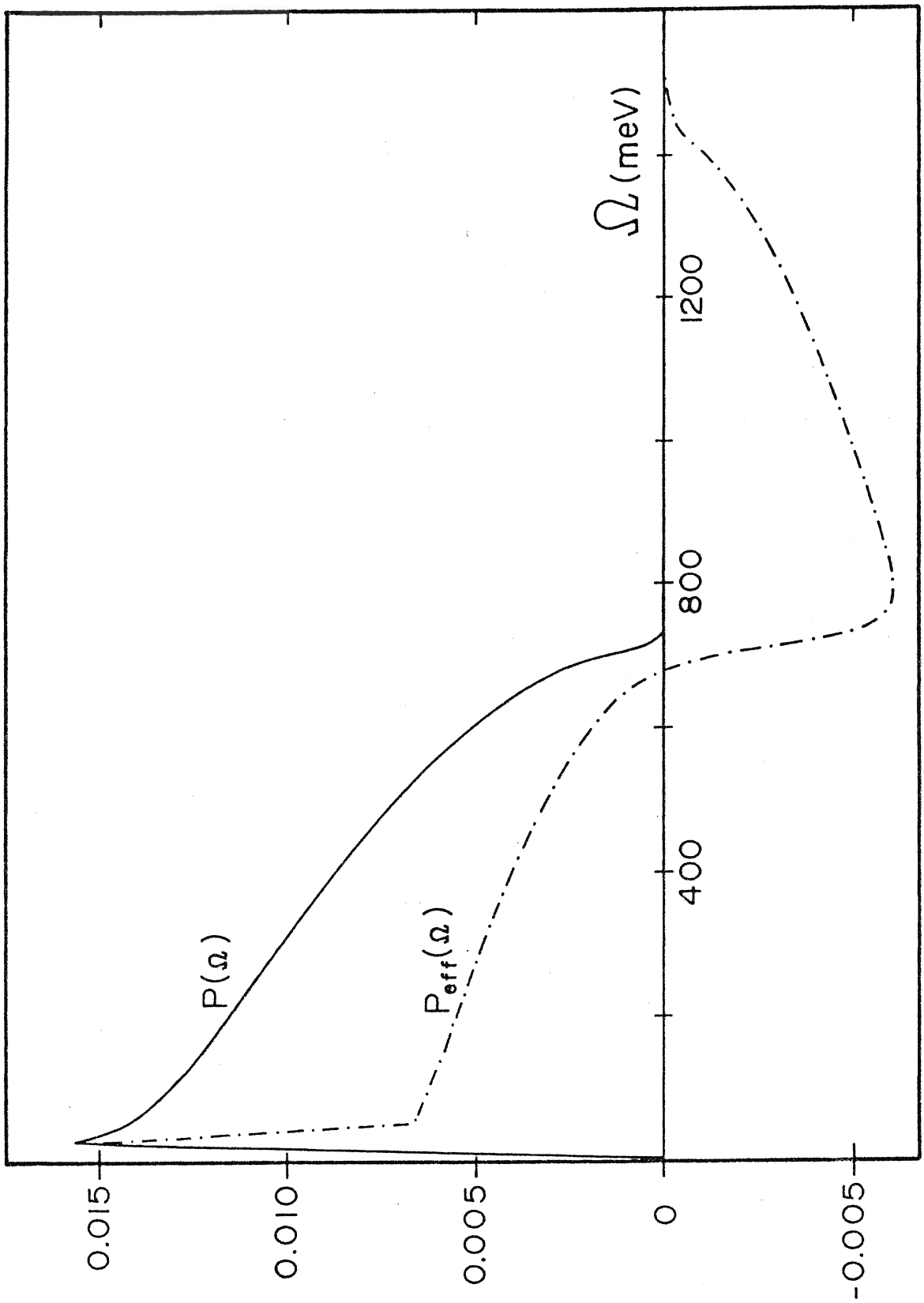
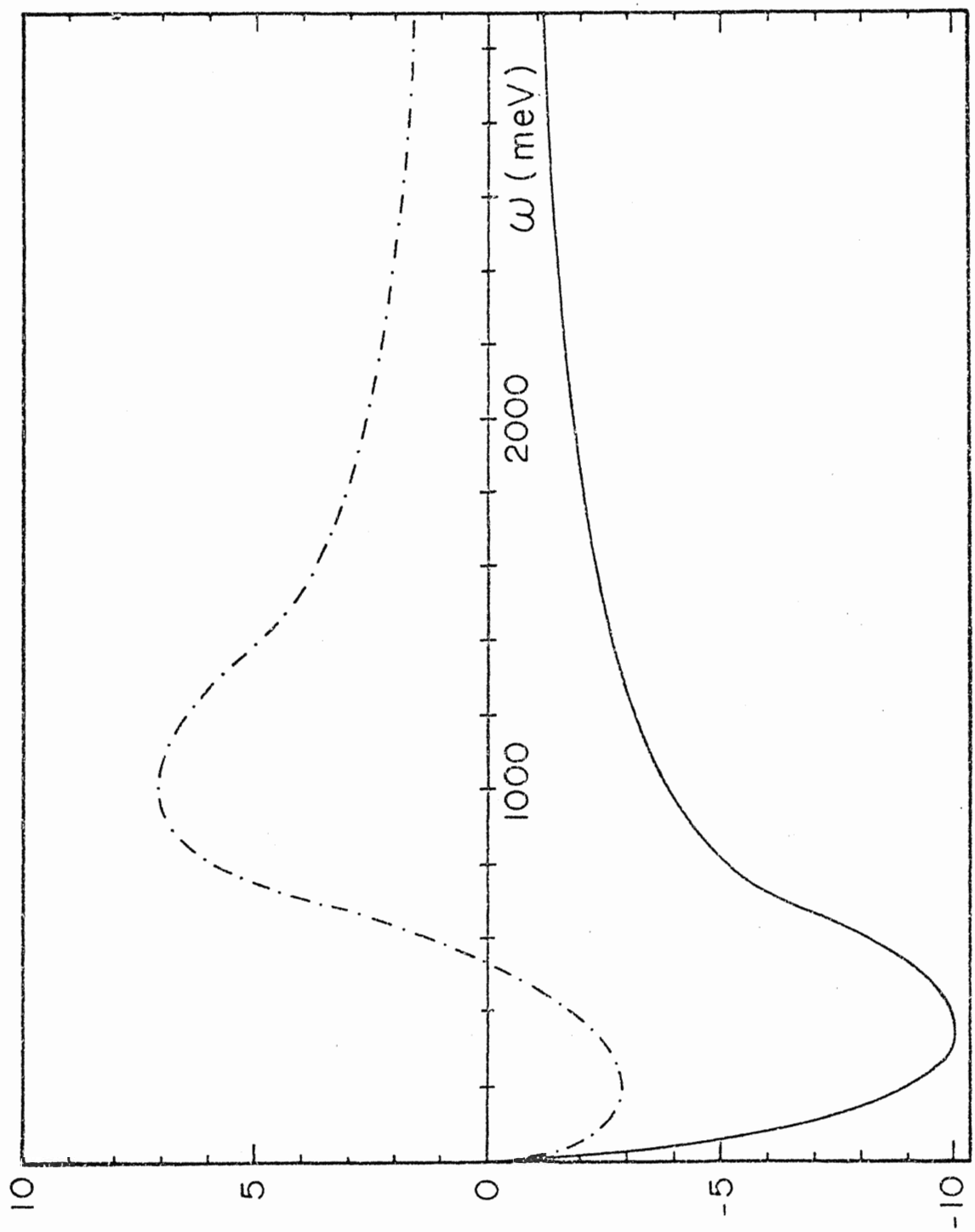

