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THIS INFINITE, UNANIMOUS DISSONANCE:
A STUDY IN MATHEMATICAL EXISTENTIALISM
THROUGH THE WORK OF JEAN-PAUL SARTRE AND ALAIN BADIOU

by

Zachary Luke Fraser

THESIS SUBMITTED IN PARTIAL FULFILLMENT OF
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In the Department of Philosophy

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*This infinite, unanimous dissonance:
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Zachary Luke Fraser
Brock University, 2008

ABSTRACT

This thesis seeks to elucidate a motif common to the work both of Jean-Paul Sartre and Alain Badiou (with special attention being given to *Being and Nothingness* and *Being and Event* respectively): the thesis that *the subject's existence precedes and determines its essence*. To this end, the author aims to explicate the structural invariances, common to both philosophies, that allow this thesis to take shape. Their explication requires the construction of an overarching conceptual framework within which it may be possible to embed both the phenomenological ontology elaborated in *Being and Event* and the mathematical ontology outlined in *Being and Event*. Within this framework, whose axial concept is that of multiplicity, the precedence of essence by existence becomes intelligible in terms of a priority of extensional over intensional determination. A series of familiar existentialist concepts are reconstructed on this basis, such as lack and value, and these are set to work in the task of fleshing out the more or less skeletal theory of the subject presented in *Being and Event*.

NOTICE

The following thesis contains, among other things, an interpretation of Sartre's *Being and Nothingness*. In the course of this interpretation, the demands of rigour have been such as to force a decisive break with certain, fairly widespread notions bearing upon the content of Sartre's philosophy and the styles of inquiry appropriate to its analysis. The general thrust of these demands is contained in the conviction that, in order to understand the significance and singularity of Sartre's philosophical undertaking, it is not enough to bathe in the light of (what might take itself to be) 'mere description' or 'evidence', nor to meditate roundly on those aphorisms that have become slogans and watchwords of existentialists everywhere. It is necessary, before all else, to unearth the basic theoretical scaffolding that gives his work its very structure, and which polarises and refracts the phenomena that he so brilliantly describes. The product of this excavation is an interpretation that lays emphasis on what could be called Sartre's '*formal ontology*', and lends little space to the customary rehearsal of the striking, descriptive vignettes for which Sartre is justly famous. I claim neither that this project is complete, nor that it should not eventually be supplemented by an investigation of the role of 'evidence' in Sartre's philosophy. I do claim that no exegesis of Sartre's thought can be sufficient without having passed through the rigours of his '*formal ontology*'.

Some Abbreviations Used in the Text

BE : Alain Badiou, *L'être et l'événement*, (Paris: Seuil, 1988). Translated into English by Oliver Feltham as *Being and Event* (London: Continuum, 2005). Citations will be of the form (BE, m#, ###/##), providing the meditation number followed by the page number in Feltham's translation, and the page number of the French original. Where the translation is my own, an asterisk is placed after the page numbers.

BN : Jean-Paul Sartre, *L'être et le néant: Essai d'ontologie phénoménologique*, (Paris: Éditions Gallimard, 1943). Translated into English by Hazel E. Barnes as *Being and Nothingness: An Essay on Phenomenological Ontology*, (New York: Philosophical Library, 1956). Citations will be of the form (BN, #.#.#, ###/##), providing the Part, Chapter and Section number of the text, followed by the English and French page numbers. Where the translation is my own, an asterisk is placed after the page numbers.

INTRODUCTION

Against all odds, I have always been concerned in a privileged way by the question of how something could still be called 'subject' within the most rigorous conditions possible of the investigation of structures. This question had an echo for me of an even older question, which I had posed at the time that I was fully Sartrean, namely, the question of how to make Sartre compatible with the intelligibility of mathematics.

- Alain Badiou¹

§ 1: Mathematical Existentialism

The wager of this thesis is that this question bears more fruit than might be expected, and that, if pursued, it offers valuable insight into the structure and trajectory of Badiou's thought, illuminating the inner workings of Sartre's ontology in the same stroke. One effect of this pursuit is that the rubric 'mathematical existentialism' comes to suggest itself quite strongly as a description of the project undertaken in Badiou's landmark text, *Being and Event*. The value of this label, though, lies more in forcing a question than in promising an adequate description. It is for this reason I chose it as a title, as the first words of this project rather than the last. It nevertheless threatens to cause some confusion, which I will try to dispel straightaway. To begin with, the title suggests that we might fruitfully place both Sartre and Badiou under this rubric. We cannot. Sartre's ontology and existentialist philosophy radically resist mathematisation, in virtue of the very structure of their concepts. This resistance, however, is as minimal as it is radical. Sartre's concepts—particularly the notions of the for-itself, consciousness and nihilation—resist mathematisation insofar as they elude identity. As Sartre formulates it, the theory of intentionality itself, which is the bedrock of his ontology, implicates an irreducible *clinamen* of non-self-identity in the very event of consciousness, which makes it *a priori* recalcitrant to the self-identical literality of anything that can be thought within the formal productions of mathematics. In this respect, the non-mathematicity of Sartrean ontology is radical. Yet, as I will

show, Sartre formulates the concept of consciousness in such a way that it comes into startling proximity with the mathematical theory of sets, a proximity that allows the manifold kinships between *Being and Nothingness* and *Being and Event* to come closer into view.

The reason for this is that Badiou's *Being and Event* announces, as its first philosophical axiom, the thesis that mathematical set theory is the science of being qua being. The ontology with respect to which Badiou articulates his entire theory of the subject, a theory which Badiou explicitly composes in view of "a determinant 'existential' motif,"² is mathematical through and through. Even in Badiou's case, however, we must avoid the confusion into which the rubric of 'mathematical existentialism' might lead us if we move too quickly. Between existentialism and mathematics, Badiou's philosophy does not produce any sort of coincidence or identity, but a strict dialectical tension. What is most 'existentialist', or Sartrean, in *Being and Event* is articulated on the ground of a mathematical ontology, but nevertheless eludes coincidence with the latter. What is most 'existentialist' in that text is Badiou's theory of the subject, which, like Sartre's, has at its core the form of a clinamen by which the subject subtracts itself from being-in-itself and so resists mathematisation and identity, into which it nevertheless comes to inscribe itself.³ Badiou's name for this clinamen is the 'event', and the resulting tension between its deviation from and reinscription in being is finds an expression in the very title of Badiou's major work, just as it does in that of Sartre's: neither *Being and Event* nor *Being and Nothingness* describe an insurmountable dualism, but a productive dissonance.

'Mathematical existentialism' names a space of tension that obtains in Badiou's work through the situation of an existentialist conception of the subject within a framework of mathematical intelligibility—the framework through which the subject is thought in its being, which is precisely that which, insofar as the subject is subject, the subject transcends. Badiou's thought exhibits this tension both internally and with respect to Sartre, whose early work—

especially *Being and Nothingness*—outlines the essential preconditions of a mathematical existentialism.

But what does ‘existentialism’ mean here? I give it no other definition than the one Sartre offers in his infamous lecture on existentialism and humanism. Against the winds of fashion that had diffused the notion of ‘existentialism’ to the point that, at the time he gave his lecture, Sartre could say that “the word is now so loosely applied to so many things that it no longer means anything at all” (EH, 25-6),⁴ Sartre insists that this “most austere” of doctrines can be reduced to the simple thesis that, with respect to the subject, “*existence comes before essence*” (EH, 26).

As is well known, Sartre himself was not the first to commit himself to this expression. It comes from Heidegger’s *Being and Time*, as Sartre fully acknowledges; accordingly, he extends the title of ‘existentialism’ to Heidegger’s thought as well, much to the latter’s chagrin. Soon after the delivery of Sartre’s lecture, Heidegger, in his “Letter on Humanism,” complains that Sartre has violently misinterpreted him. Without entering into an exegesis of Heidegger’s own interpretation of the celebrated formula, we may nevertheless gain something from looking at his interpretation of the interpretation that Sartre gives it. To begin with, Heidegger is made uneasy by what he sees as Sartre’s return to Descartes, insofar as this return blocks the thinking of being with the thought of the subject. To Heidegger, it looks as if Sartre’s claim that “*existence precedes essence*” is the claim that the entity that is the subject, understood as the Cartesian ego, is prior to any thought of being. What this obscures, Heidegger argues, is that it is *being* that lets there be thought; thought is an engagement of being, and not of *a* being. This critique can be founded only on a hasty misreading of Sartre, though one that is no doubt encouraged by the idiom of his own lecture. As we will see, the existence that precedes essence and engages itself in thought is manifestly *not* the existent subject as ego. It is consciousness that thinks, but consciousness is not an entity in the strict sense; it is the ‘there is’ by which entities may appear

as entities. The ego is merely one object among others disclosed by consciousness; it is impossible to understand Sartre's project without grasping this fundamental thesis. Consciousness is nevertheless not to be identified with what Heidegger calls being qua being; it differs from it in both presupposing the onticity of what it discloses as beings-in-themselves and in being in each case a local and singular occurrence of the 'there is' and not the 'there is' as such or in its totality. In spite of this confusion, Heidegger offers the quite useful remark that when Sartre says, "Existence precedes essence [...], he is taking *existentia* and *essentia* according to their metaphysical meaning, which from Plato's time on has said that *essentia* precedes *existentia*" (LH, 208). Within metaphysical discourse, "*Existentia* (*existence*) means [...] *actualitas*, actuality as opposed to mere possibility as *Idea*" (LH, 206-7), which would be thought as *essentia*. When *essentia* precedes *existentia*, "*Existentia* is the name for the realisation of something that is as it appears in its *Idea*" (LH, 207).

Since Leibniz, it has been customary to think these categories through the way in which they manifest themselves in syllogistic logic, which is possible to organize either with respect to the *existentia* of what falls under the terms of the syllogism or with respect to the *essentia* invoked by the terms themselves—or, as we would say nowadays, with respect to the *extension* or to the *intension* of the terms in question. To resuscitate a rather hackneyed example: in the syllogism, "Socrates is a Man / All Men are Mortal / ∴ Socrates is Mortal," we are thinking according to the essence or intension of the terms if we reach the conclusion by seeing that the essence of Mortality is contained in the essence of Man, which is contained in the essence of Socrates. We are thinking according to existence or extension if we reason that everything that is a Man is also a Mortal (that what the first term refers to is also referred to by the second), and that 'Socrates' refers to something that is referred to by 'Man'. Essence is here thought as the *Idea* that determines a class, or, as Sartre has it, as "the manifest law that presides over the

succession of its appearances, [as] the principle of the series” (BN, in.I, xlvi/12*). There is no doubt that Heidegger is correct in placing Sartre’s language within the metaphysical tradition on this point, but we cannot stop here. We know that it is illegitimate to make of the subject a simple existent or term, since it is a modality of the disclosure of terms. If we must nevertheless say of it that its existence precedes its essence, then in view of the metaphysical tradition in which Sartre establishes himself, it is reasonable to take this as meaning that the primary determination of the subject is not intensional but extensional. At present, however, we lack the framework necessary to make sense of such a statement, and how it might apply not only to Sartre’s thought, but to Badiou’s as well. In order to gain such a framework, we must sketch out the general design of the ontology underlying the two thinkers’ philosophy of the subject.

§ 2: Minimalistic Ontology

The discipline of ontology has for a long time taken its mark from Aristotle’s call for a “science that studies Being *qua* Being [ἐπιστήμη τις ἡ θεωρεῖ τὸ ὄν ἢ ὅν]”⁵. Jean-Toussaint Desanti has described this call as “an invitation to think ‘being’ intrinsically, i.e. in such a way as to determine nothing in its concept other than what properly and exclusively pertains to it” (Desanti, 59). However, as Desanti goes on to explain, “intrinsically”

can be understood in two ways: maximally or minimally. Those who choose the maximal interpretation will try to render being equal to its concept: they will try to think under the name ‘being’ the deployment of this very concept in the richness and interconnectedness of its moments.⁶ Those who choose the minimal interpretation will ask themselves the following question: what is *the least* that must be thought in order to define the status of the proposition ‘there are beings’? (Desanti, 59)

Sartre’s *Being and Nothingness* and Badiou’s *Being and Event* can both be seen as austere minimalist works insofar as this question commands the ontological inquiry pursued by each. This is not the same as calling them ontologically parsimonious in the sense of having the smallest possible ontic inventory; it is not a question of ruthlessly employing Ockham’s razor,

and trimming down the furniture of the universe (or shaving Platonic beards). Indeed, the question, “which entities exist?” is at some remove from *both* the trains of thought that Desanti sketches out, both the “minimalist” and the “maximalist” orientations. It is rather a question of determining what is to be thought when one declares that *any* entity exists. To put it another way, and one that makes direct use of Desanti’s formulation: what is at issue here is just what remains invariant throughout the class of statements “there is *x*”; as such, it makes no difference at this level whether we want to say “there are elephants,” “there are golden mountains,” “there are Ideas,” or “there are Quineans.” Determinations at the ontic level are necessarily subordinate to the delineation of ontological ground. Any such delineation will have profound effects on what is understood by affirming the existence of *any* item in the ontic inventory, and may indeed circumscribe *a priori* limits concerning what items may or may not exist, although we will find as a rule that there is a considerable degree of underdetermination at play in the ontological circumscription of the ontic.

It is interesting that in their response to the question that Desanti formulates here, Badiou and Sartre each begin with the axiomatic assumption of a concept produced in the renaissance that the pure sciences experienced at the turn of the last century. For Badiou, the concept that gives a decisive determination to the notion of the ‘there is’ is the concept of *set*, produced in the Cantorian birth of mathematical set theory. For Sartre, it is rather the concept of *intentionality*—which emerged into the light of day in the Brentanian-Husserlian birth of phenomenology—that lets the ‘there is’ be thought in its purity. In order to serve the purposes of minimalistic ontology, however, both concepts demand further purification. For the set, this purification took place in the Zermelo-Fraenkel axiomatisation of set theory (abbreviated ZF), by means of which every remaining vestige of empiricism and psychologism was stripped from the concept of set, leaving only the idea of the set as that to which other sets stand in a relation of belonging, in such a way

that the axioms regulating this relation are obeyed. For intentionality, the work of purification was accomplished by Sartre himself, and was the purpose of his critique of Husserl in *The Transcendence of the Ego*. In that text, intentionality is interpreted in such a way that consciousness is *nothing* but the disclosure of phenomena whose reality and being remain strictly external to it. Like the Zermelo-Fraenkel set, consciousness is the locus of a specific relation and nothing more.

In both intentional consciousness and the extensional set, the two philosophers find rigorously minimalistic determinations of the 'there is'. For consciousness, this minimalism is commanded by the axiom of intentionality, which states that *consciousness is consciousness of something, from which it differs*. The minimalism proper to the set is, in a parallel fashion, expressed by the axiom of extensionality: two sets are identical if they possess the same elements. There is a profound similarity between what the two axioms require. Both rid their respective forms of every speck of interiority, and every intensional determination irreducible to what lies 'outside'. In both cases, the 'there is' is rendered absolutely transparent, and yet remains perfectly intelligible; it is left as almost nothing, but neither does it dissolve into utter formlessness nor disappear from thought. In perfect lucidity the 'there is' is delivered, either to itself or to mathematical inscription. The Sartrean reduction of consciousness to *nothing* but the presentation of its noemata ("Suppose that you see a particular tree. Now, you see it just where it is: on the edge of the road, amidst the dust, alone and twisted beneath the heat, twenty leagues from the Mediterranean coast."⁷) and the set-theoretic reduction of *every* mathematical entity to a mere aggregation and presentation—a mere set—of *other* mathematical entities (the number 3, for example, becomes nothing other than the set of the numbers 2, 1 and 0),⁸ mark the same cut in different cloths.

This analogy, however vaguely outlined here, is not spurious. The similarity in structure between the Zermelo-Frankel concept of set and the Sartrean concept of consciousness can be expressed by saying that consciousness and set are ‘species’ of a common ‘genus’, which is that of *multiplicity*. We will now proceed to its definition.

§ 3: Multiplicity Defined

Here, I will propose the formal conditions necessary and sufficient for the classification of a structure as a multiplicity. These conditions allow for certain degrees of liberty with respect to their satisfaction: the precise modes in which they are satisfied are not strictly outlined, allowing for a variety of species of multiplicity to fall under the general concept. The definition nevertheless aims at a certain conceptual purity, and will result in the exclusion of some structures, such as *classes* or *intensive multiplicities*, from the general category, though as I will demonstrate, some of these excluded structures nevertheless partially participate in the general concept of multiplicity. The guiding thread in this definition will be the mathematical concept of *set*, which possesses both a degree of clarity and an ascetic minimalism that is a blessing in any attempt to formulate a general concept. I intend this definition to be compatible with the axioms of set theory (and, consequently, with Badiou’s ontology) without being restricted to them. That is to say, all sets must be recognized as multiplicities, but there may well be multiplicities that cannot be formalised as sets (*to be a set* is a sufficient but not a necessary condition for *being a multiplicity*). Our definition consists in articulating two essential characteristics.

(α) A MULTIPLICITY IS EXTENSIONALLY DETERMINED

The first is that a multiplicity must be *extensionally determined by its elements*. A multiplicity cannot be conceived as an independent being, standing in relation to its elements as one being relating to another. To use a Sartrean expression, a multiplicity cannot be in a merely

external relation to its elements. The specific being of a multiplicity is not anterior to its relation to its elements, for it is *nothing other* than this relation; it is a relation *ex nihilo*. In set theory, this relation is designated (inversely) by the sign \in . For Sartre, this relation is called ‘consciousness of’.

A consequence of this characteristic is that there cannot be two *distinct* multiplicities with exactly the same elements. Those familiar with a bit of set theory will recognize that this concept underpins the axiom of extensionality, which says that two sets are identical if they possess all the same elements. The concept itself is nevertheless a bit broader, insofar as identity is not the only possible mode of individuation.

(β) A MULTIPLICITY DIFFERS FROM ITS ELEMENTS

The second essential characteristic of a multiplicity is that while it is extensionally determined, it nonetheless *differs from its elements*. A statement about a multiplicity is not reducible a statement about its elements; it is not the same thing, for example, to say that all the elements of α belong to β and to say that α belongs to β (in set theory, the former is written $\alpha \subset \beta$, the latter, $\alpha \in \beta$). It is plain to see that failure to assume this principle leads the principle of extensional determination into contradictions. For example, suppose α is the multiplicity possessing γ and δ as elements. If α were identical to the multiplicity whose sole element is α , then it would be identical to both γ and δ , which is a contradiction since γ and δ were presumed to be distinct. In modern set-theoretical notation, this syllogism runs as follows:⁹

$$(A) \alpha = \{\gamma, \delta\}$$

$$(B) \gamma \neq \delta$$

$$(C) \{\alpha\} = \alpha$$

$$\therefore \alpha = \gamma = \delta, \text{ which contradicts (2).}$$

The being-multiple of a multiplicity is necessarily in excess of the identity of the elements. How the excess is individuated or distributed is entirely dependent on the theory in which the

multiplicity is articulated; it is a concern extrinsic to the general concept (for this reason, the fact that the above syllogism is presented in set-theoretical notation ought not to be seen as restricting the principle to set theory).

This particular characteristic of multiplicities is, in my view, of utmost importance, no less than the extensional requirement. Without this basic principle, for instance, Zermelo-Fraenkel set theory would not get off the ground. Every set in its universe of discourse is nothing other than either a set of sets, or else the void. The abjuration of the difference between a set and its elements would therefore result in a collapse of the theory's entire architecture into the void alone.¹⁰ It is a tacit axiom, not only of set theory, but of any adequate theory of multiplicity whatsoever that, in general, $\{x\} \neq x$.¹¹

It is noteworthy that we find the same requirement stated explicitly by Sartre, who writes that in order to think the consistency of a given exposition of being, "it is necessary that one be capable of holding it in view as a set [*ensemble*], that is to say, one must be capable of putting oneself *outside of being*" (BE, I.I.V, 24/59*). This very act of withdrawal is of a piece with the constitution of the thing as a multiplicity (as an *ensemble*).

By building the criterion of *extensional differentiation* into the concept of multiplicity, we may bypass (with all the advantages of theft) at least one of the objections that Bertrand Russell makes to the strictly *extensional determination* of the multiplicities with which mathematics in general, and set theory in particular, are concerned. Russell insists on univocally identifying these multiplicities as *classes*, that is, as aggregates that are intrinsically determined by a predicate satisfied by all the elements that they assemble. Classes, in contrast to sets, are *intensionally determined*. This determination, which is immanent to the class itself regardless of the elements that belong to it, gives one a positive point of leverage by which to distinguish between a class and its elements. The difference in question simply becomes a difference

between two positive terms (namely, a predicate and its extension), rather than a differentiation *ex nihilo*. The latter, to Russell's mind, is nonsense, and in Chapter XVII of *Introduction to Mathematical Philosophy*, he writes:

We cannot take classes in the *pure* extensional way as simply heaps or conglomerations. If we were to attempt to do that, we should find it impossible to understand how there can be such a class as the null-class, which has no members at all and cannot be regarded as a 'heap'; we should also find it very hard to understand how it comes about that a class which has only one member is not identical with that one member. I do not mean to assert, or to deny, that there are such entities as 'heaps.' As a mathematical logician, I am not called upon to have an opinion on this point. All that I am maintaining is that, if there are such things as heaps, we cannot identify them with the classes composed of their constituents. (183)

Unlike a heap, which has no consistency that can be disengaged from the elements of the heap themselves, and unlike a class, whose disengagement presupposes an already-given positivity, a multiplicity as we have defined it implies a pure and constitutive dimension of withdrawal, a differentiation that does not presuppose but produces its second term. The multiplicity with only one element, if it is a pure multiplicity and not an intensionally determined class, differs from its element by the very fact that it is a multiplicity. The 'null-class', stripped of its predicative support and considered as a pure multiplicity (as what is called the 'empty set', for example), is determined by nothing, but insofar as it can be considered as a being, is already differentiated from nothingness "through an existential scission between the nothing and the name"¹² by which it is grasped.



I am confident that these two characteristics capture what is essential in the notion of multiplicity, though there may be some surprise there has been no mention of 'plurality' or 'dispersion' in these remarks, such as most dictionaries provide in their definitions of the term. The reasons for this are simple. First of all, it is difficult to see how a definition of multiplicity as a plurality of things, for example, would escape being circular and trivial. I do not see how we can provide a clear notion of what 'plurality' is, without going through the concept of

‘multiplicity’, and in particular, the concept as I have defined it. The best that can be done is to say that a plurality is a multiplicity whose elements are not all identical to each other.