0.015
0.010
0.005
0
-0.005
400
800
1200
Ω (meV)
P(Ω)
P_eff(Ω)

Figure 9

The real part of the odd self-energy calculated in the finite bandwidth model (•-•-•- line) which was inverted to obtain the effective spectrum in Figure (8). The solid line is the real part of the odd self-energy calculated using the same $P(\Omega)$, but assuming a constant density of states.


Re $\Sigma_{st,odd}(\omega+i0^+)$ (meV)
10
5
0
-5
-10
1000
2000
ω (meV)

(D) Electron-Phonon Self-Energy

The electron-phonon self-energy for Pd was calculated using the calculated $\alpha^2F(\Omega)$ from Reference (9), with $\lambda_{ep} = 0.41$ (see Fig. (4)). The calculation was done using the density of states given in Equation [31], with $\epsilon_c = (\bar{p}_c^2 - 1)E_F$, $\bar{p}_c = 1.02$, and $E_F = 0.7$ eV, and for $E_F = 5.5$ eV, which is the width of the d-band in Pd. The real part of $\sum_{ep}$ is shown in Figure (10), together with the results of the calculation using a constant density of states, $N(\epsilon) = N(0)$. The electron-phonon self-energy does not have the same scaling property as the electron-spin fluctuation self-energy, because α^2F is not a function of the reduced energy $\bar{\Omega} = \Omega/E_F$, but is only a function of Ω, independent of E_F. For $E_F = 0.7$ eV, the true interacting effective mass is reduced to $(m^*/m) = 1.34$ from its value of 1.41 using the constant density of states, and the shift in the chemical potential is $\delta\mu = 1.06\% \times E_F$. For the larger fermi energy, $E_F = 5.5$ eV, $m^*/m = 1.40$ and $\delta\mu = .15\% \times E_F$. The real part of $\sum_{ep,odd}$, which enters the expression for $\sigma_{odd}(V)$, is in Figure (11).

Figure 10

The real part of the electron self-energy due to phonons, calculated assuming a constant density of states (•••••• line), and using the density of states in Equation [31] with $\bar{p}_c = 1.02$, for $E_F = 0.7$ eV (——— line) and $E_F = 5.5$ eV (----- line).

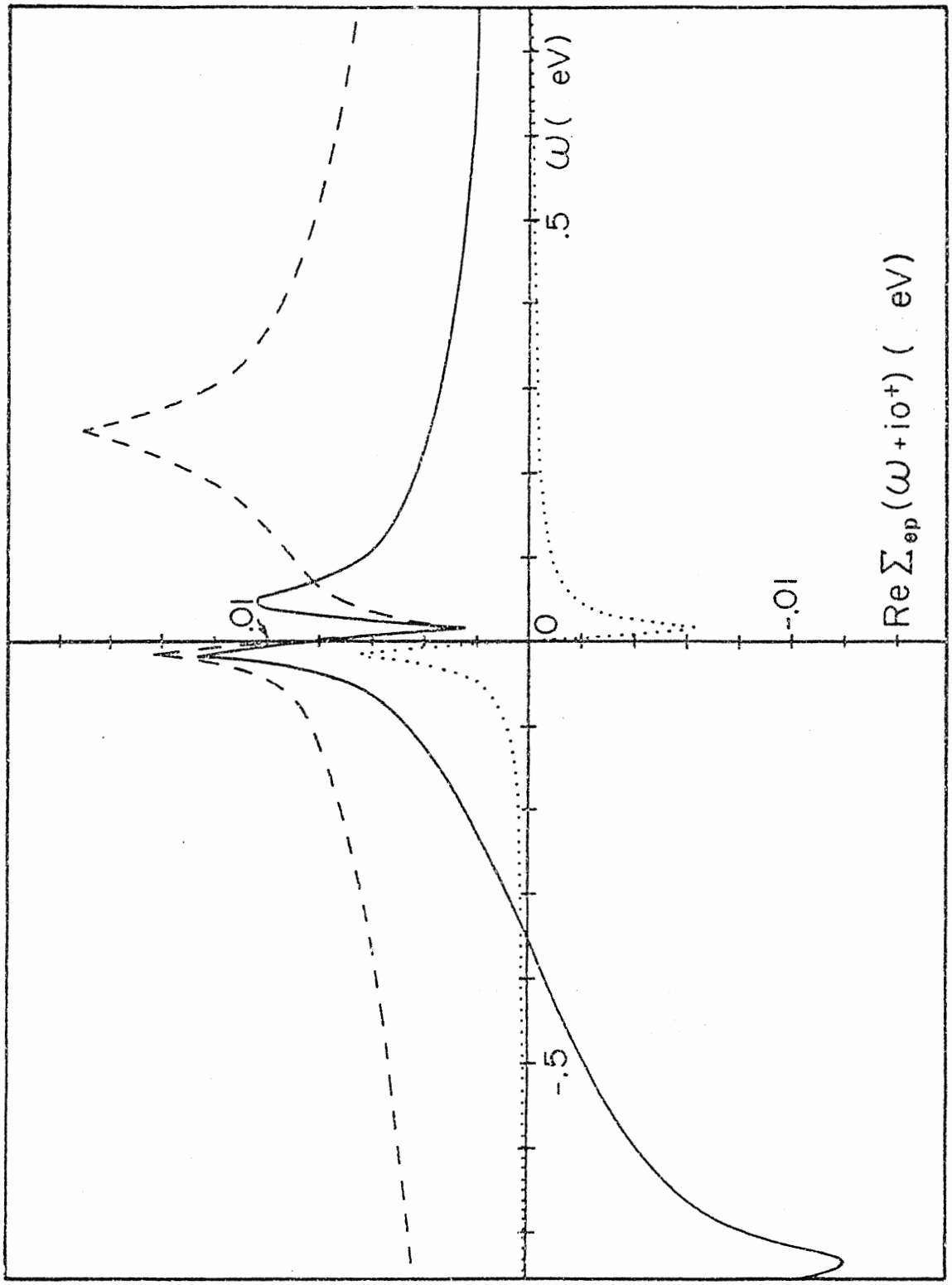
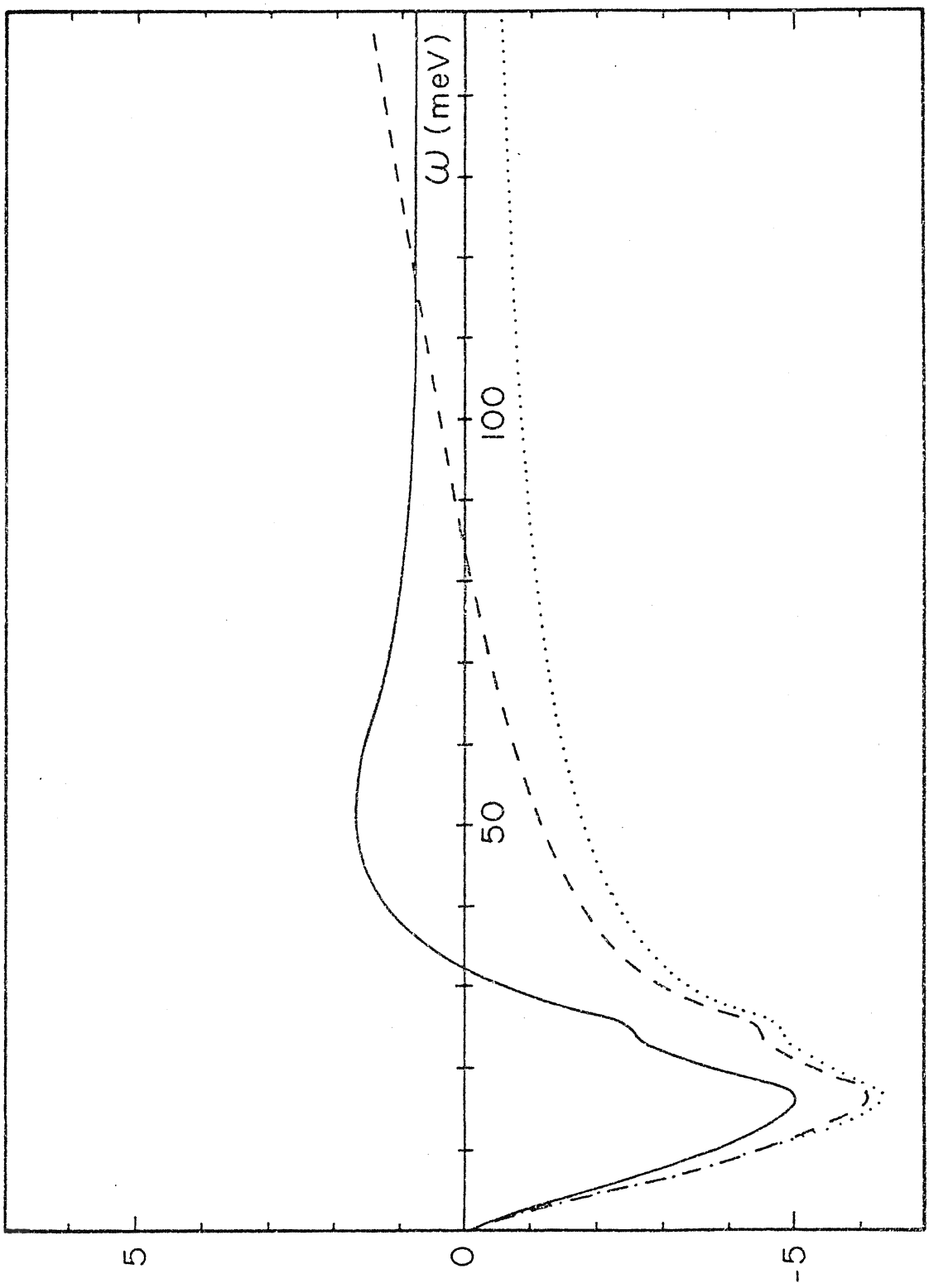

.5
ω (eV)
0
-.01
.01
-.5
Re Σ_ep (ω + iO+) (eV)

Figure 11

The real part of the odd self-energy due to phonons, calculated for the same parameters as in Figure (10).


Re $\Sigma_{ep,odd}(\omega+i0^+)$ (meV)
5
0
-5
50
100
ω (meV)

(E) Cu-I-Pd Tunneling

We will calculate the odd conductance of a Cu-I-Pd junction in order to see what effect the finite bandwidth model for spin fluctuations predicts. We have selected Cu for this calculation because $\alpha^2F_{Cu}(\Omega) << \alpha^2F_{Pd}(\Omega)$ for frequencies much less than the maximum phonon frequency in Cu. As we have seen, for $\bar{p}_c = 1.02$ the odd conductance contains a term $\Sigma_{sf,odd}(\omega + i0^+)$. The odd conductance of the Cu-I-Pd junction, after the linear background has been subtracted is

$$[79] \quad \sigma_{odd}(V) = c_1 Re\Sigma^{Cu}_{ep,odd}(eV) - c_2 Re\Sigma^{Pd}_{sf,odd}(eV) - c_3 Re\Sigma^{Pd}_{ep,odd}(eV)$$

for Pd biased positive, with $c_1 \sim c_2 \sim c_3$. The electron self-energy due to phonons for Cu was calculated using the $\alpha^2F(\Omega)$ for Cu [26] in Figure (4), assuming a constant density of states. As we have noted, the value of the parameter E_F in the paramagnon theory is uncertain. For the calculation of $\sigma_{odd}(V)$, we will use $Re\Sigma_{sf}$ and $Re\Sigma_{ep}$ calculated for $E_F = 5.5$ eV, because with $E_F = 0.7$ eV the conductance is reduced unrealistically by the narrow gap between the fermi level and the top of the band (28 meV) which is available for tunneling. The odd conductance calculated using Equation [79] with different ratios c_2/c_3 (assuming $c_1 = c_3 = 1$) is shown in Figure (12). We can compare this with the result of Rowell's experiment for the conductance of Al-I-Pd normal metal tunnel junction[17] (Fig. (13)). It is difficult to distinguish the calculated $Re\Sigma_{sf}(eV)$ from a linear background in the voltage

range which Rowell has measured. The structure in the experimental odd conductance occurs in the range of phonon frequencies in Pd, however the effect has the opposite sign from what we have calculated. We have assumed that $c_1, c_2, c_3 > 0$, although their signs are determined by the sign of $\partial\sum(\epsilon,\omega + i0^+)/\partial\epsilon$, and may be negative. The effective spectra obtained by using the inversion formula [75] to invert the odd conductances in Figure (12) are in Figure (14). Even for the largest ratio $c_2/c_3 = 10$, the effect of spin fluctuations is not evident at low frequencies against the structure from α^2F for Pd and Cu.