Furthermore, as already stated, my intent was to formulate a concept of multiplicity that would subsume that of set. It is therefore necessary to conceptualise, against Russell and with Zermelo, in what sense a singleton, or a set with one element, is a multiplicity. That it is indeed a multiplicity under the above definition can be seen by the fact that $\{x\}$ is entirely determined by its only element x , and yet the two are distinct; $\{x\}$ is *not* x . The separation the second criterion imposes between the extension that determines a multiplicity and the individuality by which a multiplicity is grasped as *a* multiplicity, is fundamentally prior to the ideas of plurality and dispersion by which multiplicity is commonly characterised. Using a metaphor that is both too spatial and too psychological to be trusted, we can say that the ‘horizontal’ dehiscence between elements of a single multiplicity is only thinkable in the light of a ‘vertical’ dehiscence that places us outside the elements being thought of together, the dehiscence symbolised by the nonidentity, $x \neq \{x\}$.

§ 4: Existence Precedes Essence: The Idea of the Subject

In *Le Siècle (The Century)*, Badiou places his theory of the subject at the extremity of a tradition which took root in the first half of the twentieth century, of which Sartre was the greatest philosophical herald. The collective project of this tradition was to conceive of the subject, not as a simple and substantial entity at the foundation of experience and thought, but as a contingent event. For Badiou as for Sartre before him, this means that the subject is a modification of the ‘there is’, conceived as consciousness or presentation. By that very fact, the subject is a particular formation of multiplicity. Sartre calls this form a project-ensemble; Badiou, calls it the local configuration, or subset, of a generic procedure.

It is now clear what it might mean to say that, with respect to the subject, *existentia* precedes *essentia*. The subject is extensionally determined. Badiou's use of a set theoretic ontology to underpin the theory of the subject, insofar as it can be thought in its facticity, is designed to give a rigorous formulation to this thought. "The subject," for Badiou,

is contingent on an event and constitutes itself solely as a capacity for truth, and since its 'matter' is a truth procedure, or generic procedure, the subject is cannot be naturalised in any way. In Sartre's vocabulary, we will say that it is without essence (this is the sense of the famous formula 'existence precedes essence'). (LS, 144)

The notion of genericity that is invoked here comes from Paul J. Cohen's work on Cantor's continuum hypothesis. In brief, a generic subset is one that can *only* be determined extensionally, one to which it is impossible to assign an intensional determinant even after the fact. By conceiving of the subject as a capacity for genericity, Badiou follows in Sartre's footsteps.

This directs us to a still more interesting problem, for both Sartre and Badiou wish to think the subject not only as that which is determined extensionally in the last instance (this is true of *everything* in these two multiplistic ontologies), but as what, in its very existence, avoids even a secondary intensional determination. The problem is that insofar as the subject is factually finite, it cannot be thought as generic; it is always possible to spin together an intensional determinant adequate to any finite set. While it is by no means sufficient, infinitude is a necessary condition for genericity. The subject must be such that, even if it is anchored in finitude, it tends toward the infinite in such a way as to outstrip essence. The difficulty is to think this tendency in a way that does not fall back on essence or intension, conceived, for example, as potentiality.

It is therefore necessary to derive the subject's dimension of possibility on the basis of its purely extensional constitution. It is at this point that we must return to the clinamen mentioned at the beginning of this introduction, and which takes the form of being-for-itself for Sartre and the event for Badiou. An attentive study of Sartre's theory of consciousness will reveal that he is

able to derive a notion of possibility as a dimension of consciousness from the structure of the latter's purely extensional determination in the light of its own non-identity. Possibility comes to be thought, not as intensional potentiality, but as a dimension of consciousness' ek-sistence or ekstasis, its 'standing outside of itself'. Contrary to Heidegger, the precedence of *existentia* in Sartre's thought does not eclipse ek-sistence, but allows it to be thought (whether Heidegger's concept of ek-sistence is ultimately homologous with Sartre's is a question that I will leave unanswered here). It will be shown that there is reason to see the same structures of ekstasis as being operational in Badiou's theory of the subject, particularly with respect to the role of the event. The event, which Badiou positions as the 'cause' of the subjective capacity for genericity, will be shown to have a structure that is strictly homologous to that of consciousness, as it is understood by Sartre.

It remains for us to elucidate these ideas in detail.

I. The Axiom of Intentionality And the Determination of Consciousness as Multiplicity

Intentionality is the theory of the 'of'.

— Louis Althusser, "Philosophy and the Spontaneous Philosophy of Scientists."¹³

It is difficult to overestimate the importance that Sartre attributed to the principle of intentionality—the principle that “all consciousness is consciousness *of* something”—whose discovery stands at the foundation of Husserlian phenomenology. Gilbert Varet is certainly correct in saying that

as the essential acquisition of Husserlian thought, as seen by Sartre, this principle becomes something like the evidently *primitive fact* of his own philosophy. It even produces something of a displacement [*glissement*] between the idea of phenomenology and a sort of *axiom* of intentionality...¹⁴

The axiomatic character of the principle of intentionality is evident to any reader of the early works of Sartrean ontology—most notably *The Transcendence of the Ego*, and *Being and Nothingness*, as is the displacement to which Varet draws our attention. At virtually every point in these texts where an ontological decision is called for, in order to cut through and reconfigure the terms of the ambiguous descriptive accounts at which Sartre excelled, it is to this axiom that he appeals. The most notable of these instances are those where Sartre breaks with Husserl himself, on the question of the *transcendental ego* and later that of the *hyle*; we will examine these instances shortly, though numerous others will appear throughout the course of our analysis. The slippage or displacement that results from the axiomatic status of intentionality has to do with the logical anteriority that this principle enjoys vis à vis any phenomenal description, in such a way that it establishes the very terms in which these descriptions take place (to borrow an expression current in the philosophy of science since Quine, the axiom of intentionality is incorporated into the theory with which every phenomenological observation is laden); from

being a mere ‘result’ of phenomenological description, it goes to become an *a priori* principle of the latter. At several points in Sartre’s text, the axiom of intentionality stands at the beginning of derivations concerning structures which by their very nature cannot be immediately given to consciousness (which nevertheless feels their ‘effects’)—much of what is included under the title of “The Immediate Structures of the For-itself” leans in this direction—and it would not be too much to say that this axiom constitutes the cornerstone on which the whole of Sartrean ontology is afterwards constructed. Klaus Hartmann observes as much in describing Sartre’s ontology as being essentially “an ontology of intentionality” itself.¹⁵ Given the importance of this axiom, there is no better way to begin a study of Sartre’s ontology than with a detailed analysis of the formulation in which Sartre accepts it as his own.

We must first take stock of the absolute character that Husserl’s literal formulation of intentionality has for Sartre. It is absolute in the sense that it is taken to express the very being of consciousness, and not simply to express the attribution of a predicate to a subject. Kirkpatrick and Williams are on the right track when they write, in the Translator’s Introduction to *The Transcendence of the Ego*, that “whereas for Husserl intentionality is *one* essential feature of any consciousness, for Sartre intentionality *is* consciousness” (22). Let us make this observation more precise. In most propositions of the form ‘A is B,’ the ‘is’ functions either as an identity or as a copula joining subject to predicate. In the first case, A and B are taken to be one and the same; in the second, B is a predicate attributed to A. In the formula for intentionality, the ‘is’ is neither a copula nor an identity, though it is closer to the latter than the former. What the formula primarily concerns is the sense in which it is possible to say that ‘consciousness is x’ where x is anything at all. “Consciousness is consciousness *of* something,” is less a predication than it is the antepredicative schema defining the only possible form of predication that consciousness can support: Whatever consciousness may be, “it is this in the mode of consciousness of being” (BN,

II.I.II, 82/118). As we will see, this stipulation has something of a recursive effect on the formula itself, so that it becomes equivalent to the proposition, “consciousness is consciousness of consciousness of something,” and so on.¹⁶ Sartre’s way of handling this regress in a way that does not lead to an infinite series of meta-consciousnesses is quite novel, but we will come to that later.

The *of* in the formula still demands clarification. Intuitively, to say that a consciousness is consciousness of *x* is to say that it *intends* *x*, or is aware of *x*, etc. But let us be more precise. The *of* in the formula designates a relation that is both determinative and differentiating. The differentiating function of the *of* is what prevents the statement “consciousness is consciousness of something” from collapsing into “consciousness is something,” which, by Sartre’s lights, is false. Consciousness’ mode of being is such that it *cannot* simply ‘be’ something; as already mentioned, it can only be something in the mode of *consciousness of being*. To put this another way, and in one which is quite important for Sartre, “consciousness is consciousness of something” means that consciousness is consciousness of something *that it is not*; as Sartre has it, “[i]t is consciousness of _____ as the internal negation of _____” (BN, II.II.I.B, 123).

It would seem that we are headed towards a startling consequence, namely that in a strict sense it is false to say that consciousness is itself, or is identical to itself. Insofar as the axiom of intentionality is the schema of all possible predication for consciousness, we cannot that consciousness is itself, but that consciousness is a *consciousness of* itself. Insofar as the ‘of’ implies an internal negation, consciousness is of itself only to the extent that it is not itself. Sartre does not back away from this consequence in the least, but makes it the focal point of his ontology. The *of* marks a difference that insinuates itself once between the *consciousness* and the *something*, and again between the consciousness and itself; hence consciousness “doubly escapes being, by an internal disintegration and by express negation” (BN, II.II.I.B, 123/158).¹⁷

So much for differentiation. Let us examine what is meant by speaking of the *of* as a determinative relation. This can be put quite simply: consciousness, which is nothing in itself, and which is not identical to itself any more than it differs from itself, is entirely determined by what it is *of*, by what it intends. Just as a set is determined by its elements, so consciousness is determined by its *noema*, by what it is present to. If we agree to call the noematic field of a given consciousness its *extension*, we may express this by saying that as a consequence of the axiom of intentionality, consciousness is *extensionally determined*. Intentionality undercuts intensionality.

This may be put just as well by saying that Sartre's ontological radicalisation of intentionality entails that nothing can be internal or immanent to consciousness; everything is exterior to it. This is why Sartre's first major statement of his theory of intentionality is set up as an attack against Husserl's notion of the transcendental ego, and why one of the several theses advanced in the Introduction to *Being and Nothingness* is that Husserl's accompanying notion of the hyle is illegitimate, "a hybrid being which consciousness rejects and which can not be a part of the world" (BN, in.IV, lix/26). Given the radical formalism of Sartre's acceptance of intentionality, it is possible to condense both arguments to their skeletal form, in the space of a few lines: Both hyle and the transcendental ego are defined as *immanent contents* of consciousness. Insofar as they are predicated of consciousness, this predication must take the form: consciousness is conscious of the transcendental ego (or: consciousness is conscious of the hyle). Already the distinction between the transcendental and the empirical ego begins to waver at this point, though hyle is not yet affected adversely since it is defined as the 'material' of immanent awareness. The differentiating dimension of intentionality offers the finishing blow to both concepts: insofar as consciousness is conscious of hyle/ego, hyle/ego must be exterior and other to consciousness, which is contradictory with their proper concepts. They are not consistent

with consciousness as defined by the axiom of intentionality, and have no place in the ontology founded upon this principle.



As I have mentioned, one of the crucial implications of the axiom of intentionality is that consciousness ‘is itself’ only insofar as it is *consciousness of itself*, and is conscious of anything only on the condition that it *differs* from it. Consequently, consciousness can only be itself while differing from itself. Clearly, this principle poses a problem concerning the individuation of consciousness. As we have seen, the principle of this individuation cannot in any way be an intensional ‘mark’ intrinsic to the consciousness under consideration. Consciousness is devoid of all intensional content. All signs point to the thesis that consciousness is *extensionally* individuated, that what makes a specific consciousness what it is, is nothing other than what that consciousness is conscious *of*. A certain analogy suggests itself with the set-theoretic axiom of extensionality, which states that ‘ α ’ and ‘ β ’ designate the same set if all the elements of α are identical to the elements of β (i.e., $(\forall x)(x \in \alpha \leftrightarrow x \in \beta) \rightarrow \alpha = \beta$). But this cannot be the case if it is entirely illegitimate to write ‘ $\alpha = \beta$ ’ when α and β purport to designate a consciousness. A consciousness is not identical to itself.

We seem to have reached an impasse. Yet there can be no doubt that Sartre himself understood the principle of individuation of consciousnesses to be that of extensionality. Not only is every other path blocked, as a consequence of the absolute denial of any intensional content of consciousness, but we find explicit statements of the extensionality thesis in Sartre’s own text, as, for instance, when we read that “it is by means of that which it is consciousness that consciousness distinguishes itself in its own eyes and that it can be self-consciousness [...]. What defines the reflection for the reflecting is always *that to which it is presence*” (BN, II.II.I, 173/209). It is here that we hit upon our answer, and see that the very proposition that gives us

the problem—that we cannot say that a consciousness *is* itself, but only that it is *of* itself, where *of* is a differentiating relation—contains the key to its solution. The extensionally individuating principle that secures the individuation of consciousness is not identity but *presence*. A consciousness ‘is itself’ precisely insofar as it is consciousness *of* itself, even while it differs from itself by this very reflexion. The argument for the identity of any consciousness is thus by necessity circular, such that when we ask, “how can a consciousness be certain that the consciousness it reflects to itself is in fact itself, if this ‘second’ consciousness is admittedly different than the ‘first’?” the only answer that we can expect to receive is that the ‘second’ consciousness is *conscious* of being the ‘first’ by the very fact that it reflects the ‘first’ to itself. Sartre emphasises that in this primary reflexivity, which he calls “non-thetic (self)consciousness”, consciousness nowhere seizes itself as being *identical* to itself in the strict sense.

Consciousness can exist only insofar as it perpetuates a non-thetic consciousness of itself. This “non-thetic reflexivity”, as I will sometimes call it, is distinct from the reflective attitude of the “I think” (“reflective, that is to say, supplied with an *I*” (TE, 46)). Rather than positing and presupposing a unitary point of reference (such as we might imagine the “I” to be), non-thetic reflexivity generates a problematic sort of unity in its very occurrence, *without needing to posit or be supplied with an I*. Throughout this essay, I will have little to say about the sort of “reflection” that posits an ego, and when I speak of the “reflexivity” of consciousness, what is meant (unless otherwise indicated) is consciousness’ non-thetic intention of “itself”. Nevertheless, it may help to clarify things if I draw attention to the fact that, even though no ego is posited and consciousness is not approached as an ‘object’ in non-thetic reflexivity, Sartre does note a certain structural ‘homology’ between the two modes of “reflection”, and writes that

It is often said that the act of reflection alters the fact of consciousness on which it is directed. Husserl himself admits that the fact ‘of being seen’ involves a total modification for each

Erlebnis. But I believe that I have demonstrated that the first condition of all reflection is a pre-reflective *cogito*. This *cogito*, to be sure, does not posit an object; it remains within consciousness. But it is nonetheless homologous with the reflective *cogito* since it appears as the first necessity for non-reflective consciousness to be seen by itself. Originally then the *cogito* includes this nullifying characteristic of existing for a witness, although the witness for which consciousness exists is itself. (BN, II.I.I, 74/110)

What this homology means, effectively, is that the non-thetic modality by which consciousness is present to itself is a modality of *intentionality*. In its bare ontological structure, we find the same principles at work that we find in the general case where there is “consciousness of something”—that is to say, we find both ‘determination’ and ‘differentiation’. Even when the reflexion is “non-thetic” or, as Sartre sometimes quite awkwardly puts it, “pre-reflective”, consciousness appears in its own extension. That it does not appear in the form of an ‘object’ or an ‘I’ is only to be expected—why would it appear as one, given its manifestly un-objectlike and non-egoic character? We must resist the temptation to fetishize consciousness as a sort of invisible ‘object’ or ‘ego’, a temptation that would lead us to ask ‘where’ in the extension the assemblage of the extension itself takes place. This temptation is all the better resisted if we refrain from confusing the formal ‘extensionality’ of consciousness with a sort of ‘spatiality’. That the term ‘extensionality’ is superficially suggestive of the Cartesian ‘res extensa’ is an unfortunate accident and nothing more. There is nothing intrinsically spatial about the idea of an ‘extension’, even if the naïve imagination has a habit of resorting to spatial constructions to illustrate extensionality to itself.

The ontological necessity of non-thetic reflexivity for the manifestation of consciousness should not be interpreted as a sort of fundamental *ground*, independent of and prior to intentionality in general or thetic intentionality in particular. While it is true that “every positional [i.e. thetic] consciousness of an object is at the same time a non-positional [i.e. non-thetic] consciousness of itself” (BN, in.III, liii/19), it is equally the case that “[i]n order to be a non-thetic self-consciousness, consciousness must be a thetic consciousness of something” (BN,

II.III, 172/208). Ontologically speaking, it is impossible to place one of these ‘dimensions’ of consciousness before the other. So far as order of exposition is concerned, of course, one may work in either direction: from the prereflective cogito to thetic intentionality, as Sartre does in *Being and Nothingness*, or the other way around, as we find in *Transcendence of the Ego*. My own strategy is to first define the intentional structure of consciousness in general, without yet distinguishing between its thetic and non-thetic modes, and then extracting both non-thetic self-consciousness and thetic consciousness-of-something as particular modes of this general structure—a strategy that, to be sure, is possible only in retrospect, as a sort of rational reconstruction performed after the phenomenological groundwork has already been constructed.

These considerations direct us to the difficult problem of exactly *how* the unity of consciousness may be accounted for without any appeal to a pre-given unitary locus of identity, such as may be provided by even an empirical and transcendent ego. This unity, if it is to exist at all, may only be secured through the dynamics of a consciousness which apprehends itself only by differing from itself. If consciousness did not unify itself in the very act of this differing, it would vanish into nothingness in the very event of its manifestation, “as might happen in a dream” (Plato, *Parmenides*, 164^d).



Sartre’s exposition of the non-thetic reflexivity by which consciousness extensionally individuates itself is as subtle and difficult as it is of the utmost importance for his ontology of the subject. To begin with, we must understand that to refer to the ‘two’ consciousnesses involved in non-thetic reflexion as ‘first’ and ‘second’ is a mere notational convenience, which we are forced to adopt only insofar as we are attempting, as far as is possible, to disclose the structure of an event without self-identity by means of manipulating manifestly self-identical marks on paper.¹⁸ Neither of the ‘two’ consciousness(es) is in any definite sense either first or

second, and the duality produced by the reflexion is one of what we may call an “immediate oscillation”: the ‘first’ is no sooner the ‘first’ than it “passes into” the ‘second’ and vice versa. The “passing into” one another of the ‘two’ consciousness(es) is comes about through their symmetrical and mutually indiscernible extensional self-determinations: one cannot involve the other in its intention without becoming the other, the difference between the one and the other becoming its own inverse—in this way the internal difference required by the axiom is maintained, even as its terms slip into one another and change places. The immediately oscillatory nature of this “phantom dyad” finds expression in the fact that for one of its terms to be a consciousness ‘of’ the other is indiscernible from its being ‘for’ the other, which is what Sartre indicates by saying that “originally the cogito includes the nullifying characterisic of existing for a witness, although the witness for which consciousness exists is itself” (BN, II.I.I, 74/110-1*). All that prevents the “immediacy” that joins the ‘two’ consciousnesses from collapsing into an undivided singularity is the ontological law demanding that consciousness cannot be itself save insofar as it is consciousness *of* itself as other than itself. The corollary of this law is that a consciousness can be unitary only insofar as it is *dual*. However, this duality is not such that its terms can be dissociated into independent unities, since each term of the duality is such only insofar as it is involved in the duality of reflexion. Accordingly, it must be understood that the structure we are attempting to explicate is

not a unity which contains a duality, not a synthesis which surpasses and lifts the abstract moments of the thesis and the antithesis, but a duality which *is* unity, a reflection (*reflet*) which *is* its own reflecting (*reflection*). In fact, if we seek to lay hold of the total phenomenon (*i.e.*, the unity of this duality [...]), we are referred immediately to one of the terms, and this term in turn refers us to the unitary organization of immanence. (BN, II.I.I, 76/112)

Sartre supports his analysis of non-thetic reflexivity (the pre-reflective cogito) with an examination of the phenomenon of *belief*, where belief serves simply as a particular case of a more general structure. The results of this examination are worth repeating here at some length:

by the sole fact that my belief is apprehended as belief, it is *no longer only belief*; that is, it is already no longer belief, it is troubled belief. Thus the ontological judgment ‘belief is consciousness (of) belief’ can under no circumstances be taken as a statement of identity; the subject and the attribute are radically different though still within the indissoluble unity of one and the same being.

Very well, someone will say, but at least we must say that consciousness (of) belief is consciousness (of) belief. We rediscover identity and the in-itself on this level. [...] But that is not true: to affirm that consciousness (of) belief is consciousness (of) belief is to dissociate consciousness from belief, to suppress the parenthesis, and to make belief an object for consciousness; it is to launch abruptly onto the plane of reflection. A consciousness (of) belief that would be only consciousness (of) belief would in fact have to assume consciousness (of) itself as consciousness (of) belief. Belief would become a pure transcending and noematic qualification of consciousness; consciousness would be free to determine itself as it pleased in the face of that belief. [...] Husserl [...] has shown the fact that only reflective consciousness can be dissociated from what is posited by the consciousness reflected-on. [...] The consciousness (of) belief, while irreparably altering belief, does not distinguish itself from belief; it *exists in order to* perform the act of faith. Thus we are obliged to admit that the consciousness (of) belief is belief. At its origin we have apprehended this double game of reference: consciousness (of) belief is belief and belief is consciousness (of) belief. On no account can we say that consciousness is consciousness or that belief is belief. (BN, II.I.I, 76-7/111)

The integral duality of non-thetic reflexivity is experienced as “troubled”—in the sense in which one speaks of “troubled waters”—owing to its palpable and perpetual act of differentiation from itself, its condition of existing “from the start as escaping itself” (BN, II.I.I, 77/111). This is the ontological basis for the familiar Sartrean theme of “bad faith,” which is to some extent inevitable so long as one defines its opposite, “[t]he ideal of good faith,” as “to believe what one believes,” since it is a consequence of belief’s ontological structure that “[e]very belief is a belief that falls short; one never wholly believes what one believes” (BN, I.II.III, 69/105). Of course, neither does one wholly disbelieve what one believes, nor wholly withdraw from the belief one is in the act of believing.