Figure 12

The odd conductance of Cu-I-Pd junction calculated using Equation [70] for different ratios c_2/c_3 (assuming $c_1 = c_3$).

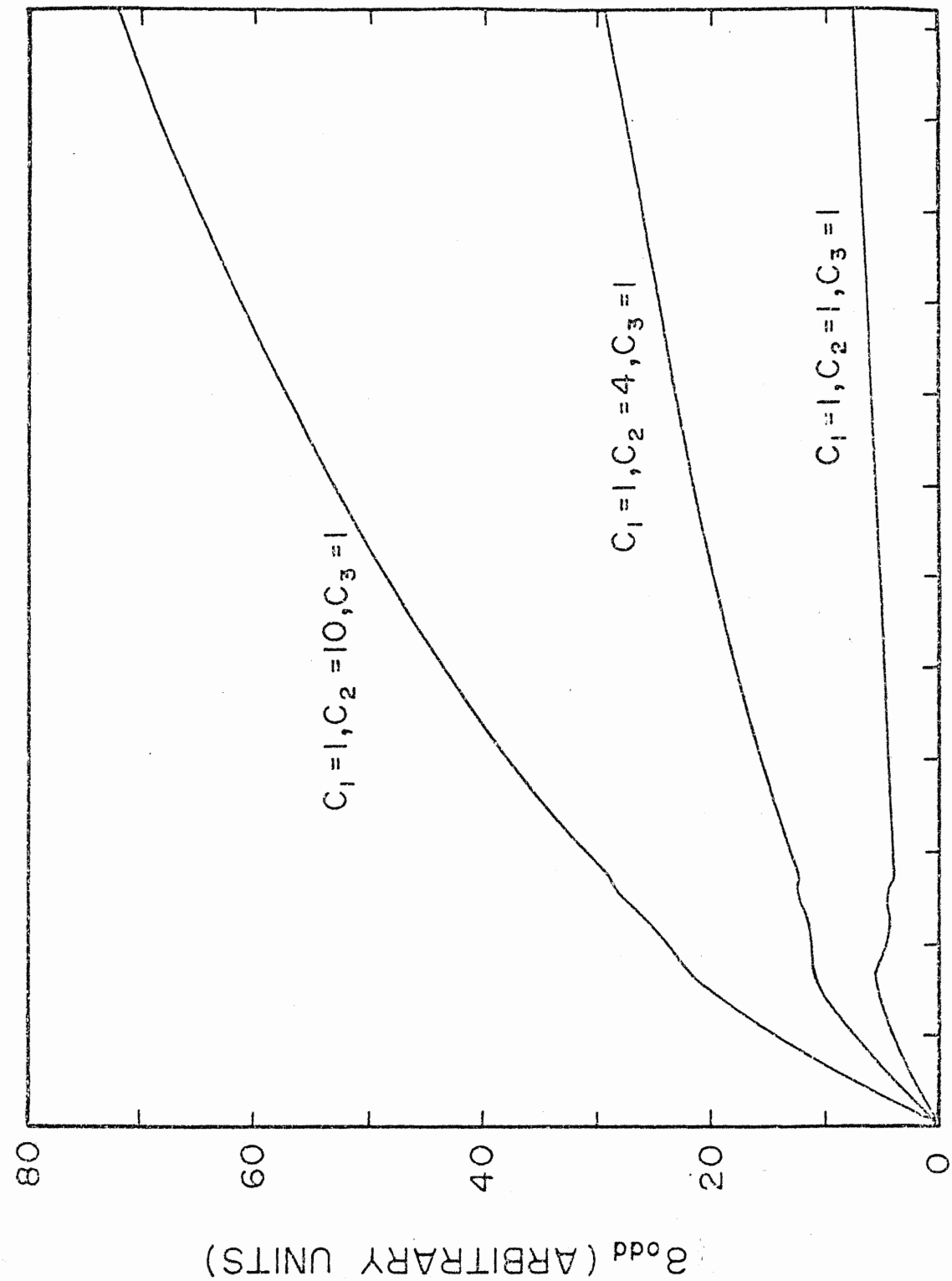

$C_1=1, C_2=10, C_3=1$
$C_1=1, C_2=4, C_3=1$
$C_1=1, C_2=1, C_3=1$
0
20
40
60
80
0
50
100
ω (meV)
$\alpha^2 F_{odd}$ (ARBITRARY UNITS)

Figure 13

The measured conductance, and derived even and odd conductances, vs voltage for Al-I-Pd from Reference (17).

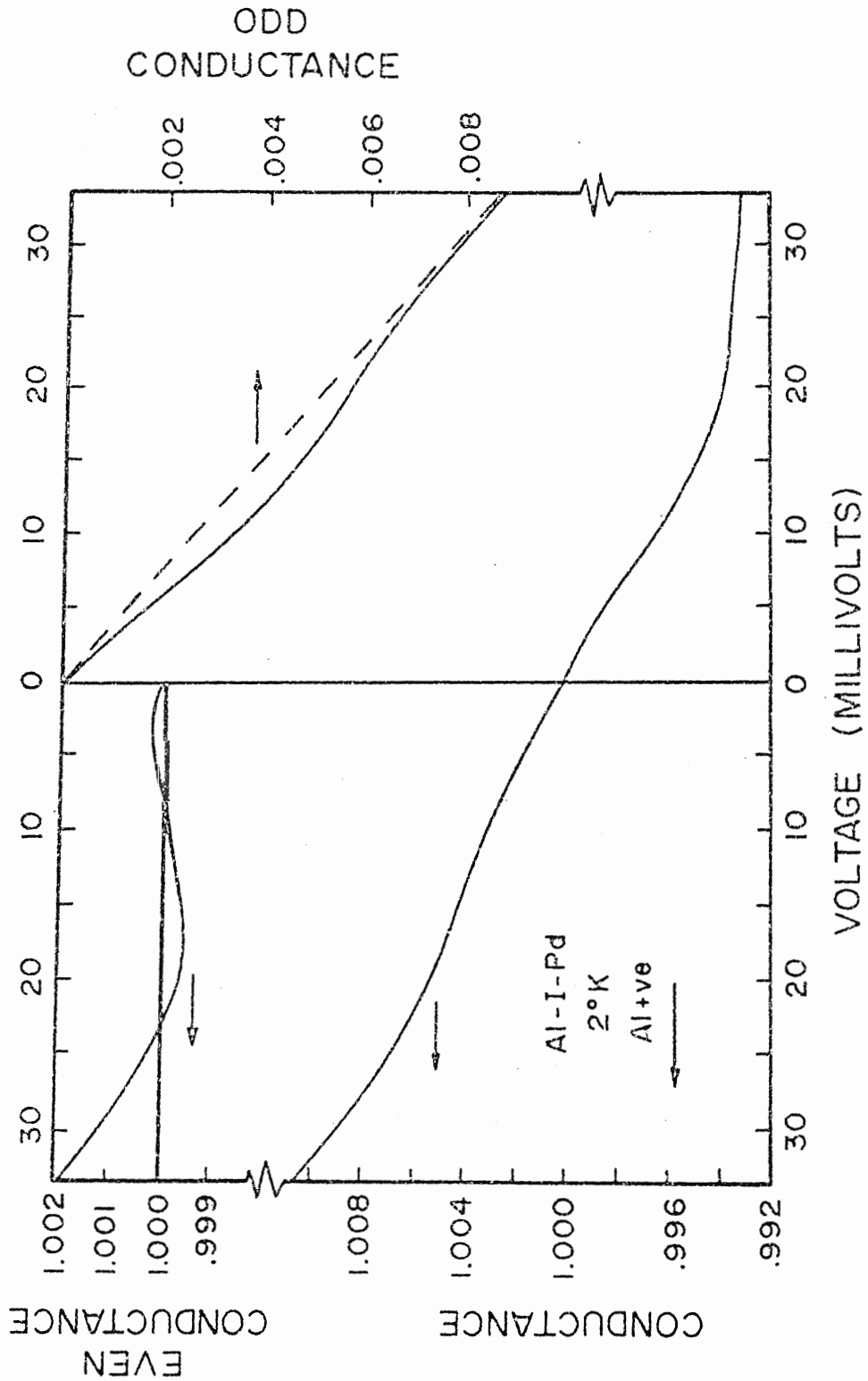
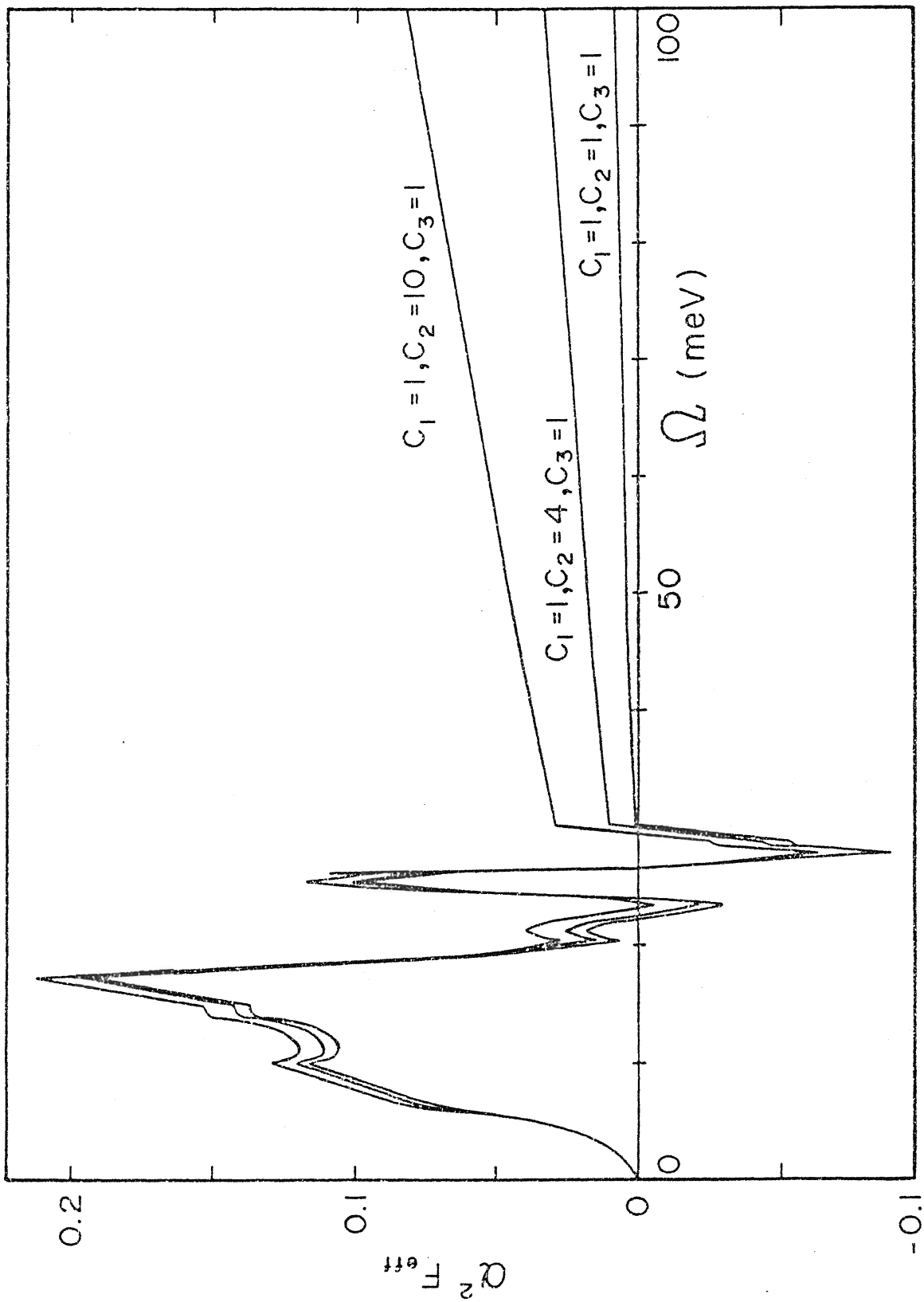

ODD
CONDUCTANCE
.002
.004
.006
.008
30
20
10
0
10
20
30
Al-I-Pd
2°K
Al+ve
1.002
1.001
1.000
.999
1.008
1.004
1.000
.996
.992
EVEN
CONDUCTANCE
CONDUCTANCE
VOLTAGE (MILLIVOLTS)

Figure 14

The effective spectra which result from using Equation [75] to invert the odd conductance for a Cu-I-Pd junction shown in Figure (12).