It is in terms of this evanescent and nihilating relation of a consciousness to itself that Sartre situates the concept of *self*, or “interiority,” as he calls it in *The Transcendence of the Ego*. These terms indicate structures to which purely intentional consciousness has given rise, as such the sense that they receive in Sartre’s thought is far removed from the more or less monadic and substantial sense which we may otherwise be inclined to give them. The self is not a substance, and its interiority cannot be conceived as inherence or containment within a substance. The term

‘self’ indicates only an evanescent relation of reflexivity in consciousness, by which consciousness unifies itself as a singular consciousness while in the same stroke maintaining an immanent distance from itself. In Sartre’s words,

The *self* therefore represents an ideal distance within the immanence of the subject in relation to himself, a way of *not being his own coincidence*, of escaping identity while positing it as unity—in short, of being in a perpetually unstable equilibrium between identity as absolute cohesion without a trace of diversity and unity as a synthesis of a multiplicity. (BN, II.I.I, 77)

In *The Transcendence of the Ego*, where the concept of consciousness’ immanent self-differentiation has not yet been developed, this configuration is anticipated under the name of “interiority,” which means “[s]imply that to be and to be aware of itself are one and the same thing for consciousness.”¹⁹ Consciousness’s interiority is said to be “pure” because it is exhausted in its act. For Sartre, this purity entails that the “absolute interiority” of consciousness “never has an outside” (TE, 84). This assertion must be carefully inspected, for here rests one of the more subtle points of Sartre’s ontology. On the surface, the claim appears to be plainly false, and contradictory with Sartre’s entire theory of intentionality—does not consciousness have *nothing but* an outside, insofar as *everything* is outside of consciousness? But “outside” means something quite different here. To say that *consciousness has no outside* is to say that there is no outside to consciousness that *is consciousness*. To pursue this spatial metaphor (we must take it as no more than this), this amounts to saying that consciousness has not ‘outer surface’. In this sense, it has no *outside* for the same reason that it has no *content*; as a pure event, it is non-substantial and without identity, nothing more than its direction upon the world. It does not determine itself as an object in the world, and has no being in excess of its appearance to itself. To say that consciousness *has* an outside would be to say that in addition to its intentional act, it would, without ceasing to be consciousness, determine itself as a being-in-itself, as a thing disclosed to the world. (It is worth recalling, in this respect, that Sartre often uses “being-in-itself” and “being-in-exteriority” as synonyms.) Consciousness would then *be* something—but

this is prohibited by the axiom of intentionality insofar as it implies that consciousness can have being *only* insofar as it is consciousness *of* being. Possessing neither exteriority nor identity, consciousness is “radically impenetrable” to the consciousness of the other, “not only refractory to intuition but to thought,” since in order for a distinct consciousness to adequately conceive of it, it “would have to think of it as pure interiority and as transcendence [*sc.* as a transcendent object] *at the same time*, which is impossible” (TE, 96). This is why, beyond an empty and formal cognition, “[a] consciousness cannot conceive of a consciousness other than itself” (TE, 96).

While intuitively sound, this thesis remains something of an enigma in *The Transcendence of the Ego*, as it does elsewhere in Sartre’s writings. Yet it is possible to shed valuable light on the problem of inaccessibility by recourse to the thesis of extensionality, in conjunction with the thesis of non-identity. The thesis of extensionality, which, as we have seen, derives from the axiom of intentionality, states that a consciousness is determined by nothing beyond what it is a consciousness *of* (*i.e.*, its extension). The thesis of non-identity, which stems from the same source, states that the extension of a consciousness supplies consciousness with its sole locus of identification. No additional identity is produced by the withdrawal of consciousness from its extension, but this withdrawal nevertheless serves to establish the *non-identity* of the consciousness with its extension. The way in which these theses bear upon the problem of inaccessibility can be explained as follows.

Let us suppose the existence of two distinct consciousnesses, ϕ and $\tilde{\phi}$.²⁰ If $\tilde{\phi}$ is to have direct consciousness of ϕ , it is clear that this can only be supplied so long as $\tilde{\phi}$ is fully conscious of the sole locus of identity of ϕ . This locus is not ‘in’ ϕ itself, but rather is wholly exhausted in the *extension of* ϕ . In order for $\tilde{\phi}$ to be conscious of the locus of identity of ϕ , $\tilde{\phi}$ would have to ‘stand in ϕ ’s shoes’ and be fully conscious of ϕ ’s extension. But, if $\tilde{\phi}$ is fully conscious of this

extension, then ϕ and $\tilde{\phi}$ would be identical-in-extension, and would not be distinct consciousnesses.²¹ They would not, of course, be *identical* to one another either, since this attribute is denied to consciousness by Sartre's radicalisation of intentionality. In the event of 'extensional identity' (wherein 'two' consciousnesses intend exactly the same extension), ϕ and $\tilde{\phi}$ could, at most, be "dual", or "co-present". That is to say, $\tilde{\phi}$ could only be directly conscious of ϕ by being conscious of itself in the face of the extension of ϕ , and at this point ϕ and $\tilde{\phi}$ would form "a duality which *is* unity" (BN, II.I.I, 76/112). Note that the same result would arise no matter how many names we give to the internally indiscernible and phantom complex of consciousness(es) that is (or are) conscious of the extension of ϕ ; if we speak of two, it is because 'two' names a sort of minimal dehiscence to which any other would be ultimately reduced. I speak of 'names' here to emphasise the fact that any supposed distinctions are purely nominal and theoretical, assumed only for the sake of argument. As we have seen, it is quite misleading to conceive of the duality given in non-thetic reflexion as a discrete "two"; the "duality" of reflective consciousness names only the perpetually unstable unification and self-differentiation that any consciousness with a single and definite extension undergoes. The radical inaccessibility between distinct consciousnesses is rooted in extensionality, and it is the reason why the reflexivity of (self)consciousness must be understood as *immediate*—that is, it must be held to take place in the "instantaneous nucleus" of a consciousness of something.²²

The physical distinction between bodies is the contingent reason why, in actual fact, the extensions of two distinct consciousnesses cannot be brought to coincide, insofar as the body situates and specifies the extensional field of its consciousness.²³ The physical impenetrability of bodies, however, can support the mutual inaccessibility of consciousnesses by virtue of the fact that the locus of identity of a consciousness is precisely the extension of that consciousness, the totality of phenomena of which that consciousness is consciousness *of*. It is possible for the

separation of bodies to entail the separation of consciousnesses insofar as intentional consciousness is extensionally determined. Conversely, it is because consciousness is purely extensional, and purely insubstantial, that it is entirely acceptable to say that if it were possible for “another” consciousness to occupy my position in its entirety, then that consciousness would be no more distinct from mine than mine is from itself. (Such thought-experiments are useful in disentangling our habitual identification of ‘the I’ with ‘consciousness’.)

To avoid misunderstanding, let us note here that we are not speaking of a ‘ghost in the machine’ when we speak of consciousness as distinct from the body which situates it. Consciousness is not ‘something else’ that needs to be synchronised with a body; it is an event that takes place precisely at the material site of the body. These are, at least, the only sites we know of where such an event might take place.²⁴



What has been demonstrated in these analyses of the consequences stemming from the axiom of intentionality is that consciousness—insofar as it is only ever consciousness *of* something, from which it differs—is a form of *multiplicity*. This is to say that consciousness satisfies both of the two criteria set forth in the definition of multiplicity outlined in § 2 of the Introduction: (1) Its individuation is strictly determined by its extension, though not in the mode of identity. (2) It nonetheless *differs* from this extension. To say that consciousness is a form of multiplicity preserves the full conceptual purity of the latter, insofar as both criteria are satisfied absolutely and *ex nihilo*: The constitutive difference is a function of nothing other than the dimension of extensional determination itself—an intention is at once a determination by and a differentiation from what is intended.



In the foregoing analysis we nevertheless seem to have taken a rather complicated route, pursuing the notion that consciousness is extensionally determined, while forbidding ourselves the theoretical convenience of defining this determination as identity. It would certainly be simpler to say that a consciousness is identical with 'another' consciousness if and only if their extensions are identical. This would, of course, bind the Sartrean theory of consciousness to an analogue of the set theoretic axiom of extensionality, as noted above. Consciousness, as Sartre understands it, cannot satisfy this axiom insofar as it cannot be conceived as a being-in-itself. Of course, Sartrean consciousness is never entirely unmoored from the in-itself, and it is always possible to isolate in thought that aspect of consciousness that is entirely specific to the in-itself alone, and which can be thought in terms of identity. This is what Sartre wishes to isolate when he speaks of consciousness "insofar as it *is*," or, alternatively, of the *facticity* of consciousness. It must be kept in mind that this concept is an abstract one, and is something of a retrospective fiction.²⁵ Facticity, which is neither consciousness nor being in itself, but that which is in-itself for a given consciousness, is never given independently. It is nevertheless a useful abstraction, and we may as well attempt to formalise it further.

In the light of the foregoing analysis of the axiom of intentionality we may put forth a structural definition of facticity: given a consciousness ϕ , we will call $F(\phi)$ the *facticity* of ϕ , and define it as a self-identical multiplicity whose extension is identical to the extension of ϕ , once we remove from this extension ϕ itself. $F(\phi)$ is therefore a *wellfounded* multiplicity. This means that there are no 'loops' or reflexivities in the membership relation defining $F(\phi)$. A facticity is a *set*, and the law of identity for facticities will simply be the set-theoretic axiom of extensionality itself, so that $F(\phi) = F(\tilde{\phi})$ if and only if $(\forall x)(x \in F(\phi) \leftrightarrow x \in F(\tilde{\phi}))$. Let us note that, for reasons explained above, $F(\phi) = F(\tilde{\phi})$ implies that ϕ and $\tilde{\phi}$ form "a duality which is unity". All of this follows from the Sartrean conception of the facticity of a consciousness as the being of that

consciousness *insofar as it is*. The operator 'F' (read: 'facticity of...') that is employed in this definition will prove somewhat useful through the course of this project, for it is implicitly utilised throughout the theory of the subject deployed in *Being and Event*. In general, if x is any multiplicity whatsoever, then $F(x)$ is the wellfounded and self-identical multiplicity minimally different from x , which is arrived at by subtracting from x all non-wellfounded elements and submitting the resulting aggregate to the regime of identity specified by the axiom of extensionality.²⁶

Sartre often speaks of facticity as being consciousness' aspect of contingency, the fact that it does not and cannot 'found' its own being. We must be careful with our terminology here, for 'found' does not mean the same thing in *Being and Nothingness* as it does in set theory, and, by extension, in *Being and Event*. Fortuitously, there is a certain degree of overlap between the two concepts. We say that a set is *founded* by one of its elements when it has nothing in common with that element. This is not to be confused with saying that a set is *wellfounded*, in which case its membership relation is without loops or infinite descents. A set may be at once founded and not wellfounded, though in Chapter V we will prove that the axiom that *every* set is founded implies that every set is also wellfounded. While it is meaningless or trivial in this sense to say that consciousness does not found itself (by definition, nothing can found itself), to say that its being is always founded by something external to it is true in both the set-theoretic and the Sartrean sense of the term. The implication of this statement in either sense entails that any ontology that begins with consciousness is committed to the existence of something that is *not* consciousness, for it implies that consciousness cannot be a *mere* conscious of itself.

II. Being-in-itself: First Descent

Le Néant parti, reste le château de la pureté.

— Stéphane Mallarmé, *Igitur*.

An extensional conception of consciousness evidently implies the existence of its extension. There is, in fact, a common custom of appealing to this implication in order to determine whether or not such and such a context can rightfully be treated as extensional. Tim Crane, for instance, explicitly focuses on this consequence of extensionality in his formulation of the intensional/extensional difference as it applies to logical and linguistic contexts (Crane, 1995, 33). He characterises an extensional context as one in which it is always possible to apply the principle of existential generalisation to its terms. Hence, a statement of the form $P(a)$ is considered to be extensional only when it is legitimate to derive from it the statement $(\exists x)(P(x))$.²⁷ He also adds a second criterion, more specifically tailored to linguistic situations: if a and b refer to the same real entity X , then it must be legitimate to infer $P(b)$ from $P(a)$.²⁸ Clearly, both criteria presuppose a certain ontological framework. Without a certain set of ontological commitments, and without a clear sense of the relation between a given ‘context’ and the real (a relation which Crane calls *reference*), the first criterion would simply be undecidable. In contexts of the sort that Crane is concerned, the validity of existential generalisation has its ground in the speaker’s commitment to a second context that is baptised as the referential target of the first; the validity of existential generalisation is only decidable in light of the postulation of both this second context and the reliability of the referential relation. Extensionality as Crane envisions it is a derivative characteristic, and one that cannot be derived on the sole basis of the context in question (as indicated, at least two other components are needed: a second context—deemed real—and a referential relation bridging the two). The situation is quite different when the principle of extensionality—qua intentionality—is taken as a Cartesian starting point, and when the

concomitant ontological commitment is taken as a consequence and not a measure of extensionality.

It is the latter of the two cases which characterises the understanding of extensionality proper to the Sartrean theory of consciousness. The extension of a consciousness consists of phenomena, or appearances (we will treat the words as synonymous, as Sartre does). Insofar as Sartre's radicalisation of intentionality entails the extensionality of consciousness, Sartre is committed to what could be called a *phenomenal realism*: that which appears, exists. Sartre is, of course, perfectly aware of this; his commitment to phenomenal realism stands as one of the primary motivations behind his formulation of the principle of intentionality—his “Intentionality” article sufficiently demonstrates this fact. Accordingly, Sartre begins *Being and Nothingness* by acclaiming the fact that “Modern thought has realized considerable progress by reducing the existent to the series of appearances which manifest it (BN, in.I, xlv/11).” What this reduction opposes itself to is what may be called the ‘radial’ or ‘representational’ model of appearances. According to this view, the existent is not identical with the multiplicity of its appearances, but is understood as being, in one way or another, the real entity ‘behind’ the appearances—the entity whose reality permits us to say that the appearances in question, which are not held to be ‘real’ in any meaningful sense themselves, may *refer* to something real. The result of this reduction is that we may now conceive of the appearance as “a full positivity,” and no longer as an illusion drawing its being from what Nietzsche called “a world behind the scenes” (BN, in.I, xlv/12). Being and appearance are collapsed into a single stratum, such that no room is left for a referential relation between appearance and reality. It is possible to situate this collapse quite clearly in terms of Crane's criteria: in the radial theory of appearances, which Sartre takes modern thought to have deposed, the world of appearances is treated as a referential context, where the relation ‘*a* is an appearance of *X*’ stands for the existentially legitimating

relation of reference, ‘*a* refers to *X*’, where *X* is a reality that does not appear, and which unlike *a* can properly be said to *be*. So long as such a relation is believed to hold, *a* may “borrow” its being from *X*.²⁹ To collapse of the dualism of being and appearance is to collapse *X* entirely into *a*, and substitute for a referential relation the immediacy of being-in-itself. The effect of this collapse is that Crane’s criterion of existential generalisation is immediately true for appearances, and the criterion of substitutability, insofar as it does not become meaningless, is now trivially valid, for “the being [the real referent] of an existent is exactly what it *appears*” (BN, in.I, xlvii/12). In order to dispel any equivocation, let us distinguish between the *mediate* extensionality that may be attributed to referential contexts and the *immediate* extensionality proper to multiplicities, such as consciousness. A *mediately* extensional context requires something else, a supplementary relation to the real, in order to be qualified as extensional. An *immediately* extensional context does not. What it presents is directly determined as real without calling upon referentiality in any way; here extensionality is determined prior to both reality and referentiality, whereas in mediate contexts it is determined only in view of these components. Evidently, a *mediately* extensional context, insofar as is not a pure transparency directly revealing its extension, is intensionally ‘coloured’ by its immanent contents. Its determination is not strictly extensional at all; rather, the attribution of extensionality to such a structure is contingent upon this stratum of intensional determination (insofar as these intensions will come to function as appearances signalling a reality beyond themselves). In the last instance, *mediately* extensional contexts are intensionally determined; their extensionality is mediated by intensionality.

Among the stakes of Sartre’s introduction to *Being and Nothingness* is a dispute with Husserl over precisely such intensional mediation. It is Husserl who is tacitly recognised as standing at the forefront of the revolution in modern thought that “has realized considerable

progress by reducing the existent to the series of appearances which manifest it (BN, in.I, xlv/11).” Sartre applauds Husserl for having restored to the phenomenon itself the full dignity of being, and for having done away with the unjustifiable reliance on a second context which would justify, via a relation of reference, the reality of the first (without, apparently, needing any justification of its own). The phenomenon itself manifests the full dignity of being, and it is on the phenomenal level that the transcendence of what appears is rooted. However, for Husserl, as Sartre reads him, this transcendence is not *immediate*. It is mediated through a complex of intensional determinants that are fully immanent to the phenomenon, and, unlike Crane’s linguistic contexts, do not reach beyond the context of the phenomenal as such (they do not refer towards a non-phenomenal reality). The relation is, again, one of *reference*, but it is a purely ‘horizontal’ structure of reference that is at stake: appearances again refer, but to other appearances, and not to a non-apparitional reality. The transcendence of what appears is simply the fact that appearances are bound up in an infinite series of other appearances. The apparitional bottle transcends me insofar as each of its appearances refer me to an infinity of others which are absent: the same bottle from behind and below, the bottle empty or full, the bottle when smashed, etc. As Sartre recounts, the Husserlian

theory of the phenomenon has replaced the *reality* of the thing by the *objectivity* of the phenomenon and that it has based this on an appeal to infinity. The reality of that cup is that it’s there and that it *is not me*. We shall interpret this by saying that the series of its appearances is bound by a principle which does not depend on my whim. But the appearance, reduced to itself and without reference to the series of which it is a part, could be only an intuitive and subjective plenitude, the manner in which the subject is affected. If the phenomenon is to reveal itself as *transcendent*, it is necessary that the subject himself transcend the appearance toward the total series of which it is a member. [...] Thus the appearance, which is *finite*, indicates itself in its finitude, but at the same time in order to be grasped as an appearance-of-that-which-appears, it requires that it be surpassed towards infinity. (BN, in.I, xlvii/13)

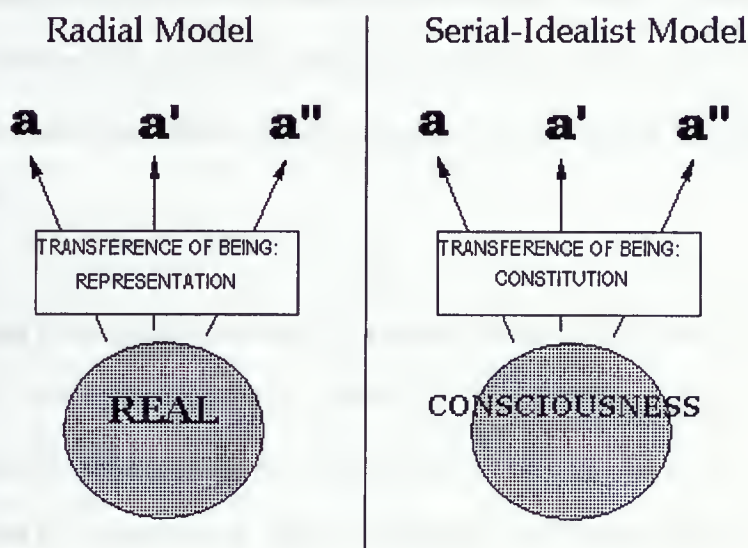
The principles which bind the apparitional series are precisely what Sartre, following Husserl, calls *essences*, and which, we have already seen, are simply another name for intensional determinants, or class predicates. By means of this complex of intensional determinations, the

phenomenon again has purchase on transcendence by way of reference, and the dualism between appearance and reality is reproduced within appearance as “a new dualism: that of the finite and the infinite” (BN, in.I, xlvii/13). The radial model is somewhat ‘flattened’, but this deformation does not change its essential structure.

Sartre reveals another similarity between Husserl’s idealist theory of appearances and its radial ancestor, which vitiates the claim of the new model to have done away with the phenomenal antirealism of its elder. As Sartre observes, the idealist appeal to infinity is something of a smokescreen. It is not the infinitude of the series as such that ensures transcendence, but the permanent gap between this infinity and the finitude of the given. The ultimate foundation of objectivity, in the end, “is defined as *lack*” (BN, in.V, lxi/27). If there were a consciousness that *could* comprehend, within its own immanence, an infinite series of appearances, then these appearances—“even if infinite in number—would dissolve in the subjective” (BN, in.V, lxi/27); they could consistently be taken as nothing more than “animations of the hyletic nucleus” (BN, in.V, lx/27). If this is the case, Sartre argues, then it is mere “slight of hand” (BN, in.V, lxi/27) to propose that we can bestow transcendent objectivity on the phenomenal series by appealing to what is, in the last analysis, simply another immanent figure of consciousness—namely, its lack of infinity, its finitude. It is this lack which endows the horizontal referential structure of the serial model with the power of objectification, and which lets it have purchase on the real. Sartre forcefully rejects this path. All that we may hope to gain from appealing to this immanent handicap is an illusory objectivity, an imagined transcendence: “the objective will never come out of the subjective nor the transcendent from immanence, nor being from non-being” (BN, in.V, lxi/27).

Sartre thus argues that by taking the idealist route, which begins from the hyletic immanence of consciousness, Husserl “makes of the *noema* an *unreal*, a correlate of the *noesis*, a

noema whose *esse* is *percipi*” (BN, in.V, lxi/27). The reality of the appearance is once again “borrowed,” secured only by a transference of being, and by a referential structure touching on an absolute entity, be it transcendental object or transcendental subject. Reduced to their bare essentials, the principle difference between the radial and idealist models is that the idealist locates the source of being at the end of the relation “x is an appearance for...” rather than “x is an appearance of...”, and calls this loan of being *constitution* rather than *representation*, but these differences pale beneath the prevailing homology.



Both models fail to draw the full consequences of the axiom of intentionality. In terms of the radial model, it is only because the appearance is taken as a *content* of consciousness that it makes sense to speak of it as distinct from something whose appearance it may be. In terms of the idealist model, it is only because the appearances are held to be *immanent* to a consciousness that the problem of reaching beyond this immanence arises. For Sartre, this agreement between the two models is of decisive importance. He accuses Husserl of letting the solution to the problem of transcendence slip through his fingers, for

Husserl defines consciousness precisely as transcendence. In truth he does. This is what he posits. This is his essential discovery. But from the moment that he makes of the *noema* an *unreal*, a

correlate of the noesis, a noema whose *esse* is *percepi*, he is totally unfaithful [*infidèle*] to his principle.

Consciousness is consciousness of something. This means that transcendence is the constitutive structure of consciousness; that is, that consciousness is born supported by a being which is not itself. This is what we call the ontological proof. (BN, in.V, lxi/28)

The ontological proof of which Sartre speaks is nothing other than a demonstration of the validity of extensional generalisation in an immediately extensional context. If consciousness is pure intentionality, if it has no immanent being outside of its transcendence towards what it is of, then it cannot be the source of the transcendent being of the phenomena that it reveals. The very revelation of the phenomena entails the trans-subjective or extra-subjective being of the phenomena. The problem of transcending subjective plenitude is done away with altogether, for Sartre, for the very reason that subjective plenitude does not exist. There is plenitude only outside the subject.



Sartre is thus faced with the necessity of creating an ontological concept adequate to the thesis that the being of the phenomenon is immediate to it, without any need to pass through the red tape of intensional determination, while nevertheless being *other* than consciousness. It is necessary to say that the appearance *is*, without mediation, but it is equally necessary to say that its being is not identical with its appearing to consciousness—its *esse* is not *percepi*. The concept that Sartre fashions for this purpose is that of the *transphenomenal being of the phenomenon*. At stake here is neither the Kantian *thing in itself* nor the Heideggerian *meaning of being*, as he is at pains to impart. Transphenomenal being is neither noumenon nor phenomenon.³⁰

“We must understand,” he writes,

that this being is no other than the transphenomenal being of phenomena and not a noumenal being which is hidden behind them. It is the being of this table, of this package of tobacco, of the lamp, more generally the being of the world which is implied by consciousness. It requires simply that the being of that which *appears* does not exist *only* in so far as it appears. The

transphenomenal being of what exists *for consciousness* is itself in itself (*lui-même en soi*). (BN, in.V, lxii/29)¹

The requirement that that which appears not exist only insofar as it appears is not a requirement for *something else that does not appear*—as is the case for the radial and idealist models of appearances—but for the independence of the being of the appearance from the fact that this appearance appears for a consciousness. That no entity other than the appearance in question is required distances the notion of transphenomenal being from the noumenon.

What distances this notion from that of the phenomenal meaning of being is precisely the distinction between the transphenomenality of the former and the phenomenality of the latter. Transphenomenal being is the non-hidden yet non-phenomenal foundation of the manifestation of phenomena; it is in each case the pure fact that the phenomenon *is*. “Consciousness,” Sartre writes, can always pass beyond the existent, not toward its being, but toward the *meaning of this being*.” If we were to identify the being of the phenomenon with the phenomenon (or meaning) of being, we would be again rooting the reality of the given in an essence; this essence, to be sure, would be common to *all that is*, but this would not save us from the difficulties that Sartre uncovers in Husserl’s serial theory of objectivity. “The passage from the particular object to the essence is a passage from homogeneous to homogeneous,” Sartre writes, “It is the same for the passage from the existent to the phenomenon of being” (BN, in.II, xlviii/15). Neither passage connects the phenomenon with its being, which nonetheless remains indifferently implicated throughout the entire phenomenal parade that probing the meaning of being brings forth:

The phenomenon of being is not being, as we have already noted. But it indicates being and requires it—although, in truth, the ontological proof which we mentioned above is *notespecially* or *uniquely* valid for it; there is *one* ontological proof valid for the whole domain of consciousness. (BN, in.VI, lxiii/29-30)

It is precisely this utterly indifferent ubiquity of transphenomenal being that Sartre seeks to formally indicate by way of a crude set of postulates. There are three in total.

It should be noted that Sartre stresses their “provisional,” “superficial” and “incomplete” character of these postulates (BN, in. VI, lxiii, lxvii/30,33), already giving them a quite different status from the apodeictically certain axiom of intentionality. He also is careful to insist upon their restriction to the being of the phenomenon in itself; the essential characteristics of being-in-itself are intransitive to the being of consciousness, which Sartre names ‘being-for-itself’ (BN, in. VI, lxiii/30). The three postulates are deceptively simple in the form in which Sartre presents them at the end of the Introduction (BN, in. V, lxv-lxvii/32-3). They are:

- (I) Being is in-itself.
- (II) Being is what it is.
- (III) Being is.

What follows is a brief summary of the meaning of these three statements.

First Postulate. One of the dangers of Sartre’s mode of exposition in his three postulates is that he writes as if he is attributing predicates to *an* entity, called Being. This is not at all the case. Being-in-itself is not a substance from which phenomena draw their ontological support—the phenomenon “is not supported by any existent different from itself; it has its own *being*” (BN, in. II, xlviii/14), or as Sartre more correctly puts it, the phenomenon “does not *possess* being, and its existence is not a participation in being, nor any other kind of relation. It is” (BN, in. II, xlix/15; e.a.). The first postulate could thus more accurately be stated, *the phenomenon is in-itself*. One aspect of the first postulate, in fact, means just what I have said here: that the phenomenon *is* in virtue of itself alone, that its being is intrinsic to it. The second sense of this postulate, closely related to the first, is that the phenomenon, insofar as it *is* (as opposed to insofar as it is revealed), maintains its being in *absolute* immanence, or rather in an absolute coincidence that is the ideal limit of immanence. Insofar as “immanence in spite of all *connection* with self is still that very slight withdrawal which can be realized” (BN, in. V, lxv/31-

2), we must distinguish this limit from immanence properly speaking. “It is an immanence which can not realize itself, an affirmation which can not affirm itself, an activity which can not act” (BN, in.V, lxv/32).

Second Postulate. The second postulate simply expresses the first in another form. Being-in-itself is here understood as *identity*. Sartre’s exposition of the second postulate consists in the main of radicalising his exposition of the first, writing that “if being is in itself, this means that it does not refer to itself as self-consciousness does. It is this self. It is itself so completely that the perpetual reflection which constitutes the self is dissolved in an identity” (BN, in.V, lxv/32). Identity is essentially a second name for the radical *enstasis* characterised by the first postulate. To conceive of this identity as a relation, as when we write $a = a$ for example, it is only to approximate it. It is, rather, a “null relation” (BN, II.I.III, 88/125), or better, an anti-relation (“the principle of identity is the negation of every species of relation at the heart of being-in-itself” (BN, II.I.I, 77/113)). It is that which, in the phenomenon, tends towards isolation. Insofar as it is (self)-identical, or enstatic, each being is “isolated in its being,” and “does not enter into any connection [*rapport*] with what is not itself” (BN, in.V, lxvi/32).

Sartre ventures some puzzling remarks in his discussion of the second postulate which have led to much confusion amongst his commentators. He writes that, due to the enstatic identity of being,

Transition, becoming, anything which permits us to say that being is not yet what it will be and that it is already what it is not—all that is forbidden on principle. For being is the being of becoming and due to this fact it is beyond becoming. It is what it is. This means that by itself it can not even not be what it is not [*il ne saurait même pas ne pas être ce qu’il n’est pas*]; we have seen indeed that it can encompass no negation. (BN, in.V, lxvi/32)

This is not to say that the world is in truth purely static, or that, as the Eleatic philosophers used to say, all change is illusory. There is becoming; things that exist in themselves do become other things. Sartre’s thesis is simply that becoming does not reach into the ontological modality of what becomes. What becomes never ceases being in absolute coincidence with itself: “It is, and

when it gives way, one can not even say that it no longer is. [...B]eing itself does not exist as a lack there where it was; the full positivity of being is re-formed on its giving way" (BN, in.V, lxvi/33).³² Sartre offers a more lyrical account of the same thesis in *Nausea*:

Of course a movement was something different from a tree. But it was still an absolute. A thing. My eyes only encountered completion. The tips of the branches rustled with existence which unceasingly renewed itself and which was never born. The existing wind rested on the tree like a great bluebottle, and the tree shuddered. But the shudder was not a nascent quality, a passing from power to action; it was a thing; a shudder-thing flowed into the tree, took possession of it, shook it and suddenly abandoned it, going further on to spin about itself. All was fullness and all was active, there was no weakness is time, all, even the least perceptible stirring, was made of existence.³³

The denial of *differences* among the in-itself must be understood similarly. Sartre clarifies this point elsewhere in *Being and Nothingness*:

The being which is what it is must be able to be the being which is not what it is not. But in the first place this negation, like all others, comes to the surface of being through human reality [...] and not through a dialectic appropriate just to being. In addition this principle [c. of non-contradiction] can denote only the relations of being with the *external*, exactly because it presides over the relations of being with what it is not. We are dealing then with a principle constitutive of *external relations* such that they can appear to a human reality present to being-in-itself and engaged in the world. This principle does not concern the internal relations of being; these relations, inasmuch as they would posit an otherness, do not exist. The principle of identity is the negation of every species of relation at the heart of being-in-itself. (BN, II.I.I, 77/113)

Being-in-itself is therefore not such that it *cannot* support differences; rather, the differences which may be grounded in the in-itself are, *qua differences*, external to the being-in-itself that they differentiate. The absence of internal relations—differentiating or otherwise—in being-in-itself signifies that being-in-itself is nowhere engaged in an act of differentiation, since in this act it would be forced to transcend its identity, a transcendence prohibited by the postulate of enstasis (first postulate) as well as by the postulate of identity (second postulate).

It is worth repeating here that in setting difference, relationality and becoming aside from being-in-itself, we are not seeking to separate a thing-in-itself in relation to which these phenomenal structures remain merely 'ideal' or illusory. "[T]he In-itself," we must recall,

is not opposed to phenomena as the noumenon is. A phenomenon is in-itself, according to the very terms of our definition, inasmuch as it is what it is, even if it is in relation with a subject or another phenomenon. Moreover, the appearance of *relation* as determining the phenomena in

connection with each other supposes antecedently the upsurge of an ek-static being which can be what it is in order to establish the 'elsewhere' and relation in general. (BN, II.III.IIB, 143/178-9*)

Third Postulate. It is clear that in the expression 'being is', the 'being' mentioned cannot refer to an entity or a substance underneath the phenomenon. The postulate "being is" may, with more accuracy and less grandeur, be rewritten "stuff is," or, alternatively, "phenomena are." As such, it seems like a rather empty statement, comparable to Quine's infamous response to the question, "What is there?" (Quine's answer: everything.³⁴) Sartre's emphasis is a bit more precise, however. His point is, as he says elsewhere, that the "in-itself is actuality [*L'en-soi est en acte*]" (BN, II.I.IV, 98/135). Being-in-itself is absolutely actual, absolutely underived, and, consequently, absolutely contingent. There is nothing higher from which it might descend, and nothing lower from which it might emerge. Its actuality is does not follow from any necessity that would precede it, for "[n]ecessity," as Sartre understands it, "concerns the connection between ideal propositions but not that of existents" (BN, in.VI, lxvi/33). (Here, we must recall that the ontological proof, which takes the existence of consciousness as its premise and transphenomenal being as its conclusion, does not materially *necessitate* the in-itself, only logically entails its reality, just as Descartes' (or Anslem's) proof does not create God.) This thesis is at the heart of Sartre's well-known conception of being as *contingent*, and is a significant component of his atheism. There is no sufficient reason for being. It is "*de trop*" (BN, in.VI, lxvi/33), unnecessary and without purpose or cause. By the same token, being-in-itself cannot subsist as *possibility*, insofar as no sufficient reason could command its actualisation. "Being-in-itself is never either possible or impossible. It *is*" (BN, in.VI, lxvi/33). Like relation, becoming, and difference, possibility exists as such only in the light of the for-itself.



As I have mentioned above, Sartre's presentation of his postulates have led to a few unfortunate misunderstandings amongst his commentators. The most disquieting of these is the

opinion according to which Sartre's vision of being-in-itself is more or less in agreement with Parmenides.³⁵ Given that this misunderstanding is suggestive enough to have led to its explicit endorsement by one of Sartre's otherwise more astute readers only increases the urgency with which it demands correction. The Parmenidean interpretation of Sartrean being-in-itself, as advanced by Klaus Hartmann, runs as follows:

This characterization of being reminds us, even down to the details, of Parmenides' account of being. Parmenidean being is identical, solid, without origin, not subject to necessity, without deficiency, and thus non-referential. Its continuity corresponds to Sartre's thesis that being is 'everywhere'. (Hartmann, 35)

Let us first clear away a handful of the more obvious errors in this comparison, before we get to the root of the problem, which concerns the equation of being and the One. For the most part, these pertain less to Hartmann's reading of Sartre than his reading of Parmenides.³⁶

(α) It is true that Sartrean being-in-itself is not subject to necessity, but it is simply false to say the same of Parmenidean Being. In the *Way of Truth*, Parmenides insists that Being is such that "overmastering Necessity holds it in the bonds of a limit that fences it about."³⁷ Similar formulations recur throughout the poem, sometimes with the word 'Necessity', sometimes 'Destiny.' It must be understood, of course, that Parmenidean Being is *self-necessitating*, insofar as it is *causa sui* and *self-limiting*. I will later propose that this is precisely the concept of the One that is at stake for both Sartre and Badiou (who both reject it): the identity of limit and limited. The identity of Being and Necessity in the One may mitigate Hartmann's apparent error of claiming that Parmenidean Being is "not subject to necessity," since the One, at least, is not subject to any *other* necessity, but this is all the more reason to distance Parmenides from Sartre. Parmenidean Being, if it resembles anything in the Sartrean ontology, is closer to what Sartre calls the *ens causa sui*, the impossible synthesis of the in-itself and the for-itself. I will return to this figure in Chapter IV and again in Chapter VII, but suffice it for now to note that Sartre considers such an entity, which provides for the necessity, the ground, the unity and the identity

of its own existence, to be *impossible*. It takes its place in Sartre's ontology as a real but inexistent region of non-being, on the basis of which Sartre constructs his theory of value.

(β) Hartmann's comparison between Sartre and Parmenides on the basis of "continuity" and fullness does, indeed find superficial support in the texts. The most dramatic parallel is between many of Sartre's remarks on the in-itself at the end of the Introduction to *Being and Nothingness* and elsewhere and lines 22-25 of Parmenides' eighth fragment, where Parmenides writes,

Nor it is [*sc.* Being] divisible, since it is all alike (homogeneous); nor is there something more here than there, that might hinder it from holding together, nor some part weaker, but it is all full of what is. Therefore it is all continuous; for what is is close to what is. (*op. cit.*)

This certainly finds an apparent echo in Sartre's pronouncement that

being is what it is. In the in-itself there is not a particle of being which is not wholly within itself without distance. When being is thus conceived there is not the slightest suspicion of duality in it; this is what we mean when we say that the density of being of the in-itself is infinite. It is a fullness. (BN, II.I.I, 74/110)

The above-cited passage from *Nausea* also seems to lend support to Hartmann's comparison.

But this parallel is far more superficial than Hartmann takes it for. While Parmenidean fullness is a presence of all of being in all of being, Sartrean 'fullness' refers to the enstatic density of being of each infinitesimal instance of the in-itself, hence Sartre's equation of the 'fullness' of identity with the absolutely 'exteriorised' isolation that permeates being. This isolation is so far from the holism of the Parmenidean One that in order for the in-itself to even *come together* as a multiplicity, "a being must arise which is simultaneously present to each in-itself isolated in its own identity" (BN, II.II.II.A, 137/172). The formation of multiplicity is not presented here as a fragmentation of the in-itself, but as a synthesis; the infinite density of the in-itself is *beneath* multiplicity and unity. This comes to the fore when we find Sartre appealing to the radical isolation of the in-itself in order to explain the infinite *divisibility* of what appears. The 'fullness' of the Sartrean in-itself, insofar as it is tantamount its isolation and infinite density,

is diametrically opposed in this respect to Parmenidean 'continuity', which Parmenides took to express the *integral indivisibility* of Being.³⁸ Among the most dramatic expositions that Sartre gives of this principle, in *Being and Nothingness*, is to be found in his remarkable paragraph on the *corpse*, where he writes:

in so far as it [sc. the corpse] appears at present as a pure in-itself, it exists in relation to other 'thises' in the simple relation of indifferent exteriority: the corpse *is no longer in situation*. At the same time it collapses into itself in a multiplicity of sustaining beings, each maintaining purely external relations with the others. [...] From the outset physiology is condemned to understand nothing of life since it conceives life simply as a particular modality of death, since it sees the infinite divisibility of the corpse as primary, and since it does not know the synthetic unity of the 'surpassing towards' for which infinite divisibility is the pure and simple *past*. (BN, III.I.II, 348/388-9)

The equivalence that Sartre wishes to draw in this passage, and in several others like it, seems clear enough: insofar as we think a thing in terms of its infinite divisibility, we think it qua being-in-itself.³⁹

Incidentally, when Sartre comes to formulate his own concept of continuity, he does so at the furthest remove from Parmenidean thought: continuity, he reasons, comes to pass only when the in-itself is affected by an internal negativity:

We may recall here the famous definition of Poincaré: a series a, b, c , is continuous when we can write $a = b, b = c, a \div c$ [sc. Poincaré himself writes ' $a < c$ ', which is more intelligible, but Sartre's meaning is clear]. This definition is excellent in that it gives us a foreshadowing of a type of being which is what it is not and which is not what it is. By virtue of an axiom [sc. the transitivity of identity], $a = c$, by virtue of continuity itself, $a \div c$ [sc. $a < c$]. [...] [O]nly the For-itself could thus exist in the ekstatic unity of self. [...] [I]f the chronological order is continuous, it could not be symbolised on the order of identity, for the continuous is not compatible with the identical. (BN, II.III.II.A, 134-5/170-1*)

Continuity, for Sartre, is so far from being an attribute of the in-itself in its entirety that it is essentially *incompatible* with the in-itself's fundamental characteristic, which is identity.

Needless to say, continuity thus understood could not, in principle, be predicated of the Parmenidean One, which is innocent of every negativity.

(Y) We now turn to the heart of the matter. In the last instance, Hartmann authorises his comparison between Sartrean being-in-itself and Parmenidean Being by reference to Sartre's

assertion that the in-itself is “undifferentiated” and everywhere coincident with itself in an absolute (self-)identity. In particular, he invokes Sartre’s proposition that the in-itself “cannot even not be what it is not” (BN, in. VI, lxvi/33; quoted in Hartmann, 35). For Hartmann, it follows from this that being-in-itself is ultimately reduced to a monolithic and absolute Unity, whose every ‘aspect’ or ‘part’ (if it could support such structures) is identical to every other. Unsurprisingly, Hartmann finds such a monolith insufficient for its purpose of providing the “material content” of the phenomenal world. “The difficulties of regarding being as identical and undifferentiated plenitude of content come to the fore once we remember that the same qualities do not appear everywhere” (Hartmann, 40), he writes. There is likely something of a joke in this remark: it is doubtful that Hartmann suspects that Sartre actually ‘forgot’ that the same qualities do not appear everywhere. Lapses such as these tend to be rather rare. The implication seems to be that if Sartre wishes to claim that diversity does exist in the world, this diversity could not be grounded in the in-itself; given the limited options available to his bicameral ontology, Sartre would be committed to restoring to consciousness all the constitutive power that he sought to wrest away from it in his dispute with idealism. The in-itself would become little more than a blank page on which consciousness would draw the world entire.

The problem with Hartmann’s interpretation is that he ignores the radical exteriority that perfuses the Sartrean in-itself. ‘Exteriority’ here does not refer to the *ek-static* standing-outside-onself that Sartre attributes to the for-itself alone, but its very opposite. The in-itself is without *any* capacity to reach beyond itself *at any point*, and it is for this reason that it is devoid of internal relations. There is no *difference* amongst being-in-itself, but this is because, insofar as it exists, difference qua difference is a negativity; difference cannot be seized in itself in such a way that it would simply be what it is. The exteriority of the in-itself is an expression of its radical *enstasis*. Would this not instate an even more radical One? A One more akin to that of the

Neo-Platonists, of which we cannot even predicate extension (as we can of the Parmenidean sphere)? No, not unless we begin by postulating the existence of unity, which Sartre does not do. “Identity,” the chief attribute of the in-itself, “is the limiting concept of unification: it is not true that the in-itself has any need of a synthetic unification of its being; at its own extreme limit, unity disappears and passes into identity” (BN, II.I.I, 74/110). We must resist the temptation of thinking of this identity as a perfection of unity (hence as perfect unity), in order to grasp it as the transcendent limit of unification. The in-itself is not the most unified of things, but is foreign to unity as such. Any ambiguity that remains between the two interpretations is, to my mind, dealt a swift blow by the *reductio* that follows from the first. Even the weakest attribution of unity to the in-itself—the attribution of mere ‘consistency’, for instance—leads us to the absurd position that Hartmann takes to be Sartre’s own. If we postulate that there exists, for being-in-itself, an All or a Whole that is already everywhere in communion with itself, and which forms a Totality, then, certainly, the denial of internal difference could not fail to necessitate a collapse into perfect singularity. If we do not postulate an All of being-in-itself, then the radical isolation entailed by its absolute identity has the very opposite effect: no ‘particle’ of being-in-itself can “enter into any connection with what is not itself” (BN, in.VI, lxvi/32). Even here, the image fails to convey the concept insofar as particles are still unities.

The real threat to the concrete complexity of the world arises from the very opposite direction that Hartmann suspects. Being-in-itself is so unlike the One of Parmenides that it leans more closely towards the being-without-the-One that Plato imagines in the dialogue that bears the monist’s name. There, Plato relates, through the mouth of Parmenides, that if the One is not, then beings

cannot be singular but every particle of them is infinite in number; and even if a person takes that which appears to be the smallest fraction, this, which seemed one, in a moment evanesces into many, as in a dream, and from being the smallest becomes very great, in comparison with the fractions with which it is split up. (Plato, *Parmenides*, 164)

Hartman does not arrive at this equally dire conclusion because he petitions the principle of unity from the outset. He considers Sartre's undifferentiated being as an undifferentiated All, from which the Parmenidean conclusion naturally follows.