$\alpha^2 T_{eff}$
0.2
0.1
0
-0.1
0
50
100
Ω (meV)
$C_1=1, C_2=10, C_3=1$
$C_1=1, C_2=4, C_3=1$
$C_1=1, C_2=1, C_3=1$

IV. Conclusions

MacDonald[13] found that with the inclusion of the finite width of the band, the mass enhancement was greatly reduced from its value in the standard paramagnon theory for the same Stoner enhancement. We have seen that in addition the finite bandwidth model reduces the characteristic spin fluctuation frequency Ω_{sf} in the spin fluctuation spectral function $P(\Omega)$. Depending on the choice of the parameter E_F, Ω_{sf} may be on the order of Ω_{ep} for Pd, which is in agreement with the results of the dynamic susceptibility calculations for Pd.[12] The magnitude of $P(\Omega)$ is much smaller for a nearly full band than in the standard spin fluctuation theory, and is an order of magnitude smaller than $\alpha^2F(\Omega)$ for Pd. For the nearly full band, the first order vertex corrections are small, which justifies our method for calculating the self-energy. However, the momentum dependence of the self-energy due to spin fluctuations is also small (as is the momentum dependence of the self-energy due to phonons) and so $\sum_{sf}$ and $\sum_{ep}$ enter the conductance of the Cu-I-Pd tunnel junction with approximately the same weight. The effect of spin fluctuations on the conductance is masked by the phonons.

It may be more productive to use normal metal tunneling to find the effect of spin fluctuations in a material which has a less full d-band than Pd. For example Sc which has S = 4 and a d-band which is 25% full could have a much greater effect of spin fluctuations in the normal state tunneling than Pd in spite of the smaller Stoner factor. This is because $P(\Omega)$ has a larger magnitude for a less full band, and because the momentum dependence of the self-energy due to spin fluctuations may be much larger than the momentum

dependence of the electron-phonon self-energy. However, the characteristic spin fluctuation energy is also much larger for the less full band, and it may be difficult to separate the effect of spin fluctuations from the linear background. Also the vertex corrections become more important in calculating the self-energy for the less full band, and the normal state tunneling may contain the momentum dependence of the self-energy which is not proportional to the momentum independent self-energy.

Appendix

The appendix contains the equations for evaluating the Lindhard function, $u(\vec{q},\omega + i0^+)$, at $T = 0$.

(A) No Momentum Cut-off

For $p_c \to \infty$, the Lindhard function is

$$u(\vec{q},i\omega_\nu) = \frac{1}{\beta N(o)} \int \frac{d\vec{p}}{(2\pi)^3} \sum_{\omega_\ell} G^{(o)}(\vec{p},i\omega_\ell)\, G^{(o)}(\vec{p}+\vec{q},i\omega_\ell+i\omega_\nu)$$

where

$$G^{(o)}(\vec{p},\, i\omega_n) = \frac{1}{i\omega_n - \epsilon_{\vec{p}}} \quad .$$

Performing the sum over ω_ℓ yields

$$[A1] \qquad u(\vec{q},i\omega_\nu) = \frac{-1}{N(o)} \int \frac{d\vec{p}}{(2\pi)^3} \frac{f(\epsilon_{\vec{p}}) - f(\epsilon_{\vec{p}+\vec{q}})}{i\omega_\nu + \epsilon_{\vec{p}} - \epsilon_{\vec{p}+\vec{q}}} \quad .$$

This can be analytically continued by replacing $i\omega_\nu \to \omega + i0^+$. Relabeling the variable of integration $\vec{p} \to \vec{p} - \vec{q}$ in the second term gives the result

$$u(\vec{q},\omega+i0^+) = \frac{-1}{N(o)} \int \frac{d\vec{p}}{(2\pi)^3} f(\epsilon_{\vec{p}}) \left[\frac{1}{\omega+\epsilon_{\vec{p}}-\epsilon_{\vec{p}+\vec{q}}+i0^+} - \frac{1}{\omega+\epsilon_{\vec{p}+\vec{q}}-\epsilon_{\vec{p}}+i0^+} \right] .$$

At T = 0, the fermi function restricts the integration to the interior of a sphere of radius p_F. The integral can be done in spherical polar coordinates, and the result for Re $u(\vec{q},\omega + i0^+)$ is

$$[A2] \quad \text{Re } u(\bar{q},\bar{\omega} + i0^+) = \tfrac{1}{2}\{1 - \frac{1}{2\bar{q}} [1 - (\frac{\bar{\omega}}{2\bar{q}} - \frac{\bar{q}}{2})^2] \ln \left| [1 + (\frac{\bar{\omega}}{2\bar{q}} - \frac{\bar{q}}{2})]/ \right.$$

$$\left. [1 - (\frac{\bar{\omega}}{2\bar{q}} - \frac{\bar{q}}{2})] \right| + \frac{1}{2\bar{q}} [1 - (\frac{\bar{\omega}}{2\bar{q}} + \frac{\bar{q}}{2})^2] \ln \left| [1 + (\frac{\bar{\omega}}{2\bar{q}} + \frac{\bar{q}}{2})]/[1 - (\frac{\bar{\omega}}{2\bar{q}} + \frac{\bar{q}}{2})] \right| \}$$

where $\bar{q} = q/p_F$ and $\bar{\omega} = \omega/E_F$. Also note that Re $u(\bar{q},\bar{\omega} + i0^+)$ is an even function of $\bar{\omega}$. The limit of Re $u(\bar{q},\bar{\omega} + i0^+)$ for small $\bar{\omega}$ is

$$\lim_{\bar{\omega}\to 0} \text{Re } u(\bar{q},\bar{\omega}+i0^+) = \tfrac{1}{2}\{1 + \frac{1}{\bar{q}}[1 - (\frac{\bar{q}}{2})^2] \ln \left| (1 + \frac{\bar{q}}{2})/(1 - \frac{\bar{q}}{2}) \right| \}.$$

At $T = 0$,

$$\text{Im } u(\bar{q},\bar{\omega} + i0^+) = \frac{1}{N(o)} \int \frac{d\vec{p}}{(2\pi)^3} \theta(p_F - |\vec{p}|)\ \theta(|\vec{p} + \vec{q}| - p_F)$$

$$[\delta(\omega + \epsilon_{\vec{p}} - \epsilon_{\vec{p}+\vec{q}}) - \delta(\omega + \epsilon_{\vec{p}+\vec{q}} - \epsilon_{\vec{p}})] \ .$$

Note that Im $u(\bar{q},\bar{\omega} + i0^+)$ is an odd function of $\bar{\omega}$. For $\bar{\omega} > 0$, only the first delta function will be non-zero. For fixed $\vec{q} = q\hat{z}$, the integration is restricted by the first θ-function to the interior of a sphere of radius p_F, and by the second θ-function to the exterior of a sphere of radius p_F shifted by $-\vec{q}$ from the first sphere. The delta function further restricts the integration to the plane

$$\vec{p}\cdot\vec{q} = \frac{\omega}{q} - \frac{q}{2} \ .$$

The results of this integration are:

(i) for $0 < \bar{q} \leq 2$

$$\text{[A3]}\quad \text{Im } u(\bar{q},\bar{\omega} + i0^+) = \begin{cases} \dfrac{\pi\bar{\omega}}{4\bar{q}}, & \text{for } 0 \leq \bar{\omega} < 2\bar{q} - \bar{q}^2 \\[2ex] \dfrac{\pi}{4\bar{q}}\left[1 - \left(\dfrac{\bar{\omega}}{2\bar{q}} - \dfrac{\bar{q}}{2}\right)^2\right], & \text{for } 2\bar{q} - \bar{q}^2 \leq \bar{\omega} < 2\bar{q} + \bar{q}^2 \\[2ex] 0, & \text{for } 2\bar{q} + \bar{q}^2 \leq \bar{\omega} \end{cases}$$

(ii) for $\bar{q} > 2$

$$[A3] \quad \text{Im } u(\bar{q},\bar{\omega} + i0^+) = \begin{cases} 0, & \text{for } 0 \le \bar{\omega} < \bar{q}^2 - 2\bar{q} \\ \frac{\pi}{4\bar{q}} \left[1 - \left(\frac{\bar{\omega}}{2\bar{q}} - \frac{\bar{q}}{2}\right)^2\right], & \text{for } \bar{q}^2 - 2\bar{q} \le \bar{\omega} < 2\bar{q} + \bar{q}^2 \\ 0, & \text{for } 2\bar{q} + \bar{q}^2 \le \bar{\omega} \quad . \end{cases}$$

(B) Finite Bandwidth

Within the finite bandwidth model, the non-interacting Green's function is

$$G^{(o)}(\vec{p},i\omega_n) = \frac{\theta(p_c - |\vec{p}|)}{i\omega_n - \epsilon_{\vec{p}}} ,$$

and the expression [A1] for the Lindhard function changes accordingly. When $\bar{q} \le \bar{p}_c - 1$, ($\bar{p}_c = p_c/p_F$) there is no change in the expressions for $u(\bar{q},\bar{\omega} + i0^+)$ from the case with no momentum cut-off. When $\bar{q} \ge \bar{p}_c + 1$, $u(\bar{q},\bar{\omega} + i0^+) = 0$. When $\bar{p}_c + 1 > \bar{q} > \bar{p}_c - 1$, the integration is most easily done in cylindrical coordinates. The result of the integration for Re $u(\bar{q},\bar{\omega} + i0^+)$ is

$$[A4] \quad \mathrm{Re}\, u(\bar{q},\bar{\omega} + i0^{+}) = \frac{-1}{4\bar{q}} \{1 - \bar{p}_c^{\,2} + \bar{p}_c\bar{q} - \bar{q}$$

$$+ [(\frac{\bar{\omega}}{2\bar{q}} - \frac{\bar{q}}{2})^2 - \bar{p}_c^{\,2}] \ln|2\bar{p}_c\bar{q} - \bar{q}^2 + \bar{\omega}|$$

$$+ [(\frac{\bar{\omega}}{2\bar{q}} + \frac{\bar{q}}{2})^2 - \bar{p}_c^{\,2}] \ln|2\bar{p}_c\bar{q} - \bar{q}^2 - \bar{\omega}|$$

$$- [(\frac{\bar{\omega}}{2\bar{q}} + \frac{\bar{q}}{2})^2 - 1] \ln|2\bar{q} - \bar{q}^2 - \bar{\omega}|$$

$$- [(\frac{\bar{\omega}}{2\bar{q}} - \frac{\bar{q}}{2})^2 - 1] \ln|2\bar{q} - \bar{q}^2 + \bar{\omega}|$$

$$+ (\bar{p}_c^{\,2} - 1 + \bar{\omega}) \ln|\bar{p}_c^{\,2} - 1 + \bar{\omega}|$$

$$+ (\bar{p}_c^{\,2} - 1 - \bar{\omega}) \ln|\bar{p}_c^{\,2} - 1 - \bar{\omega}|\}.$$

For the imaginary part of $u(\bar{q},\bar{\omega} + i0^{+})$, when $\bar{p}_c+1 > \bar{q} > \bar{p}_c+1$, there are three ranges of q-values to distinguish.

(i) $\bar{p}_c - 1 < \bar{q} \leq 2$ and $\bar{p}_c - 1 > 2\bar{q} - \bar{q}^2$

$$[A5]\ \text{Im}\, u(\bar{q},\bar{\omega} + i0^+) = \begin{cases} \dfrac{\pi\bar{\omega}}{4\bar{q}}, & \text{for } 0 \leq \bar{\omega} < 2\bar{q} - \bar{q}^2 \\ \dfrac{\pi}{4\bar{q}}\left[1 - \left(\dfrac{\bar{\omega}}{2\bar{q}} - \dfrac{\bar{q}}{2}\right)^2\right], & \text{for } 2\bar{q} - \bar{q}^2 \leq \bar{\omega} < \bar{p}_c^{\,2} - 1 \\ \dfrac{\pi}{4\bar{q}}\left[\bar{p}_c^{\,2} - \left(\dfrac{\bar{\omega}}{2\bar{q}} + \dfrac{\bar{q}}{2}\right)^2\right], & \text{for } \bar{p}_c^{\,2} - 1 \leq \bar{\omega} < 2\bar{q}\bar{p}_c - \bar{q}^2 \\ 0, & \text{for } 2\bar{q}\bar{p}_c - \bar{q}^2 \leq \bar{\omega} \end{cases}$$

(ii) $\bar{p}_c - 1 < \bar{q} \leq 2$ and $\bar{p}_c^{\,2} - 1 \leq 2\bar{q} - \bar{q}^2$

$$[A5]\ \text{Im}\, u(\bar{q},\bar{\omega} + i0^+) = \begin{cases} \dfrac{\pi\bar{\omega}}{4\bar{q}}, & \text{for } 0 \leq \bar{\omega} < \bar{p}_c^{\,2} - 1 \\ \dfrac{\pi}{4\bar{q}}\left(\bar{p}_c^{\,2} - 1\right), & \text{for } \bar{p}_c^{\,2} - 1 \leq \bar{\omega} < 2\bar{q} - \bar{q}^2 \\ \dfrac{\pi}{4\bar{q}}\left[\bar{p}_c^{\,2} - \left(\dfrac{\bar{\omega}}{2\bar{q}} + \dfrac{\bar{q}}{2}\right)^2\right], & \text{for } 2\bar{q} - q^2 \leq \bar{\omega} < 2\bar{q}\bar{p}_c - \bar{q}^2 \\ 0, & \text{for } 2\bar{q}\bar{p}_c - \bar{q}^2 \leq \bar{\omega} \end{cases}$$