But is this really such a fallacy? Is it truly illegitimate to consider *all* being-in-itself when reasoning about its character? In the context of Sartre's ontology, it is unquestionably so. Sartre repeatedly asserts that "totality can come to being only by the for-itself" and that "it is the for-itself in its presence to being which causes there to be an *all of being*" (BN, II.III.II, 180/216), or again: "the presence of the For-itself is what makes being-in-itself exist as a totality" (BN, II.II.I, 121/157). To surreptitiously import the notion of the All when considering the in-itself qua in-itself is tantamount to confusing two meticulously separated ontological categories. The All arrives only with the advent of consciousness, and as soon as consciousness is present, then the exteriority and isolation of the in-itself takes on the form of external relations of difference, as I have indicated above, in the discussion of the second postulate.

What, then, of the Platonic vision of being-without-the-One? Is this the inevitable condition of being-in-itself? If so, then while Hartmann's objection as it stands may well be invalid, its equally disastrous inversion would then have to be accounted for: how can the objectivity of the phenomenal world be secured, without being constituted part and parcel by the subject, if being-in-itself has *no* innate unity? Rather than a blank sheet of paper, being would be resolved into an infinitely dispersed vapour on the basis of which consciousness, as the sole principle of unification, would be responsible of moulding a world. This latter scenario is no more acceptable than the former. It is, however, the problem that Sartre sees himself as confronting, as can be seen in his texts. It particularly informs his later concept of *inertia*, which, in many ways, is the concept of the in-itself in its theoretical maturity. In the *Critique of Dialectical Reason*, he speaks of the threat of dispersion that haunts every unity or totality

effected by human praxis, writing, in an illuminating passage that “[t]hrough its being-in-exteriority, the inertia of the in-itself gnaws away at [the] appearances of unity; the passive totality is, in fact, eroded by infinite divisibility” (CDRvI, in.II.II, 45).

To begin with, let us note that the problem of dispersion has been poorly put, for many of the same reasons that Hartmann’s criticisms miss their mark. The image of ‘vaporization’ is misleading, insofar as it applies a certain negativity to the in-itself in the form of a internal ‘distancing’ of the in-itself from itself. This may be suggested by Sartre’s use of the term ‘exteriority’, but we must be careful not to interpret this term incorrectly. The exteriority of the in-itself is *identical* to its enstasis: the spatial metaphors that animate these terms must be exorcised, if Sartre is to be properly understood. The material structure of the in-itself is no different than that of the phenomenon whose being-in-itself it is—the two are rigorously identical! As Sartre repeatedly emphasises, being-in-itself is simply the transphenomenal being of the phenomenon, and not a noumenon standing behind it. Being, insofar as it *is*, is simply ‘indifferent’ to this configuration. It is this indifference that makes it a threat to the unities that thought and action impose upon it, for these unities are only sustained so long as they are actively imposed. It is this utter indifference that makes the metaphor of *inertia* such a fine one.



As contingent, enstatic and self-identical, the Sartrean in-itself offers little to thought. It is for this reason that many commentators—Catalano among them—have seen fit to use the term “brute existence” as a synonym for being-in-itself. The postulates of the in-itself do not function much like the axioms of a theory. Rather than laying the foundation for a systematic extrapolation of intelligible structures, they seem to bar the in-itself from intelligibility. When they enter into rational constructions, it is only with respect to the operations of the for-itself, whereupon they adopt an essentially negative function—they effectively force the for-itself to

pick up the slack (*i.e.*, relationality, temporality, possibility, negativity, differentiation). One of the points where Badiou breaks most significantly with Sartre concerns the question of being's transparency to thought. In a brief article dedicated to his first great mentor on the tenth anniversary of his death, Badiou recounts the scope and significance of this break. "For a long time," he says,

I cared little of being, for I rejoiced, with Sartre, only in the sense-bestowing functions of Nothingness. Being was the wearisome weight of the chestnut tree's root, it was the massive, the excessive [*l'en-trop*], the practico-inert. What brought me out of all this—what awoke me from my Sartrean slumber—was an interminable meditation on set theory, and especially on its two existential extremities: the axiom of the empty set and the axiom of infinity. The decision to take the historical corpus of mathematics as the very one in which being *qua* being could be enunciated, and hence as ontology in the strict sense, sums up the renunciation of the congested metaphors of massive and finally unthinkable being ('without *raison d'être*' said Sartre, and 'without relation with any other being'). In confiding being to the care of the pure multiple, such as it is secured by the matheme, one disposes it, on the contrary, to the most subtle and ramified thought there is, at the same time that one subtracts it from all experience. Being as thought by mathematics is neither contingent (as Sartre declared), nor necessary (as the classics say). It is infinitely exposed to thought, and subtracts itself from the same. This is why mathematics is at once immense and incompletable, proceeding by axiomatic decisions (*as if* it were contingent) and by constrained demonstrations (*as if* it were necessary). (SDF, 19)

We now turn to the consequences of this crucial decision.

III. Mathematical Ontology

Mathematics is not the rigid and petrifying schema, as the layman so much likes to view it; with it, we rather stand precisely at the point of intersection of restraint and freedom that makes up the essence of man itself.

— Herman Weyl, "The Current Epistemological Situation in Mathematics."⁴⁰

While Sartre begins by submitting the 'there is' to the phenomenological discovery of intentionality, and the investigative horizon that it opens, Badiou initiates his own enterprise by placing the 'there is' under the mathematical invention of the *set*, and the axioms that govern its deployment. Sartre's fidelity to Heidegger's pronouncement that "*Only as phenomenology is ontology possible*,"⁴¹ is matched and opposed by Badiou's declaration that it is mathematics, and mathematics alone that is adequate to the thinking of being *qua* being. The symmetry we find here, which is reinforced by a certain stylistic confluence and a mutual predilection for axiomatics, is broken on a number of points.



The first, perhaps most significant and pervasive source of dissymmetry has to do with the role of the *axiom* as such. The divergence between Badiou and Sartre on this point essentially echoes a shift of the place of the axiom in mathematics itself, one which took place near the end of the nineteenth century and the beginning of the twentieth (the era, incidentally, from which both philosophers draw their founding principles). Sartrean philosophy maintains itself in the older, 'Euclidean' (or Cartesian) understanding of axiomatics, for which the assertion of a proposition as an *axiom* requires that that proposition be of *apodeictic certainty*: the axiom is the seal of necessity.⁴² There is no difficulty in seeing the axiom of intentionality as an only slightly belated inheritor of this tradition; Sartre repeatedly attributes to it the full certainty of the *cogito*.⁴³

Without delving too deeply into a detailed history, we may note that it was precisely this understanding of axiomatics as apodeictically certain that prepared the ground for the tectonic shift that mathematics, and with it the very notion of the axiom, would undergo. As Kline recounts, when held up to the light of this norm, a “slight blemish seemed to mar the collection of axioms” of Euclidean geometry (453): the parallel postulate, which stated “that if two lines m and l meet a third line, n , so as to make the sum of angles 1 and 2 less than 180° , then the lines m and l meet on that side of the line n on which the angles 1 and 2 lie” (453), seemed somewhat less “self-evident” than the others. Efforts were made in the seventeenth and eighteenth centuries to either replace it with another, more clearly certain axiom, or else to derive it from the others, thereby showing that this doubtful ‘axiom’ was, in fact, a theorem. The efforts towards replacement managed to bear some fruit: John Playfair succeeded in providing a more concise, if no more self-evident version of the postulate, stating “that given a line l and a point P not on that line, there is one and only one line m in the plane of P and l which passes through P and does not meet l ” (455). The efforts towards derivation, however, proved barren—with one notable exception resulting from the work of Girolamo Saccheri. What was exceptional about Saccheri’s work was not its success (it did not succeed) but the remarkably fruitful character of its failure. Saccheri attempted to demonstrate the parallel postulate indirectly. Aiming at a *reductio ad absurdum*, he removed the parallel postulate (or rather, Playfair’s version thereof) from the axioms of Euclidean geometry and put statements implying its negation in its place: one being that no parallels to l through P exist, the other being that there exist more than one such parallel. The former did produce a contradiction with the other axioms, while “from the second alternative (there is more than one parallel to l through P) Saccheri derived a number of strange theorems but no contradiction” (454). To cut a long story short, Saccheri, satisfied with having established the falsity of the negation of the parallel postulate by its very strangeness, published

his findings under the title *Euclid Vindicated From All Defects*, and died the same year. Mathematics would have to wait nearly a hundred years before mathematicians—spearheaded by Lobachevsky and Gauss—would begin “to draw the conclusion which Saccheri should have drawn” (455): that Saccheri’s ‘proof’ did not demonstrate the truth of the postulate, but its utter contingency. Other geometries were possible in which the postulate failed, geometries which were no less complex and rigorous than their Euclidean forbearer. The parallel postulate, once its contingency and the contingency of its alternatives had come into the full light of day, ceased to be exceptional and became the rule. In the non-Euclidean geometries that followed the revolution that began with Lobachevsky and Gauss, the axiom would take its place as the trace of a contingent decision rather than a testimony to a necessary truth.

This new vision of geometry was yet to receive its Euclid. He appeared only in the last year of the nineteenth century in the figure of David Hilbert. In 1899, Hilbert published *The Foundations of Geometry*, which undertook to rebuild Euclidean geometry from scratch, no longer founded on appeals to self-evident propositions and basic intuitions, but on manifestly contingent arrangements of axioms prescribing the operations that may be carried out on undefined symbols.⁴⁴ Definitions, as such, were virtually absent from the text, and the axioms themselves came to be treated as objects of analysis, allowing for inquiries into their independence, mutual consistency and compatibility with other groups of axioms.⁴⁵ This text, among a handful of others with which it was more or less contemporary, marked the beginning of the formalist movement in mathematics, with which Badiou has long been sympathetic (without giving way on his Platonist convictions, as we will see). Under the exigency of compressing the formalist orientation in mathematics into a nutshell, we might say the following: what distinguishes this movement is, first and foremost, a commitment to the autonomy and freedom of the axiomatic method (it is without any norm other than consistency), and the

identification of mathematical propositions with their formal inscriptions (a sort of immanently mathematical materialism).⁴⁶

The effects of this paradigmatic shift in the notion of axiomatics will become visible repeatedly throughout the following study of Badiou's thought. Axiomatic ontology will be grasped as a contingent and material situation, and (true to the Althusserian tenet that there is always a considerable temporal lag between the transformation of scientific concepts and the philosophical categories to which they are wedded⁴⁷) the category of truth will be reconfigured to be once again compatible with the post-Hilbertian concept of axiomatics. The better part of the difficulty involved in passing from Sartre's work to Badiou's has to do with the effort necessary to reorient oneself according to principles that are no less foundational than they are manifestly contingent (for all Sartre's emphasis on contingency, he at least presents his foundations as being no less apodeictic).



Another dissymmetry, related to the first, becomes apparent when we reflect that while it is manifestly obvious that phenomenology offers a theoretical account and analysis of the 'there is' in its concrete situations, it is quite difficult to say the same for set theory—especially in view of the modern separation of axiomatics from the norm of metaphysical truth (see above). Not once in set theory's formal inscriptions do we find any mention of the 'there is' or presentation as such. This disparity necessitates a certain "stratification of discourse," which is present in Badiou's work but absent for Sartre, who sees his work as being continuous with the phenomenology that gives him his axioms: *Being and Nothingness* is explicitly written as "An Essay on Phenomenological Ontology," and the majority of the text is devoted to producing *new* phenomenological analyses, and tilling the ground for new phenomenological axioms and principles. We do not find this continuity in Badiou's work. *Being and Event* is explicitly "not an

ontological presentation, a treatise on being, which is never anything other than a mathematical treatise” (BE, 13/20). *Being and Event* is not a treatise in ontology, but in what Badiou calls *metaontology*: the speculative extension of ontology whose task, among other things, includes the very identification of mathematics as ontology—an identification that is no business of mathematics itself (as most mathematicians will no doubt agree). In *Being and Event*, this identification proceeds, first of all, through the philosophical determination of the concept of presentation in a way that is, in some sense (which is not always transparent), homogeneous with set theory. To a large extent, this involves interpreting the axioms and theorems of set theory in such a way that they speak to us of presentation itself.

The identification of set theory and ontology often raises a particularly troublesome misunderstanding, which springs from a confusion of mathematical ontology with mathematical cosmogony, such as we find with the Pythagoreans. In claiming that mathematics is ontological, Badiou expressly “does not in any way declare that being is mathematical, which is to say composed of mathematical objectivities” (BE, 8/14). What is thought by mathematics—what is thought in the material situation of mathematics—is not “the same thing” as what occurs in other situations, and is not the primitive stuff out of which other situations are materially composed. Leaning more heavily towards Plato than Pythagoras on this point, Badiou insists that mathematics as ontological thought attests to a separability of the intelligible mathematicity of the world. Ontology, the practice of determining the intelligible structure of presentation itself, thinks what is separable and intelligible with respect to the minimal determinations of any situation whatsoever, yet “it is not by itself the thinking of a concrete situation” other than itself.⁴⁸

The work of metaontological speculation then becomes a sort of philosophical labour carried out on the formal productions of mathematical thought, comparable, perhaps, to the

Kantian operation of Schematism, which submits the manifold of the given to the concepts of the Understanding through a process that is not itself 'conceptual'. Here, however, it is less a matter of bridging the conceptual and the aesthetic, than the formal and the conceptual. Another difference, of course, is that Kant's account is analytical (in the sense of it being a 'transcendental analytic') and concerns a state of affairs presumably given in advance, and so as long as we understood that it is doing what it must, Schematism can function while being happily enshrouded in mystery, its inner workings unbeknownst to us. But Badiou is not performing a transcendental analysis;⁴⁹ he is explicitly constructing a theoretical apparatus whose structure is not presumed to lie dormant in either the mind or the world. The operations required for his 'schematism' are precisely what need to be constructed.



Two other difficulties deserve our attention. The first concerns Badiou's misleading characterisation of the ontological situation—set theory—as the "presentation of presentation". Such an expression naturally leads one to expect that, whatever 'presentation' might mean (as we will see, Badiou gives us little to work with on this point), it must be a structure 'native' to the set theoretical situation. One may even, a bit more cautiously, content oneself with saying that even if presentation as such is external to set theory,⁵⁰ only in set theory do we find the presentation of presentation—elsewhere reigns the presentation of 'things'.

The most obvious problem with this interpretation is that it fails to take account of the stratification of discourse between ontology and metaontology: metaontology alone speaks of presentation; set theory speaks only of sets. More significantly, metaontology speaks of presentation in such a way as to attempt to mirror, at least in its formal structure, the axiomatic deployment of sets within set theory; this constraint forbids it from speaking of the presentation of *anything* that is not itself a presentation (the constraining analogue being that all sets are sets

of sets), again, with the exception of the void.⁵¹ Metaontology cannot claim that some presentations are presentations of presentation and some are not.

A better interpretation of Badiou's characterisation of set theory as the presentation of presentation would be that we should understand set theory as the situation in which the formal structure of any given presentation is presented, and that these formal structures are, simply, sets. We would then replace our faulty assumption that set theory is uniquely the presentation of presentations and say, instead, that set theory is the presentation of *sets*, which display the formal structure of presentation in general. Set theory would be like a microcosm in which the lineaments of the macrocosm would be reflected. This is, at least, a *less* erroneous reading than the first. It raises the question, however, of how one is to establish a correspondence between sets, on the one hand, and actual presentations on the other.

This was, in essence, the question that arose from my own initial reading of Badiou's text. It seemed to me that there could be no hope of set-theoretic ontology providing the groundwork of a theory of situations and subjective actions without at least the possibility of a bridge between the two being elucidated. The question itself was intertwined with another one, which essentially was that if it is not a question of drawing a 'correspondence' between presentations and sets, if presentations simply *are* sets, then what are we to make the statement in *Being and Event* that reads:

The thesis that I support does not in any way declare that being is mathematical, which is to say composed of mathematical objectivities. It is not a thesis about the world but about discourse. It affirms that mathematics, throughout the entirety of its historical becoming, pronounces what is expressible of being qua being. (BE, 8/14)

This statement seems to directly contradict any supposition that Badiou claims sets and presentations to be precisely the same thing. But if their correlation is not one of identity, what is it? Badiou responded to my questions as follows:⁵²

The difficult point in your question is the sense of the word 'correspondence'. In my theoretical apparatus [*dispositif*], I believe that the question, 'what is the correspondence between a being

and its set' is deprived of signification. In effect, only the notion of set gives sense to that of being, in the context of an ontology—so that it must be posited that 'all being' [*dout l'être*]] is thought in and by the set. This does not at all mean that a given multiplicity is 'the same thing' as the set, but only that insofar as one thinks it in its being, one thinks it as a set.

Everything hinges here on a subtle displacement: whereas I posited sets as being a certain kind of presented thing—so that we have one situation populated by sets and others populated by dogs, thieves, stars and syllables, for example—Badiou implies that the set is *not* one kind of thing among others, but neither is it the substance of all things. It is precisely a mode of thought; the aspect that is produced of each thing when that thing is thought in its being is exactly a *set*. No situation, no presentation is composed of sets; rather, set theory deploys the means by which any presentation may be thought *as* a set, and this mode of thinking is by definition 'ontological'.

This interpretation offers us a means of approaching the relation between the interior and the exterior of the ontological situation that is more or less consistent with Badiou's idiosyncratic commitment to mathematical Platonism. Against the caricaturised 'platonism' frequently favoured by anthologies and textbooks in the philosophy of mathematics, Badiou insists that a genuine Platonism is not concerned with establishing the external reality of mathematical objects, but in the way in which mathematics altogether dissolves the difference between exterior and interior, transcendent and immanent (as might be anticipated, this very aspect of mathematicity makes Badiou's localisation of mathematics in a single situation extraordinarily problematic). For Badiou, Plato's assertion "[t]hat mathematics thinks means in particular that it regards the distinction between a knowing subject and a known object as devoid of pertinence" (TW, 54). When this sort of Platonism is brought to bear on mathematical ontology, the consequence is that "[i]n so far as it touches on being, mathematics intrinsically thinks. By the same token, if mathematics thinks, it accesses being intrinsically" (TW, 50). In other words, there is no need to establish a correspondence between sets and presentational forms, rather, presentational forms only become intelligible as such once a "a co-ordinated movement of

thought, co-extensive with being” (TW, 54), ordains what *will have been* these forms, in accordance with set theoretic structures. This is what is meant by Badiou’s remark that “insofar as one thinks [something] in its being, one thinks it as a set”.

In order to see what this could mean concretely, we must ask what the ontological situation might be—if it is not, strictly speaking, a presentation of sets. Metaontologically speaking, it is a presentation of presentations, like everything else. This of course tells us nothing; and the real question (couched in terms that will be explained in the next chapter) concerns how these presentations are structured, or counted-as-one. Roughly speaking: what do we find there, and how would we recognise it? The answer now seems simple enough: Faithful to Hilbert, we will say that it is not at all a presentation of ‘sets’, but of *signs*. Paraphrasing Quine, we will then say: *to be thought in its being is to be thought as the value of an ontological variable*.⁵³ To think a particular structure “in its being”, that is, according to its pure presentational form, is simply to think it in the symbolic language of set theory, which is one and the same as thinking it “in and by the set.” Hence, it is not the case that “sets” are brought into correspondence with situations or “structures”, but that structures are brought into correspondence with *symbolic expressions*, both terms belonging to the same general category (the category of the existent; that is, of the presented, the consistent, the counted-as-one). The concept of ‘set’ itself is, qua concept, only quasi-mathematical—as Badiou notes, nowhere in set theory itself does the concept of ‘set’ play any operative role. It arises only as a sort of phantasm whenever the axioms of set theory are satisfied in some domain (a phantasm that is no doubt both useful and misleading in guiding mathematical intuition).⁵⁴ For example, since the demonstration of the famous Löwnheim-Skolem theorem, we have known that it is possible to interpret the axioms of ZF in such a way that they are satisfied in a model constructed solely out

of natural numbers. In such circumstances, this numerical domain is truly thought in and by the set.

If this is how we are to understand things, what this means is that the relation between ontology and the various concrete situations that it ‘thinks’ is to be conceived as the relation between a syntactical apparatus and the models that satisfy it. This seems to be how Badiou’s use of mathematical ontology plays itself out in *Being and Event*: in his employment of both Gödel’s notion of constructibility and Cohen’s notions of genericity and forcing, Badiou explicitly treats the situations in question as *models*. Now, this is a topic on which Badiou has devoted an entire book (his first, as it happens), entitled *The Concept of Model*. In this book, written many years prior to his identification of mathematics with ontology, Badiou indicates what proves to be a significant difficulty for the position we have taken. The point to be made is that it is only possible to treat a domain as a model for a mathematical syntax if that domain is *already mathematical*—in the present state of the science, this means that the domain must already be organised along set theoretic lines. In *The Concept of Model*, Badiou shows how

the concept of model is strictly dependent, in all its successive stages, on the (mathematical) theory of sets. From this point of view, it is already inexact to say that the concept connects formal thought to its outside. In truth, the marks ‘outside the system’ can only deploy a domain of interpretation for those of the system within a *mathematical envelopment*, which preordains the former to the latter. [...] Semantics here is an *intramathematical* relation between certain refined experimental apparatuses (formal systems) and certain ‘cruder’ mathematical products, which is to say, products accepted, taken to be demonstrated, without having been submitted to all the exigencies of inscription ruled by the verifying constraints of the apparatus⁵⁵

The problem that confronts us here is clear: The ontological/mathematical formalisation of a situation is possible only in the light of its pre-ontological mathematisation. This is something that Badiou hints at from time to time, but which he has never philosophically thematised. This notion seems implicit, for instance, when he speaks of a “horizon of mathematicity” in which *any* situation is disclosed, and upon which mathematical physics, for example, operates.⁵⁶ We find ourselves led back to Badiou’s Platonism at this point, insofar as we are compelled to

conclude that, for Badiou, mathematical ontology's relation to the multiplicity of concrete situations can be thought as neither a relation in immanence (no set or presentation contains all others; ontology is no exception) nor as one of transcendence (the outside of ontology can only deploy a domain of interpretation within an ontological envelopment). At the same time, we see that Badiou cannot help but undermine his assertion that his thesis is not about the world, but about discourse; he undermines it by rendering the distinction between the two both unintelligible and indispensable.⁵⁷

The question remains as to how we are to bridge the 'gap' between what manifestly pertains to the ontological situation and some other concrete situation. To a large extent, *The Concept of Model* was written as a polemic against the empiricist uses of model-theoretic concepts that overlook the essentially intramathematical character of formal semantics and fill the gap between the mathematical and the extra-mathematical with specious and analogical operations. As Badiou sees it, such a conception of mathematical formalisation "enslaves" science to the "detestable philosophy" of logical empiricism which envisages scientific mathematisation as "an imitative craft."⁵⁸ We find such a critique delivered in Badiou's reading of Von Neumann and Morgenstern's *Theory of Games and Economic Behaviour*,⁵⁹ where Badiou targets

passages where a relation as weak as resemblance is explicitly evoked, such as, for example: "[the models] must be similar to reality in those respects which are essential in the investigation at hand."⁶⁰ Or: "[s]imilarity to reality is needed to make the operation significant."⁶¹ One sufficiently sees how external analogy and simulation are here summoned to reduce the initial gap between the inert opacity of the facts and the constructive activity of models.⁶²

After reading passages such as these (and there are several), one may be quite surprised to turn to the twelfth meditation of *Being and Event* and read that

Set theory, considered as an adequate thinking of the pure multiple, or of the presentation of presentation, *formalises* any situation whatsoever insofar as it reflects the latter's being as such; that is, the multiple of multiples which makes up any presentation. If, within this framework, one wants to formalize a particular situation, then it is best to consider a set such that its characteristics—which, in the last resort, are expressible in the logic of the sign of belonging

alone, \in —are comparable to those of the structured presentation—the situation—under consideration. (BE, m12, 130/149*)

Is there any difference at all between the plainly analogical operations called for here and those that receive a blistering critique in *The Concept of Model*? Against the charge of employing vague and metaphorical operations in the place of rigorously mathematical or scientific ones, no honest defence is possible, except, perhaps, one which tries to defuse the charge by arguing that no other options are available.⁶³ As for the more serious charge—that proceeding in this way, and drawing analogical connections between mathematical structures and empirical realities makes science (here ontology) into an ‘imitative craft’ against all its intentions—this charge can be easily absorbed by the same murky waters of Platonism that were evoked earlier. Following this tack, one might argue, as Badiou seems to in his letter, that it is not so much a question of establishing a correspondence (analogical, mathematical or otherwise) between the syntactic-ontological and concrete-presentational structures, but of determining the latter as intelligible through their very mathematisation. Insofar as we can speak of a correspondence, it is a correspondence between formulae and models whose constitution becomes thinkable in terms of sets (on the ontological register) or presentations (on the metaontological register) only once this correspondence is enacted. This seems to be in line with Badiou’s remark that “*only* the notion of set gives sense to that of being.” In other words, there is no *being* of the situation, dissociable from its ontic concreteness, that can be thought prior to its set-theoretic formalisation. This solution seems to have a strongly idealistic character about it, in contrast to Badiou’s frequently professed ‘materialism’; this impression may be corrected, however, by insisting that everything involved in the Platonic correlation between ontology and its ‘exterior’ is strictly material—even mathematical thought itself is, for this Platonic formalist, a material configuration of signs. My own position on this matter, however, is that Badiou’s philosophy is constructed in such a way as to make the distinction between idealism and materialism strictly irrelevant to it. What content

can the opposition between materialism and idealism have for one who follows Parmenides in affirming the *identity* of thinking and being?⁶⁴

The point to which this brings us is this: To the extent that a mathematical ontology of concrete situations is possible, it must be possible to treat these as ‘models’ of set theory. Accordingly, these situations must be apprehended as being *already mathematical* in some sense, however crudely or vaguely understood. To the extent that ontology avoids the ‘empiricist’ mandate of being an ‘imitative craft’ (a characterisation against which Badiou rails in *The Concept of Model*), the correspondence between the ontological situation and its outside can be classified as neither a relation of transcendence nor immanence, but must be thought as a point of indiscernibility between the two. This is the source of all the obscurity attributable to the ‘Platonist’ position of metaontology, which forces us to ask, as Brassier does,

Where is Badiou speaking from in these decisive opening meditations of *Being and Event*? Clearly, it is neither from the identity of thinking and being as effectuated in ontological discourse, nor from within a situation governed by knowledge and hence subject to the law of the One. [...]metaontological discourse seems to enjoy a condition of transcendent exception vis-à-vis the immanence of ontological and non-ontological situations.⁶⁵

This obscure position is one we must nonetheless occupy, and we will attempt to do so in the next few chapters, whose goal will be to set up the fundamental concepts of Badiou’s metaontology in order to prepare the groundwork for his theory of the subject. While doing this, we will aim to elucidate these concepts in such a way that their kinship with the concepts of Sartrean ontology, where it exists, will be made evident.

IV. The One, the Multiple, Presentation and the Count-as-one

All that participates unity is both one and not one.

- Proclus, *Elements of Theology*, proposition 2.⁶⁶

From wherever it may be that Badiou is speaking in the first meditations of *Being and Event*, the first words to come to us are these:

The experience from which ontology, ever since its Parmenidean institution, has made the portico of a ruined temple is the following: what *presents* itself is essentially multiple; *what* presents itself is essentially one. (BE, m1, 23/31*)⁶⁷

What is being said here, in these two formulae that escape contradiction only by a difference in emphasis? The appeal to 'experience' clarifies nothing on its own. There is nothing in this passage that summons up the least phenomenological evidence. As for the formulae on which this passage rests, to all appearances, these are the sort of statements that can be deciphered only if they are already understood. Proceeding perhaps a bit coarsely, we could destroy the symmetry between the two invocations of '*what presents itself*,' hold the second to be speaking of the presented entity (the *quid* given in presentation), and the first to be speaking of the act of presentation itself (the *there is* or *il y a*). In other words, *what* there is is in every case *one*, while the *there is* itself is fundamentally multiple (or, to put it another way, to speak of an instance of the *there is* is always to speak of a multiplicity).

This 'experience' confronts the ontologist with a dilemma: do we begin from the multiplicity of presentation (qua presentation), or the unity of the entity (qua entity)? The identification of entity and unity has had a long life in the history of philosophy, which it would be pointless to attempt to summarise here. One can find explicit identifications between the two running as far back as Parmenides, if not Pythagoras himself, surfacing in different forms in Aristotle and Proclus, and—to some extent—in Plato (problematically, though, since Plato was

also among the first to speak of the being of ‘things’ that could not be understood as unities in any clear sense, such as the Other in the *Sophist*, the $\chi\omega\rho\alpha$ in the *Timaeus* and the ‘nothing’ that survives the One in the *Parmenides*; significantly, these ‘Eleatic’ dialogues are framed as a fundamental dispute with the Parmenidean tradition). Over two millennia later, we find the thesis on the identity of entity and unity resurrected and given renewed vigour through the work of Leibniz. It was Leibniz who, in a letter to Arnauld, encoded this perennial thesis in its tersest and most impenetrable form, giving us the axiom: “ce qui n’est pas *un* être n’est pas un *être*” (an expression that is not nearly so eloquent in English: we have to settle for the conceptually accurate but unpleasantly awkward: “what is not *one* being is not a *being*”).⁶⁸

Next to this well-enshrined coordination of entity and unity, the second formula invoked at the beginning of this chapter seems to be painfully obscure. Though there have been certain foreshadowings (notably in Plato himself), it is only relatively recently in the history of philosophy that we find explicit efforts made to speak of ‘the *there is*’ as a notion distinct from entities as such, and seldom have there been efforts to connect the question of the ‘there is’ with the classical opposition between unity and multiplicity—an opposition which, in contrast to the opposition between entity and presentation, has been a perennial and explicit theme for philosophical ontology. It seems, moreover, that in trying to speak of the ‘there is’ in a way that is independent from the unity always paired with the entity, language threatens to befoul our efforts. How can we speak of *a* presentation that is not *one* presentation? The simple answer is that we cannot—but this is something of a false problem so far as concerns Badiou’s philosophy. It is a false problem, because its formulation overlooks the fact that when Badiou says that the One is not, he is not declaring unity’s abolition, but its scission. Immediately following his assertion that “the one *is not*,” Badiou adds that

It is, however, not a question of disposing with what Lacan pinned to the symbolic as its principle: *there is* some One [il y a *de l’Un*]. Everything hinges on mastering the gap [*l’écart*]

between the supposition (which must be rejected) of a being of the one, and the thesis of its 'there is' [*« il y a »*]. [...] What must be declared is that the one, which is not, exists only *asoperation*. Or again: there is no one, there is only the count-as-one. (BE, m1, 23-4/31-2*)

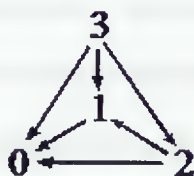
This splitting of unity into operation and effect is integral to the thinking of presentation and the metaontological delivery of the formal thinking of presentation to mathematics, and specifically, to set theory. It is set theory itself that formalises this split, and provides us with a figure of multiplicity adequate to the thinking of presentation, and, more dramatically, to the univocal determination of the existent as presentation (and so of presentation as the presentation of presentations). To speak of *a* presentation is to speak of presentation affected by an operation of the count-as-one, and not of a presentation solidified according to the intrinsic unity characteristic of entity. The unity of a presentation is always extrinsic.

In what follows, I aim to give an explanation of the joint concepts of presentation and the count-as-one. One side of my analysis will be conducted in the light of the preceding exposition of the Sartrean concept of impersonal consciousness, and aim to outline formal similarities between the two structures; the other will look forward to the determinations imposed on the concept of presentation by its set-theoretic conditioning. It is from this side that the prevailing homology between consciousness and presentation is interrupted and we encounter certain modifications which, though they be strictly internal to the general concept of multiplicity, cause the notion of presentation to diverge from that of consciousness.

I will begin with the notion of the count-as-one, since it is this notion which provides us with the metaontological machinery necessary for speaking of presentations. The simplest way to proceed would be situate the notion of the count-as-one with respect to the definition of multiplicity proposed above, and then to derive the notion of the One as something secondary to the count. By proceeding in this way, we may show how the count-as-one is not antithetical to multiplicity, but is one of its immanent structural characteristics. We will show how speaking of

the count-as-one need not presuppose *the* One whose existence is disavowed. On the contrary, so long as we cleave to the definitions provided below, speaking of the One presupposes the operation of the count.

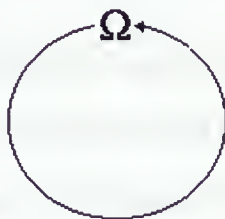
In the foregoing analysis of the concept of multiplicity, I distinguished between two dimensions of the multiple, the separation of which is maintained in accordance with the Second Criterion (that a multiplicity must differ from its elements). Corresponding to the elements of the multiple (its determinant extension), we have multiplicity's *first dimension*, which we may call the *material* of a multiplicity; this is its first dimension. Corresponding to the 'multiplicity as such', insofar as it is separated from its elements by a pure and empty differentiation, we have what we may call the *form*, or *formal unity*, of the multiple—its second dimension. What I am suggesting is that the count-as-one is nothing more and nothing less than this dimension of formal unity (correlatively, we may speak of the unified material as the 'counted-as-one'). It is easy to provide a simple representation of this distinction using the graph notation that is sometimes employed in set theory. It is possible to represent sets by graphs composed of nodes and arrows.⁶⁹ For example, the set expressing the Von Neuman ordinal 3, which in standard notation is written $\{\emptyset, \{\emptyset\}, \{\emptyset, \{\emptyset\}\}\} = \{0, 1, 2\} = 3$, is given by the graph:



In such a graph, it is possible to see the arrows as designating the operation of formal unification. In this example, we may say that *insofar as it is the source of an arrow*, 3 formally unifies (counts-as-one) 0, 1 and 2. Insofar as they are the targets of arrows, 1 and 2 are material for this unification, but they too enact formal unifications: 2, of 1 and 0; 1, of 0 alone. 0, or the empty set, unifies nothing. In these graphs, the arrows represent nothing other than the inverse of the

set-membership relation \in : $\alpha \in \beta$ becomes $\beta \rightarrow \alpha$. Depending on the context, we may read this as ‘ β contains α ’, ‘ β counts α as one’, or ‘ β presents α ’.

For our purposes, I propose to schematise the One as an instance in which formal unification and unified material are *identical*. Following a certain custom, let us use the sign Ω for the One thus defined.⁷⁰ Its graph is simple:



Using standard notation, we may also designate such an instance of the One by writing: $\Omega = \{\Omega\}$.⁷¹ Speaking more generally, the statement “the One exists” could be written:

$$(\exists x)(x = \{x\}).$$

In passing, let us note that Parmenidean Being, characterised above as being itself the limit to which it finds itself subject, is aptly schematised by the definition of the One provided here as $\Omega = \{\Omega\}$. Aristotle emphasises just this point in his critique of Parmenides in the *Physics*. He sees a manifest contradiction in Parmenides’ claim that Being is both limited and indivisible. If the Being of Parmenides “is one as indivisible, nothing will have quantity or quality, and so the one will not be [...] limited, as Parmenides says, for though the limit is indivisible, the limited is not” (*Phys.*, 185^b, 15-20).⁷² Aristotle thus reasons that an equivocation, or identification, of the limit with the limited is at the heart of the Parmenidean ontology of the One.

To insist, as Badiou does, that to think presentation without the One requires that one master the ‘gap’ between the being of the one and its ‘there is’ means just that one must maintain a gap within the structure of unity itself. One must conceive of the ‘there is one’ in such a way

that it does not immediately collapse into ‘what there is’; that is, in such a way that the formal unity of the ‘there is’ does not collapse the material that it unites in presentation. When such a collapse takes place, formal unity fuses with its unified material and we have a reinstatement of the One (perhaps only in the form of the entity, or any entity whatsoever). Conversely, the thesis that the One is not is simply the thesis that it is a law of being that such a collapse cannot take place.⁷³

The activity of separating the ‘there is’ from the ‘what there is’ is one with the act of determining the unity of the ‘there is’ as operational and the being of the ‘there is’ as multiple. In light of this thesis, whose demonstration I will soon offer in full, it is apparent why it is that Badiou may say, not only that the One is not, but that “the One is the non-being of being-multiple” (BE, m3, 42-3/54*), or, elsewhere: “It the same thing to say that the one is not, and to say that there is a law of the multiple, in the double sense of being what constrains the multiple to manifest itself as such, and what rules its structured composition” (BE, m1, 25/33). The specific ‘law of the multiple’ that illuminates these propositions most clearly remains only implicit in both Badiou’s text and the ZF axiomatic, but figures prominently in the definition of multiplicity under which we are working. It is the Second Criterion, the law that a multiple unconditionally differs from its elements, and whose logical form is written:

$$(\forall x)(x \neq \{x\})$$

By the classical square of opposition, this law is *logically equivalent* to the thesis that the One is not, as we have formalised it here; namely:

$$\sim (\exists x)(x = \{x\})$$

If there is a clear sense in which the One is the non-being of being-multiple, it is this: that the law asserting the negation of the existence of the One is equivalent to the law that opens the dimension in which multiplicity may take shape as the universal form of presentation.⁷⁴

If we like, we may express the law of the multiple/count-as-one in a more familiar style: to say that the One is not is to say that *every unity is a unity of something, from which it differs*. The analogy being drawn here is exact, and we will find the same consequences here that we did earlier in our analysis of the axiom of intentionality. Like consciousness, the count-as-one is ‘in itself’ perfectly empty, and receives all of its ontological determination from its extension, its unified matter. We have already seen how crucial the notion of a constitutive withdrawal is to the structure of the count-as-one. The conception of the ‘there is’ that Badiou is deploying here, riven as it is by its ‘intentional’ structure, is to a remarkable extent homologous with the ‘there is’ as it is determined by the Sartrean concept of impersonal consciousness. For Badiou, the name of the ‘there is’ itself is *presentation*, which is only the obverse of the count-as-one.

This notion of obverse is a bit peculiar, however, and would benefit from some explanation. Badiou’s exposition of metaontology in *Being and Event* suffers somewhat from a terminological overdetermination of a few of its key concepts, such as *presentation*, *count-as-one* and *situation*. We have already explained the count-as-one, and the notion of presentation has been invoked often enough by now that the reader at least has a sense of what it may mean; the exact distinction between the two, however, is far from clear. Badiou’s concept of situation has not yet received any attention at all. The name ‘situation’, Badiou tells us, is given to

any presented multiplicity. Granted the effectiveness of the presentation, a situation is the place of taking-place, whatever the terms of the multiplicity in question. Every situation admits its own particular operator of the count-as-one. This is the most general definition of a *structure*; it is what prescribes, for a presented multiple, the regime of its count-as-one. (BE, m1, 32/24)

To speak of a presentation, then, is the same as to speak of a situation, but *presentation* itself is also the act that gives situations (or presentations). Badiou’s remark that a situation is “the place of taking place, *whatever the terms* of the multiplicity in question” informs us that when we speak of a situation, our mind is directed more to the formal unity of the situation than the material situated within it (the metaphor of place works well here).

The mutual obscurity that remains between the concepts of presentation, situation and count-as-one was indicated pertinently by Oliver Feltham in a 1999 interview with Badiou. The relevant question was: “Can one ask what counts-for-one a situation? Does it makes sense to ask what *performs* the operation of the count-for-one of the situation? Is there an agent?” To this, Badiou responded, “The operation is the situation itself. The operation is not distinct from the multiplicity itself. There is no presentation of multiplicity *and* the operation. The operation *is* the same thing as the presentation.”⁷⁵ The three terms form a perfect trinity, identical in everything but sense.

The difference in sense that remains between the morning star of presentation and the evening star of the count-as-one is codified in Badiou’s distinction between *consistent* and *inconsistent* multiplicity. This duality is “established in the distribution [*partage*] of the count-for-one, between upstream inconsistency and downstream consistency” (BE, m1 25/33*).⁷⁶ *Inconsistency* is the aspect taken by multiplicity when considered in abstraction from its unification by the count; it is the character of a multiplicity such as it could be imagined to have existed “before” it was counted as one; to put in another way (one which recalls a distinctly Sartrean metaphor) inconsistency is “the *inertia* retroactively discernible on the basis of the fact that the operation of the count-for-one must effectively operate so that there may be one” (BE, m1, 25/33*; e.a.). The emphasis on “retroactive discernment” is significant, given that the thought of inconsistency is necessarily abstract: the inconsistent dimension of a multiple does not, in any actual sense, ‘pre-exist’ the consistency of the same multiple. A multiplicity only exists insofar as it possesses both dimensions. The second dimension, here called consistency, is the aspect of multiplicity that is brought to our consideration insofar as we consider a multiplicity to be *a* multiplicity, that is, insofar as we consider it as a ‘single thing’ (or as many ‘single things’, depending on how far ‘upstream’ or ‘downstream’ we position ourselves). The

consistency of a multiplicity is its dimension of formal unity. “Situation” is the term by which Badiou designates a presented multiplicity insofar as it is seized in its consistency: a situation is a presentation counted-as-one. Seized “upstream” of its consistency, the same multiplicity is conceived as “presentation” (which is said to be “reciprocal with ‘inconsistent multiplicity’” in the glossary appended to *Being and Event* (BE, 555).).

In parentheses, we should note that Badiou’s use of the terms “consistent” and “inconsistent” differs significantly from both Cantor’s use and their traditional acceptance in mathematics. In a now famous letter to Dedekind, Cantor wrote that

If we start from the notion of a definite multiplicity (*Vielheit*) (a system, a totality) of things, it is necessary, as I discovered, to distinguish two kinds of multiplicities (by this I always mean *definite* multiplicities).

For a multiplicity can be such that the assumption that *all* of its elements ‘are together’ leads to a contradiction, so that it is impossible to conceive of the multiplicity as a unity, as ‘one finished thing’. Such multiplicities I call *absolutely infinite* or *inconsistent multiplicities*.

As we can readily see, the ‘totality of everything thinkable’, for example, is such a multiplicity; later still other examples will turn up.

If on the other hand the totality of the elements of a multiplicity can be thought of without contradiction as ‘being together’, so that they can be gathered together into *one* thing’, I call it a *consistent multiplicity* or a ‘set’.⁷⁷

For Cantor, the distinction between the inconsistent and the consistent has to do with whether it is *possible* to think their elements ‘together’, or to count them as one. Badiou adopts weaker criteria: a consistent multiplicity is one whose elements *are* thought of as “together”, while an inconsistent multiplicity is one whose elements *are not* thought of as “together”. Cantor’s criteria are *de jure*, while Badiou’s are *de facto*.

The sheerly metaphorical character of Badiou’s reference to an “upstream” and a “downstream” to the count must nevertheless be kept in mind here. What they indicate is a certain hierarchical orientation that is immanent to a ‘universe’ consisting of nothing but multiplicities of multiplicities—a point that we may shed better light upon when we come to examine the set-theoretic schematisation of this ‘universe’. Given Badiou’s insistence that the One is not, and that every unity is only ever the unity of a multiplicity that is “in itself”

inconsistent, we may pursue Badiou's metaphor and say that, if this river has a source, it can only be the extreme limit of inconsistency, the unique place at which being is absolutely *in itself*, unsupported by any relation to anything prior to it. There, we would return to the scene described in that well-worn passage from Plato's *Parmenides* where we are told that where the One is not, "the Others will not be one, but neither will they be many. For if they are to be many, there must be a one among them, since if none of them is one thing ($\epsilon\nu$), they will all be no-thing (οὐδέν), and so not many either" (165^e). At the source of the river, if the river has a source, would be the void, the "prime matter" of pure multiplicity. This is a consequence that Badiou fully accepts and to which I will return later.

If we wish to fill out the concept of presentation beyond what has been said so far, we will find scarce resources in *Being and Event* for this task. So far, we have seen that presentation, like consciousness, is understood as the dimension of the 'there is' that may be abstracted from the question of *what* there is. Like consciousness, presentation is a form of multiplicity, being in every instance strictly determined by what is presented or counted-as-one, and non-identical with that which is presented/counted. We also know that, for Badiou, the intelligible form of presentation is given its essential structure through the mathematical figure of the set, insofar as this figure is deployed and determined through the Zermelo-Fraenkel axiomatic of set theory. From this thesis, we have drawn only the slightest and most general consequences, the totality of which is immeasurable. Central among these consequences, for our project, are those which cause Badiou's concept of presentation to differ sharply from the Sartrean concept of consciousness. Before entering into the mathematics behind these difference, let us first approach them on a strictly metaontological level. The primary distinction concerns the theme of *presence*.