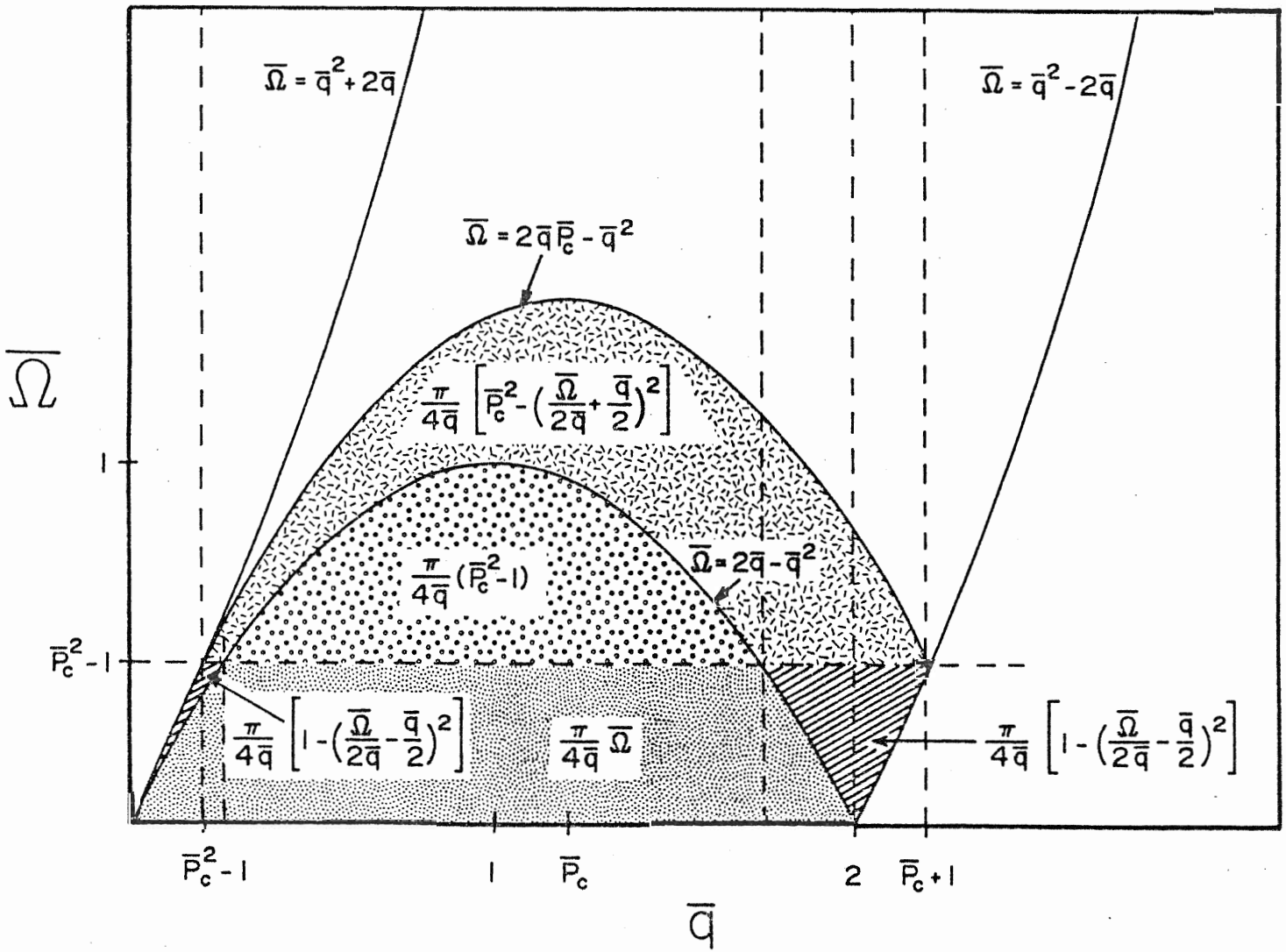
(iii) $2 < \bar{q} \leq \bar{p}_c + 1$

$$[A5] \quad \text{Im } u(\bar{q},\bar{\omega} + i0^+) = \begin{cases} 0, & \text{for } 0 \leq \bar{\omega} < \bar{q}^2 - 2\bar{q} \\ \frac{\pi}{4\bar{q}} \left[1 - \left(\frac{\bar{\omega}}{2\bar{q}} - \frac{\bar{q}}{2}\right)^2\right], & \text{for } \bar{q}^2 - 2\bar{q} \leq \bar{\omega} < \bar{p}_c^{\,2} - 1 \\ \frac{\pi}{4\bar{q}} \left[\bar{p}_c^{\,2} - \left(\frac{\bar{\omega}}{2\bar{q}} + \frac{\bar{q}}{2}\right)^2\right], & \text{for } \bar{p}_c^{\,2} - 1 \leq \bar{\omega} < 2\bar{q}\bar{p}_c - \bar{q}^2 \\ 0\ , & \text{for } 2\bar{q}\bar{p}_c - \bar{q}^2 \leq \bar{\omega} \end{cases}$$

See Figure (15) which shows the regions of the $(\bar{q},\bar{\omega})$-plane where Im $u(\bar{q},\bar{\omega} + i0^+)$ is given by the various expressions.

Figure 15

The $(\bar{q},\bar{\Omega})$-plane showing the different domains for $\mathrm{Im}\, u(\bar{q},\bar{\Omega}+i0^{+})$, which is zero except in the shaded regions.


$\overline{\Omega} = \overline{q}^2 + 2\overline{q}$
$\overline{\Omega} = \overline{q}^2 - 2\overline{q}$
$\overline{\Omega} = 2\overline{q}\overline{P}_c - \overline{q}^2$
$\frac{\pi}{4\overline{q}}\left[\overline{P}_c^2 - \left(\frac{\overline{\Omega}}{2\overline{q}} + \frac{\overline{q}}{2}\right)^2\right]$
$\frac{\pi}{4\overline{q}}(\overline{P}_c^2 - 1)$
$\overline{\Omega} = 2\overline{q} - \overline{q}^2$
$\frac{\pi}{4\overline{q}}\left[1 - \left(\frac{\overline{\Omega}}{2\overline{q}} - \frac{\overline{q}}{2}\right)^2\right]$
$\frac{\pi}{4\overline{q}}\,\overline{\Omega}$
$\frac{\pi}{4\overline{q}}\left[1 - \left(\frac{\overline{\Omega}}{2\overline{q}} - \frac{\overline{q}}{2}\right)^2\right]$
$\overline{\Omega}$
1
$\overline{P}_c^2 - 1$
$\overline{P}_c^2 - 1$
1
$\overline{P}_c$
2
$\overline{P}_c + 1$
$\overline{q}$

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