It may seem to throw a wrench into the conceptual machinery we set up to homogenise the two ontologies when, after extensively studying Sartre's characterisation of consciousness as

presence to x, we find Badiou insisting that “presence is the exact contrary of presentation” (BE, m1, 27/35)! But contrary in what sense? We need not encounter any resistance on this point if we can show that the opposition in question takes place within the general concepts underneath which are seeking to subsume *both* presentation and consciousness (*i.e.* ‘presence’). These concepts, of course, are multiplicity and the ‘there is’—two concepts which are, in truth, two sides of a single coin: the ‘there is’ is only the concept of multiplicity operating in an ontological (or metaontological) context, insofar as to speak of an instance of the ‘there is’ is simply a clumsy way of speaking of the multiplicity *of* what there is in some particular instance, the distinction between the ‘there is’ and the ‘what’ being sufficiently safeguarded by the notion of multiplicity itself, as we have already seen.

For the sake of uncovering a conceptual (and not yet mathematical) explanation of the sense in which presentation is the contrary of presence, we need to leave Badiou’s text for a moment and turn to one of its immediate predecessors—namely, Lyotard’s *The Differend*, from which Badiou claims to have “borrow[ed] the word ‘presentation’ in this sort of context” (BE, 484/523). The relevance of this text for Badiou’s mathematical ontology is underscored in his text “Custos, quid noctis?”, his 1983 review of Lyotard’s work. In this work, Badiou offers great praise for the rudimentary ontology of presentation that Lyotard has begun to chart out in *The Differend*. On the other hand, the epistemology that Lyotard proposes, couched almost entirely in the language of jurisprudence, comes in for a harsh critique, especially where it aims to debase what have consistently been Badiou’s three great intellectual passions: mathematics, dialectics and the philosophy of the subject. What is most interesting about this short review, however, is that it is there that we find, for the first time, Badiou’s assertion that mathematics should be understood as ontology itself—that mathematics, and not the (Lyotardian/Jakobsonian) theory of phrases, has the resources required for the thinking of presentation. There, we read that

“mathematics, throughout its history, is the science of being qua being—that is, of being insofar as it is not—the science of impresentable presentation. I will prove it one day.”⁷⁸ Let us leave aside for now the notion of the impresentable,⁷⁹ and the notion of non-being that are raised here; we will return to them later. We will leave aside, also, the fact that Badiou never offers us a proof, but opts instead for a series of axioms (with all the advantages of theft over honest toil, as Russell once said).⁸⁰ What is significant here is that Badiou sees something in Lyotard’s exposition of the notion of presentation that naturally suggests his own later set-theoretic conditioning of this same concept—the conditioning under which presentation is rendered (or remains) contrary to presence.

What does Lyotard say about presentation, then, that could be relevant for us here? His most useful remarks in this respect come (unsurprisingly) from the chapter of *The Differend* entitled “Presentation.” Here, we find that the purpose of the notion of presentation is, to a large extent, to free the notion of the given from its anchorage in the subject. “The idea of [an immediate] given,” Lyotard writes “is a way of receiving and censuring the idea of a presentation. A presentation does not present a universe to someone; it is the event of its (inapprehensible) presence” (Lyotard, 1988, 61). Givenness, argues Lyotard, presupposes an *I* to whom the given is given; there is no such presupposition in the case of presentation. Rather, in order for there to be an *I*, a presentation must already be in effect. In an important passage of *The Differend*, singled out by Badiou in his review, Lyotard seems to echo Sartre with remarkable fidelity:⁸¹ in retracing the path of Descartes, Lyotard insists that “what resists absolutely the radical doubt is not, as Descartes believed, the ‘I think’, but the ‘There has been [*il y a eu*] this phrase:⁸² I doubt’.”⁸³ Indeed, Sartre seems to arrive at the same conclusion with respect to the Cartesian doubt, when he insists that

the transcendent *I* must fall before the stroke of the phenomenological reduction. The *Cogito* affirms too much. The certain content of the pseudo-‘Cogito’ is not *I have* consciousness of this chair,’ but ‘There is [*il y a*] consciousness of this chair’. (TE, 53-4)

To the bare “*il y a*”, Lyotard gives the name, “presentation”, and Sartre, “consciousness”. What is intended by both names is an “absolutely impersonal” (TE, 37) occurrence of the “*il y a*”, by which the muddled notion of givenness is distilled by ridding it of any subordination to an *I* to whom the given would be given.

But we have moved too quickly here, and have missed the point of our inquiry. It is in fact *not* the same proposition of which Sartre and Lyotard are certain. Sartre affirms the certainty of the ‘there is’, while Lyotard affirms the certainty of the ‘there has been’ (the ‘*il y a*’ and the ‘*il y a eu*’, respectively). The reason for this slight difference has already been suggested in Lyotard’s characterisation of the presenting event as ‘inapprehensible’. The event, for Lyotard, has, in every case, already occurred by the time that one can be certain of it, whereas for Sartre consciousness apprehends the event that it is in its very occurrence. The inapprehensibility of presentation may lead one to think that ‘presence’ is not quite the word for this event; Lyotard, never too timid to contradict himself,⁸⁴ agrees: “The *There is* [*il y a*] takes place, it is an occurrence (*Ereignis*), but it does not present anything to anyone, it does not present itself, and it is not the present, nor is it presence.”⁸⁵ When one can finally say that one is certain of a presentation having taken place, the presentation of which one speaks is no longer the presentation in which that speech is presented: “the phrase that presents the presentation itself entails a presentation which it does not present” (ibid., 70). One is always running a bit late.

Presence is nothing other than a reflexivity in presentation, or, in Sartrean terms, in consciousness. It is because consciousness is consciousness of being consciousness of something that we may say that it is a presence (specifically, a presence to *something*). This definition of presence is equally sustained in Lyotard’s text (despite occasional terminological carelessness), and there is no reason to think that it has not been carried over into Badiou’s. What would falsify

this suggestion, of course, would be any indication that presentation—the exact contrary of presence—admitted of reflexivity: specifically, a reflexivity in the relation constitutive of its multiple structure. In Sartrean ontology, this relation goes by the name ‘...is conscious of...’ In Badiouian metaontology, we call this relation ‘...presents...’ or ‘...counts-as-one...’ In the language of set theory, this relation is designated (inversely) by the epsilon⁸⁶ (\in), or, in the graphs drawn above, by the arrow (\rightarrow). The question then becomes, does Badiou’s set-theoretic ontology permit us to declare the possibility of a presentation that presents itself presenting? Does it allow us to declare the existence of a set α such that $\alpha \in \alpha$? We know that the limit case of such reflexivity is banned from Badiou’s philosophy by its initial axiom that *the One is not*. (Similarly, such a limiting case is banned from Sartrean ontology by the thesis that God, the *ens causa sui*, is not—but we will come to this later.) But what of multiplicities, or sets, which present themselves along with others—sets such as $\alpha = \{\alpha, \beta\}$?

The answer, in brief, is that such multiplicities, or forms of presentation, are prohibited by the Zermelo-Fraenkel axiomatic of set theory—specifically, by the axiom of foundation—and *this* is the decisive formal difference between the Badiouian theory of presentation and the Sartrean theory of sets. I say that it is *decisive* because virtually all of the other differences that can be drawn between the two structures in some way or another either follow from or crystallize around this original difference. In order to understand why this is the case, we need to first direct our attention to the questions of identity and ground.

V. Being-in-itself: Second Descent

All things proceed from the void and are borne towards the infinite. Who can follow these astonishing processes?

–Pascal, *Pensées*, §84.⁸⁷

Syntax and the Void, ladies and gentlemen, that's all there is!

- Traditional toast of the Society for Exact Philosophy.⁸⁸

Once seized in its multiple-formation, it becomes evident that a presentation, no less than a consciousness, is incapable of sustaining its existence in and of itself. This is a strict consequence of extensional determination. In order to ground the existence of a presentation or a consciousness, and in order to apprehend its specificity, we must appeal to what that structure discloses as its 'elements', to the extension that determines it. Both Badiou and Sartre acknowledge this exigency, though the consequences it yields in the two systems are dramatically different. One of the essential reasons for this difference has to do with the radical univocity of Badiou's set-theoretic ontology, in contrast to the necessarily equivocal ontology of *Being and Nothingness*. The univocity of being, for Badiou, is intimately related to the syntactical form of set theory. It is from this corner that we ought to approach it.



In general, commitment to ontological univocity is closely wedded to the minimalist orientation, despite Sartre's adherence to the latter but not the former (even here we could observe that Sartre's inadherence to univocity is not a principled rejection, but an inescapable consequence of his system). There is not so much a necessary connection between the two as there is a general, mutual motivation of one commitment by the other; there are, nevertheless, exceptions (Deleuze comes to mind as a univocal maximalist). The task of Badiou's metaontology, to cite Desanti once more, is to determine the "*the least* that must be thought in

order to define the status of the proposition ‘*there are beings*’,” and to do this by stretching the fabric of the ‘there is’ across the axiomatic framework of set theory. His election of set theory as the mathematical apparatus to serve this function is not altogether arbitrary; to a large extent, set theory has already accomplished within classical mathematics what Badiou wishes it to do elsewhere: it has provided a univocal language capable of distilling the *least* that must be thought in order to define the status of virtually any proposition of the form ‘there is a mathematical structure such that...’, at least so far as the classical corpus of mathematics is concerned.⁸⁹ In its Zermelo-Fraenkel axiomatisation, set theory offers us something like a *universal language* in which almost any mathematical proposition can be rewritten, whatever its field of origin, using only the primitive logical signs (the operators \rightarrow , \sim , \exists , $=$, and an alphabet of free variables),⁹⁰ and the sign for set-membership (\in). Virtually any mathematical proposition can therefore be rewritten in the form “there exists a set x such that...” or “there does not exist a set x such that...”

The univocity that set theory brings to mathematics as a whole is matched by a more profound univocity that is proper to it, and which could be called the *ontological* univocity of set theory. Set theory is written in such a way as to admit one and only one type of variable, which is determined uniquely by the relation of belonging \in . As Badiou observes, “That there is only one type of variable means: all is multiple, everything is a set,” but this is determined through an “inscription without concept.” “If indeed,” Badiou writes,

the inscription without concept of that-which-is amounts to fixing it as what can be bound, by belonging, to the multiple, and if what can be thus bound cannot be distinguished, in terms of the status of inscription, from what it is bound to—if, in $\alpha \in \beta$, α only has the possibility of being an element of the set β inasmuch as it is of the same scriptural type as β , that is, a set itself—then that which is is uniformly pure multiplicity. (BE, m3, 44/55-6)

Set theory’s syntactic homogeneity imposes a strict univocity on Badiouian metaontology.

Schematising the set as presentation, Badiou opens a horizon in which, in the last analysis,

nothing exists other than presentations. Clearly, that there is nothing but presentations means that every presentation is a presentation of presentation.⁹¹

There is therefore a sort of regress, where presentations present presentations, which present presentations, etc. In Sartrean ontology, this regress is blocked after its very first step, by virtue of that ontology's bivocal structure. The reality of the extension is guaranteed by the fact that consciousness, being absolutely intentional, exists only insofar as its extension does, and this ontologically secondary existence is given immediately and with Cartesian certainty. For Sartre, however, the structure of extensional determination does not give way to any regress, since the ontological descent from consciousness to extension stops at the level of the transphenomenal being of the phenomenon, which Sartre (axiomatically) declares to be what the phenomenon is *in itself*, or the phenomenon insofar as it is what it is. The identity of the in-itself, unlike that of presentation or consciousness, is immediate, and owes nothing to any form of relational determination, of which extensional determination is a particular species. For Badiou, however, the identity of what is presented can almost never be said to be immediate. In a certain sense, this could just as well be understood to be a consequence of three related metaontological theses: that, against Sartre, ontological identity and ontological unity are the same thing,⁹² that "*what presents itself is essentially one*" (BE, m1, 23/31(?)), and that unity is only ever operational (unity *of* something). Where the Sartrean notion of consciousness throws us immediately "into the dry dust of the world, onto the rough ground, amongst things" (I, 33), and ultimately onto the bedrock of being-in-itself, the Badiouian notion of presentation opens onto an abyss, and the presented, rendered in the pure transparency of its presentational form, disappears from sight.

The best way of approaching this abyss is through its set-theoretic formalisation, particularly through the criterion of set-theoretic identity, the axiom of extensionality. This axiom states that two sets α and β are identical if and only if each of their respective elements are

identical. Now, as we know, these elements are themselves sets, and so we must evaluate their identity in exactly the same way. Hence, the identity of α and β is contingent upon the establishing an identity between all the γ 's in α , and all the δ 's in β . In the same way, the identity of each γ and δ is contingent on the identity of each η in γ , and each θ in δ , etc. Only once (and if) this downward regress comes to a halt, or, to use Badiou's fluvial metaphor, only once we reach the source of the stream of presentation, can the identity of any set be stabilized. This is an important fact about the theory of sets and indicates a problem that must be elucidated here.

In order to make things more precise, let us introduce some new terminology: we will call an \in -descent (or \in -chain) any series of sets $\delta_0, \delta_1, \delta_2, \delta_3, \dots \delta_i$, etc., such that $\delta_n \in \delta_{n-1}$ for every element of the series. If it were possible for infinite \in -descents to exist, then there would be some sets whose identity would be indefinitely deferred. These would be sets δ_0 from which an \in -descent could be traced in such a way that if δ_n belongs to the descent, then so does δ_{n+1} . That such sets do not exist, and that every set legitimated by set theory therefore has an assignable identity, is ensured by Zermelo's axiom of foundation.⁹³

The axiom of foundation states simply this: every non-empty set possesses at least one element with which it has nothing in common. Formally, this is expressed:

$$(\forall \alpha)(\exists \beta)(\beta \neq \emptyset \rightarrow (\beta \in \alpha \ \& \ \beta \cap \alpha = \emptyset))$$

This axiom has the peculiar characteristic of being rather marginal and obscure as far as the working mathematician is concerned, while being, at the same time, of great theoretical importance for both the integrity of the ZF axiomatic and its metaontological schematism. In *Set Theory and the Continuum Hypothesis*, Paul Cohen introduces this axiom with the observation that it "is a somewhat artificial one and we include it for technical reasons only. It is never used

in conventional mathematics” (Cohen, 53).⁹⁴ Badiou echoes this sentiment, observing that the axiom is “supernumerary with regard to the needs of the *working mathematician*”⁹⁵ and hence with regard to historical ontology” (BE, m18, 187/208-9*), but immediately adds that “its scope is thus rather reflexive or conceptual. The axiom indicates an essential structure of the theory of being, although it is not required for [*sc.* most of] the particular results of this theory. [...] Its usage is largely meta-theoretical” (BE, m18, 187/209*). The essential ontological structures indicated by this axiom are numerous, but the one that presently interests us is the capacity that it grants to the axiom of extensionality to decide on the identity of any consistent multiplicity. This capacity is secured by a simple lemma, which follows directly from the axiom.

The lemma is this: if the axiom of foundation is adopted, no infinite descending \in -chains exist. It can be demonstrated as follows:⁹⁶

We begin by constructing a denumerable sequence of sets, which we will call the δ -sequence. We will assign a natural number to each element of the sequence, and construct it in such a way that, beginning with an arbitrarily chosen set δ_0 , we have $\delta_{i+1} \in \delta_i$ for any element δ_i in the sequence (so that δ_1 is an arbitrarily selected element of δ_0 and so on).

Without any appeal to the axiom of foundation, we can be certain that even if this sequence is infinite, it is no longer than can be measured by the *smallest* infinite ordinal ω_0 .⁹⁷ If the contrary were true, then there would have to be a link in the \in -chain δ indexed by ω_0 (an \in -descent terminating with this link would have a length of ω_0+1). By definition of an \in -descent, in order for δ_{ω_0} to occupy a place in the chain, there must exist a δ_j such that $\delta_{\omega_0} \in \delta_j$; but for this to be the case, j must be an ordinal whose successor is ω_0 , and by definition of ω_0 , no such ordinal exists (ω_0 is such that if $n < \omega_0$ then $n + 1 < \omega_0$, and so there can be no $n \in \omega_0$ such that $n + 1 = \omega_0$, for we would then have $n + 1 < \omega_0$ and $n + 1 = \omega_0$, which is a contradiction).⁹⁸ Because the assumption that an \in -descent of a length greater than ω_0 implies the existence of a link indexed by ω_0 itself, it is proven that if there exists a \in -descent of infinite length, then its length must be ω_0 , meaning that for every ordinal n lesser than (or belonging to) ω_0 there exists a δ_n .

In order to establish that every \in -descent is of *finite* length (i.e. terminates with a δ indexed by a finite ordinal), we must have recourse to the axiom of foundation. We will proceed by constructing a second set Δ whose elements are the links of the δ -sequence (the \in -descent in question). If the δ -sequence is of

length ω_0 then there are ω_0 elements in Δ . At this point, it is useful to recall the definition of the set ω_0 . ω_0 is the first infinite ordinal number, and as such it contains as elements *all* the finite ordinals. This means that if n is an element of ω_0 , then $n + 1$ is also an element of ω_0 . Now, we have said that Δ contains all the sets of the δ -sequence up to, but not including, δ_{ω_0} . Accordingly, if $\delta_n \in \Delta$ then $\delta_{n+1} \in \Delta$ as well. Now, recall that for every number n , $\delta_{n+1} \in \delta_n$. As a result, for every $\delta_n \in \Delta$ there exists a set $\delta_{n+1} \in \Delta$ such that $\delta_n \in \delta_{n+1}$.

The contradiction arises when we bring the axiom of foundation to bear on the set Δ . This axiom assures us that Δ has at least one element with which it has no elements in common. We may call this the *founding element* of Δ . Since all the elements of Δ have been numbered by the finite ordinals, we will call this founding element δ_f where f is an unknown finite ordinal. The problem is that, f being a finite ordinal, we know that δ_{f+1} is also an element of Δ . But, by definition, δ_{f+1} is also an element of δ_f . δ_f and Δ share the element δ_{f+1} , and so δ_f cannot be a founding element of Δ . Given that every element δ of Δ is indexed by a finite ordinal, it makes no difference which δ_f is presumed to be the founding element. The consequence is that Δ possesses *no* founding elements whatsoever, and this is in a direct contradiction with the axiom of foundation. This contradiction can only be avoided if we abandon our original hypothesis. *Infinite \in -descents do not exist.* \square ⁹⁹

The non-existence of infinite \in -descents has a number of interesting consequences in addition to legitimating the regime of identity described by the axiom of extensionality. The first consequence is that circular \in -descents are likewise non-existent. This turns out to be a trivial corollary as soon as one realises that there is no difference whatsoever between a circular \in -descent and a repetitive infinite \in -descent. If a multiplicity is an element of itself, then we say that it is characterised by self-membership, or a circular \in -descent of length 1. Another corollary that follow from this is thus that the axiom of foundation is inconsistent with the existence of the One, insofar as the One is that which satisfies the formula $\Omega = \{\Omega\}$. Hence, the axiom ratifies one of the principle characteristics of the multiple: that if x is a multiple, $x \neq \{x\}$.

We are not yet finished with \in -circularity, however. Badiou employs precisely this structure of self-membership in order to deploy a quasi-mathematical characterisation of the

event. Accordingly, the axiom of foundation consummates the formal exclusion of events (thus characterised) from the field of ontology. Badiou is, of course, aware of this consequence, which is, in part, responsible for the divided character of his magnum opus (*Being and Event*). The theory of the event that he proposes takes us admittedly “*outside* the field of mathematical ontology” (BE, m18, 184/205*), although Badiou continues to situate it in relation to ontology and ontological structures. On the basis of the preceding, however, we may already foresee that the event, should it exist, is not only “outside the field of mathematical ontology”, but outside the regime of identity that governs this field. As a non-wellfounded occurrence, the event cannot be said to be identical to itself in any ‘ontologically’ (mathematically) meaningful sense. Like the Sartrean event of consciousness, its mode of individuation must be something other than identity. We will return to this in the next chapter.



With the axiom of foundation, Badiou remarks, “set-theoretic ontology affirms that, while presentation may certainly be infinite, it is nevertheless always marked by a finitude *with regards to its origin*” (BE, m18 187/208). But what is the ‘origin’ of these manifold descents? Badiou’s answer is in perfect conformity with his set-theoretical infrastructure, but presents us with certain metaontological difficulties: the origin of every descent is the void, understood as the unique and empty set \emptyset .

Set-theoretic ontology secures the place of the void, where all of presentation is grounded and in which the identity of every consistent set is rooted, with the axiom of the empty set. This axiom states that a set exists which possesses no elements. In formal notation, it reads:

$$(\exists \alpha)[\sim(\exists \beta)(\beta \in \alpha)]$$

This proposition is unique among the axioms of set theory in that it unconditionally asserts the existence of its object. Every other axiom is of a form that we may call ‘hypothetical’, having

more or less the syntax: “Let α be a set whose existence is presupposed. There then exists another set β , constructed on the basis of α in such and such a fashion” (BE, m5, 62/75*). With respect to identity, the empty set is unique insofar as the axiom of extensionality assures its identity *without mediation*. No recourse is made, via the epsilon, to any other set on which its identity would depend. This can be readily seen by a comparison of the process by which one establishes the identity and unicity of the empty set with the manner in which other sets are identified. The evaluation of the equation $\{\alpha\} = \{\beta\}$, for example, is mediated by an investigation of the \in -descents of the (two) set(s): only once we know that α and β are identical can we know whether or not $\{\alpha\}$ and $\{\beta\}$ are; and, of course, α and β can be identified only after their respective elements are, and so on. Every element in the \in -descent is another layer of mediation between a set and the affirmation of its identity. The situation with \emptyset is quite different, and Badiou sums this up nicely:

Mathematicians tend to say, with some levity, that the empty set is unique ‘after the axiom of extensionality’. Yet this is to proceed as if ‘two’ voids can be identified like two ‘somethings’, which is to say two multiples of multiples, whilst the law of difference is conceptually, if not formally, inadequate to them. The truth is rather this: the unicity of the void set is immediate because nothing differentiates it, not because its difference can be attested. An irreducible unicity based on in-difference is herein substituted for unicity based on difference. (BE, m5, 68/82*)

It is because the empty set is *immediately* identical to itself that it can found every mediate identity, and anchor the endless profusion of sets, all of which are only ever sets of sets if they are not the void itself.



In the preceding, we have sought to explicate a few of the essential points involved in Badiou’s set-theoretic formalisation of the concept of presentation. The axis on which these points are plotted is what we may call the ontological regime of identity. In Zermelo-Fraenkel set theory, this regime is condensed into three axioms: the axiom of extensionality, in which the primary idea of set-theoretic identity is expressed; the axiom of foundation, which ensures the

universal pertinence of this idea over the universe of sets; and the axiom of the empty set, which grounds every identity in the *immediate* identity, or in-difference, of the empty set. These three axioms also allow us to delineate three crucial points of departure by which the Badiouian category of presentation detaches from the Sartrean category of pre-personal consciousness. In its metaontological schematism, the axiom of extensionality determines every normal (*i.e.* non-evental) presentation as identical to itself.

As is obvious, this axiom satisfies the First Criterion of multiplicity for the set, insofar as it is little more than a formalisation of the principle of extensional determination in the mode of identity. According to our definition, however, the principle of extensional *differentiation* is equally essential to the concept of multiplicity. This principle is presupposed by the axiom of extensionality (in that its application depends on discerning the difference between the set to be identified and the elements upon which this identity will be founded), but it is only *secured* by means of the axiom of foundation and the axiom of the empty set. The axiom of foundation operates by explicitly concentrating extensional differentiation in at least one element of the set (or presentation) in question: its ‘founding element’. This element is *absolutely* distinct from the set to which it belongs, or the presentation that presents it.¹⁰⁰ By necessitating the existence of a at least one founding element in each existent set, the axiom of foundation, by consequence, maintains a general distinctness between a set and its elements, insofar as it prohibits the formation of the One.

The axiom of the empty set furnishes us with the third and final element of the ontological regime of identity. It asserts the primitive and underived existence of the void itself, situating it as the prime matter of which every set is a “modality according-to-the-one” (BE, m4, 57/70). This manoeuvre seems to take us quite far from Sartrean ontology, but not so far that we cannot detect certain structural similarities between the two doctrines. Like Sartrean being-in-

itself, the void is the ground of immediate identity on the basis of which every presentational identity takes root. The recursive nature of the process of identification prescribed by the axiom of extensionality, and the hierarchical dimension given to the universe of presentation by the axiom of foundation, entails that ontological identity is only possible insofar as it can be traced back to the unmediated identity of the void. We see an analogous treatment of identity in the Sartrean doctrine of the in-itself, which is the underived and unmediated ground of the former, as well as the basis for any presented (*i.e.*, phenomenal) existence.

An important point of divergence between Sartrean being-in-itself and the Badiouian void (insofar as the latter is named and schematised by the empty set) concerns the joint problems of unicity and distinction. While the in-itself is immediately ‘self’-identical, it cannot in any meaningful sense be said to be unique. It is, as we have seen, without any immanent principle of individuation. From the point of view of consciousness, at least, there are infinitely many distinct being-in-themselves, even if this distinctness can only be established ‘retrospectively’ as an “external relation” (BN, II.I.I, 77/113). From the point of view of set theory, there is only one \emptyset . We may understand this divergence as a result of Sartre’s separation of the principle of unification from the principle of identity (see Chapter II). That the in-itself is characterised by absolute identity does not entail, for Sartre, that there is any trace of unity to be found amongst the in-itself, considered in a purely transphenomenal sense (as paradoxical as this thesis may seem). Being-in-itself is not composed of unities nor is ‘all’ of being-in-itself unitary (nor, we should add, is there an ‘all’ of being-in-itself). By contrast, for Badiou, in mathematics as “in every situation in thinking, the problem of unity and of identity are indiscernible. [One must] elaborate the question of identity from the question of the unity of the multiplicity—it’s the same thing. The unity of the multiplicity *is* the ontological identity” (OP, 170). For Badiou, identity is uniquely distributed by means of the operation of unification, that is, by means of the

formation of sets—and all of this on the basis of the primordial unification that is the naming of the void. Outside of these operations, there is no basis for separating identities. There is no “unthinkable residuum” that would make that which is emptied of every unity distinct from another posited emptiness.

In a sense, it could be said that while Sartre’s vision of the in-itself appears to follow the line of reasoning in the *Parmenides* whereby so long as the one is not (so long as consciousness does not bestow unity), “each mass of [being-in-itself] must be without limit of multitude” (164^D) and disperses in an inertia of exteriority, Badiou’s set-theoretic ontology pursues this line to the very end of the same dialogue, to where Plato states that “if there is no One, [nor] can the Others be or appear like or unlike, or the same or different, or in contact or apart” (166^C). The final result of this abolition of every distinction is that “if the one is not, nothing is” (166^C; quoted in BE, m2, 31/41). Badiou strongly emphasizes this point in Plato’s dialogue, drawing attention to the abolition of all distinction in the absence of the one, and to the ambiguity of the Greek text, which by his lights allows of being read not simply as a denial of every particular existence, but as the affirmation of the being of the nothing itself.¹⁰¹

And yet it is entirely possible to completely reverse this opposition between Sartre and Badiou, between the disunity of the in-itself and the indistinctness of the void. It is evident in the very imagery that Sartre frequently uses to describe the concept of the in-itself that it is equally possible to think the latter as an absolutely compact block, as pure massivity and as an “inherence in itself without the least distance” (BN, in.VI, lxv/31). It is likewise possible to draw a distinctly Platonic equivalence between absolutely inconsistent multiplicity and an undifferentiated Nothing, as in the finale of the *Parmenides* where the absolute proliferation of difference is shown to be indiscernible from the absolute cancellation of difference. It is clear that Badiou is keenly aware of the profound equivalences, which, despite all appearances, make

what is explicitly intelligible of both void and in-itself more or less coincide—this much is evident when he says that,

At bottom, to the figure of compactness given in *Being and Nothingness*, I will substitute the figure of absolute pulverization [*pulvéulence absolue*]. They are not antinomical: absolute opacity, absolute pulverization... in a certain sense, there is a point where they are indistinct.⁰²

In the second Meditation of *Being and Event*, Badiou explicitly connects this indistinct and absolute ontological figure to the *Parmenides*' "aporetic conclusion," which he renders in French as "Si l'un n'est pas, rien [n']est," and Feltham translates as "If the one is not, (the) nothing is" (BE, m2, 36/46); as has already been mentioned, what is intended in both expressions is not the annulment of being (Badiou insists on the distinction between being-nothing and non-being¹⁰³) but the identification of being-in-itself with a pure void:

'If the one is not, (the) nothing is' also means that it is only in completely thinking through the non-being of the one that the name of the void emerges as the unique conceivable presentation of what supports, as unrepresentable and as pure multiplicity, any plural presentation, that is, any one-effect. (BE, m2, 36/46)

Symmetrically, the very possibility of the absolute compactness/pulverisation of the in-itself rendering itself to thought exclusively as void persistently haunts the Sartrean doctrine of transphenomenal being, as our analysis of Hartmann's critique has shown (Chapter II): if the three postulates are taken to be *adequate* to the in-itself, then for all intents and purposes it becomes an unintelligible void. If Sartre avoids this conclusion, it is only by insisting that the postulates are *not* adequate. They are merely "three characteristics which the preliminary examination of the phenomenon of being allows us to assign to the being of phenomena. For the moment it is impossible to push our investigation further" (BN, in. VI, lxvi/33). One is left to wonder about the nature of this impossibility—is it out of caution, uncertainty or foresight that Sartre says that *for the moment* it is impossible to proceed further? If this impossibility were absolute—if the postulates could not be extended because they were immediately adequate to being—then we would indeed have a problem that is an inverted, but otherwise identical, version

of the problem sighted by Hartmann. It could be argued that one of the key differences between the Sartrean doctrine of the in-itself and the Badiouian doctrine of the void is effectuated by Badiou's insistence that the axiomatic determination of the void as void is *absolutely* adequate to the being that it inscribes. Badiou himself seems to claim as much when he writes that,

because the void is indiscernible as a term (because it is not-one) its inaugural appearance is a pure act of nomination. This name cannot be specific; it cannot place the void under anything that would subsume it—this would be to re-establish the one. The name cannot indicate that the void is this or that. The act of nomination, being a-specific, consumes itself, indicating nothing other than the unrepresentable as such. [...] The consequence is that the name of the void is a pure *proper name*, which indicates itself, which does not bestow any index of difference within what it refers to, and which auto-declares itself in the form of the multiple, despite there being *nothing* which is numbered by it. (BE, m4, 59/72)

The crucial point of departure between the Sartrean and the Badiouian axiomatisations of the in-itself, then, does not consist in opposing emptiness to massivity, but in opposing, to the manifest but unintelligible brutishness of being, the Parmenidean doctrine according to which “being and thinking are the same,” but with the subtractive caveat that this sameness is fundamentally grounded in what Bachelard called “the first clear thought.”¹⁰⁴ As we proceed, however, the clarity of this thought becomes more and more difficult to discern.

VI. Nothingness and Event

La première pensée claire c'est la pensée du néant.

- Gaston Bachelard, *La Dialectique de la Durée*.

Things are complicated by the fact that, despite everything, the void adopts several dissimilar aspects in Badiou's philosophy. To begin with, we have the notion of the void as the ultimate foundation or ground of presentation. This notion is supported by an axiomatic skeleton consisting of the axiom of extensionality, the axiom of foundation and the axiom of the empty set, and accompanies the fluvial metaphor sketched out in Chapter IV. It is further accompanied by a harmonisation of two Platonic concepts: that of the *πληθος* of others-without-the-One illustrated at the end of the *Parmenides* (BE, m2, *passim*), and that of the *receptacle* or *χωρα* into which the Ideas are inscribed and thereby affected by the "errant cause" of *αναγκη*, as recounted in the *Timaeus* (BE, m4, 55/68).¹⁰⁵ The Sartrean metaphor of *inertia* is also invited into this ensemble of philosophemes, to much the same effect as the Platonic "errant cause". The void, according to this view, is something like the inexhaustible prime matter from which every presentation is composed, so that every presentational multiplicity is conceived as "a modality-according-to-the-one [*selon-de-l'un*] of the void itself" (BE, m4, 57/70). Now, the void thus conceived is not uniquely located at the root of the tree of presentation, but, without losing its basic unicity, thoroughly infiltrates the tree's every branch: it is that aspect of multiplicity that is still-uncounted and which inheres in every presentation; the void names the inconsistency of multiplicity, the being-upstream-of-the-count, in general. For,

Insofar as the one is a result, by necessity 'something' of the multiple does not absolutely coincide with the result. To be sure, there is no antecedence of the multiple which would give rise to presentation because the latter is always already-structured such that there is only oneness or consistent multiples. But this 'there is' leaves a remainder [...] a must-be counted [...] which

causes the structured presentation to waver towards the phantom of inconsistency. (BE, m4, 53/66)

Badiou expresses this inherence using a favourite image of Sartre's, and says that the void "haunts" every situation (BE, m8, 94/110). This vision of the void as haunting or inhering in every situation without presenting itself as such does admit of a valid set theoretic formalisation, in the form of the theorem that the empty set is a subset of every set, a theorem that will be explained and demonstrated in Chapter VIII.

Beyond these 'grounding', 'abyssal' and 'haunting' figures of the void, Badiou proposes another: this time, the void is thought in terms of the nothingness of the count-as-one itself, its sheer *Unselbstständigkeit*. For Badiou, however,

it comes down to exactly the same thing to say that the nothing [sc. the void] is the operation of the count—which, as source of the one, is not itself counted—and to say that the nothing is the pure multiple upon which the count operates—which 'in itself', as non-counted, is quite distinct from how it turns out according to the count.

The nothing [sc. the void] names that undecidable of presentation which is its unrepresentable, distributed *between* the pure inertia of the domain of the multiple, and the pure transparency of the operation thanks to which there is oneness [*d'où procède qu'il y ait de l'un*]. The nothing is as much that of structure, thus of consistency, as that of the pure multiple, thus of inconsistency. (BE, m4, 55/68; e.a.)

As can be seen here, the thought of the void as the operation of the count soon resolves itself into the thought of what insists between the count and its effect. The void is "the imperceptible gap, cancelled then renewed, between presentation as structure and presentation as structured-presentation, between the one as result and the one as operation, between presented consistency and inconsistency as what-will-have-been-presented" (BE, m4, 54/67*). This gap is precisely what is indicated by the non-identity $x \neq \{x\}$, the non-identity between a multiplicity and its elements, or—this is a crucial point—between a consciousness and its noemata. It is not immediately evident how this conception of the void is related to the void as empty set. It is nevertheless this notion of the void that is most important for the theory of the event, and it is the only one that allows us to make productive and non-trivial sense of Badiou's thesis that

for the void to become localisable at the level of presentation, and thus for a certain type of intrasituational assumption of being qua being to occur, there must be a dysfunction of the count, resulting from an excess-of-one. The event will be this ultra-one of chance, on the basis of which the void of a situation may be retroactively discerned (BE, m4, 56/69*).

It is to the event that we now turn.

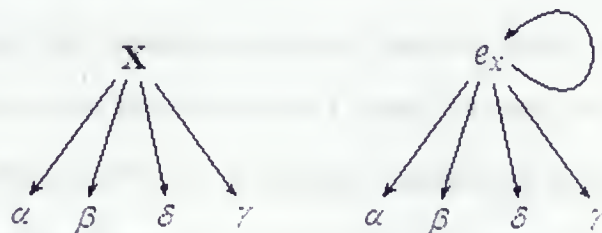


The axiom of foundation assures us that every nonempty set—every situation—contains at least one element with which it has nothing in common.¹⁰⁶ When this element is not empty, we have what Badiou calls an *evental site*. This site is understood metaontologically as “that which is *almost* withdrawn from the situation’s regulation by an immanent norm, or its state. In a situation (in a set), it is like a point of exile where *it is possible* that something, finally, might happen.”¹⁰⁷

What is odd about this placement of the event is that the event itself violates the very axiom that assures us of the existence of such sites. An event, a figure “outside the field of mathematical ontology” (BE, m18, 184/205*), has the form of a reflexive multiplicity, consisting of the elements of the site on the one hand, and itself on the other. As Badiou has it,

as multiple, the event’s figure mobilizes the elements of the site, delivering them from the axiom of foundation. Subtracted from this axiom, and thus unfounded, the multiple of the elements of the site is going to act in a peculiar manner, namely, by immanentizing its own multiplicity.¹⁰⁸

In other words, if the site where the event occurs is X , then the event e_x is described by the multiple, $(\{e_x\} \cup X)$. If the elements of X are α , β , γ and δ , for example, then the respective graphs for X and e_x would look like this:¹⁰⁹



Here, one may ask why such an ontologically illegitimate construction is called for. One possible, but ultimately unwarranted, explanation is that the reflexive structure of the event is motivated by empirical observations pertaining to the nature of what Badiou wishes to call ‘events’. The ontological illegitimacy would then be the inconvenient but ultimately necessary price that must be paid in order to save the phenomena in question. Neither assumption is justifiable. For one, Badiou clearly asserts that an event is not legible as such on the basis of empirical data alone, but “can only be *thought* by anticipating its abstract form, and can only be *revealed* in the retroaction of an interventional practice which is itself entirely reflective [*réfléchie*]” (BE, m17, 178/199*). Furthermore, the judgment as to whether or not an event has taken place is always held to be essentially *undecidable*, and so in principle cannot be inferred on the basis of any available empirical information. This position is continuous with Badiou’s frequent assertions to the effect that “the pure analysis of phenomenal appearing cannot decide between divergent orientations of thought. We need, in addition, to make explicit the axioms of thought that *decide* an orientation.”¹¹⁰ The decision as to whether there are or are not events which break with the essential structure of presentation is a paradigmatic example of such an undecidable point of divergence.

Moreover, set theory as it stands is sufficiently supple enough to approximate virtually any merely empirical descriptions insofar as they can be formalised with the sole sign of belonging (and no sign is available in the formulation of the ‘matheme of the event’ that is not available in ontology).¹¹¹ Yet, even if we grant that certain situations or occurrences necessitate a mathematical formulation that violates the axiom of foundation, there is still no justification for the inconsistency that sits at the centre of Badiou’s edifice, for there exist alternative set theories that allow for such constructions without producing a contradiction. One such set theory, Peter Aczel’s ZF⁻+AFA,¹¹² has been successfully exploited by Jon Barwise to produce a *mathematical*

theory of situations, a fact that is not lost on Badiou, who devotes an endnote of *Being and Event* to Barwise, writing that

It is quite remarkable that the Anglo-Saxon school of logic has recently used the word 'situation' to attempt the 'real world' application of certain results which have been confined, up till the present moment, within the 'formal sciences'. A confrontation with set theory then became necessary. A positivist version of my enterprise can be found in the work of J. Barwise and J. Perry. There is a good summary of their work in J. Barwise, 'Situations, sets and the Axiom of Foundation', *Logic Colloquium '84* (North Holland: 1986). (BE, 484/523)

As the title indicates, the work cited here directly addresses the restrictive character of the axiom of foundation and, without positing any suspension of the laws of being, proposes a theory of situations (a 'metaontology' in Badiou's jargon) rooted in a different axiomatic. Badiou's decision to theorise the event as an exception from the laws of being cannot, therefore, be considered either an accident or an unstable compromise between mathematics and empirical data.¹¹³

If we wish to understand the motivating reason for the event's \in -reflexivity and inconsistency with the axioms of ZF, then, rather than trying to justify this structure empirically, we ought to look towards the *consequences* of positioning a reflexive multiplicity within a universe whose ontological structure is governed by the Zermelo-Frankel axioms. In doing so, we see that the group of axioms whose operation is most directly affected by the reflexivity of the event are those which we have called the set-theoretical regime of identity. The effects with respect to the axiom of foundation are obvious enough to spare further analysis—this axiom is simply contradicted, or suspended. The effects with respect to the axiom of extensionality are more interesting: The event, being subtracted from foundation, cannot be identified by the axiom of extensionality. Its \in -descent is infinite. The paradox, of course, is that the proposition that the axiom of extensionality (AE) cannot determine the self-identity of e_x is true only if e_x is, in fact, 'identical' to itself, otherwise the matheme of the event would not entail the circularity of the event's \in -descent: we need some assurance that when we write $e_x = \{x \in X, e_x\}$, that e_x

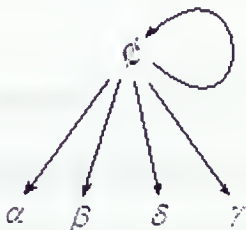
designates a self-identical structure, in order for us to be sure that the circle rejoins itself.

Formally, we seem to have something like the proposition:

$$\sim[AE \rightarrow e_x = e_x] \leftrightarrow [e_x = e_x \ \& \ e_x \in e_x]$$

Albeit, the ‘identity’ demanded here is purely nominal, and does not reach into the ontological composition of e_x itself. The point to be grasped here is that precisely insofar as e_x *is* what it is, its mode of individuation cannot be ontological in the strict sense.

It is difficult not to notice that the structure that we are approaching bears a striking resemblance to the one that Sartre attributed to impersonal consciousness, for which we may provide a diagrammatic approximation that is virtually identical to the graph of the event. Putting \mathcal{C} for a given consciousness, and $(\mathcal{C} \rightarrow \alpha)$ for ‘ \mathcal{C} is present to α ’, or ‘ α belongs to the extension of \mathcal{C} ’, we have:



This structural similarity is significant, for it is on the heels of this structure’s occurrence that both Sartre and Badiou introduce the formation of the subject. For Badiou, the subject first appears in the operation of *intervention*, which, he tells us, is necessary in order to bind the ephemeral and inconsistent occurrence of the event to the situation in which it takes place, in order, that is, to affirm the *belonging* of e_x to its situation. Before this is possible, however, the individuation of e_x must be established. Intervention, for Badiou, thus has two aspects: the recognition of the evental form of e_x (the recognition that it is in fact e_x *itself* that is presented in e_x) and the decision on the belonging of e_x to the situation. This recognition, however, has no basis in anything but its own operation. As Badiou remarks,

An event of the site X belongs to itself, $e_x \in e_x$. Recognizing it as a multiple supposes that it has *already* been named—for this supernumerary signifier e_x to be considered as an element of the one-multiple that it is. The act of nomination of the event is what constitutes it, not as real—we will always posit that this multiple has occurred—but as susceptible to a decision concerning its belonging to the situation. (BE, m20, 203/225)

This ‘reality’ that is supposed to precede the subjective act of nomination is, in fact, quite difficult to place, and we have every reason to suspect that it is without content. If the act of nomination is what effects the reflexivity of the event, then the only reality prior to this act would be the bare ‘facticity’ of the event, which is nothing other than its site ($F(e_x) = X$). But this is clearly not what Badiou wishes to say; to “always posit that this multiple has occurred,” is to *deny* that “nothing will have taken place but the place.”¹¹⁴ The affirmation of the prior reality of the event is little more than a placeholder for a theoretical insufficiency, or the wish to avoid a sort of arbitrary ‘decisionism’ with respect to the event.¹¹⁵ This insufficiency vanishes if we attend to things more carefully, but first let us turn our attention to another deficiency with which this one is closely connected.

Badiou notes early on in the meditation on intervention that “the procedure of decision,” with respect to which the event is recognised, “requires a certain degree of preliminary separation from the situation, a coefficient of unrepresentability” (BE, m20, 201/223), since the event is by its very definition not something that can be ratified by the laws of presentation. At first glance, it seems that the preliminary separation that will permit the intervention to occur could be nothing other than the event itself, while by the same measure, “It seems that the event, as interventional placement-in-circulation of its name, can only be authorised on the basis of that other event, equally void for structure, which is the intervention itself” (BE, m20, 209/231). For Badiou, this constitutes a vicious circle, which must be broken. “There is actually no other recourse against this circle,” he writes,

than that of splitting the point at which it rejoins itself. It is certain that the event alone, aleatory figure of non-being, founds the possibility of intervention. It is just as certain that if no intervention puts it into circulation within the situation on the basis of an extraction of elements

from the site, then, lacking any being, radically subtracted from the count-as-one, the event does not exist. In order to avoid this curious mirroring of the event and the intervention—of the fact and the interpretation—*the possibility of the intervention must be assigned to the consequences of another event*. It is evental recurrence which founds intervention. In other words, there is no interventional capacity, constitutive for the belonging of an evental multiple to a situation, save within the network of consequences of a previously decided belonging. An intervention is what presents an event for the occurrence of another. It is an evental between-two. (BE, m20, 209/231-2)

Badiou is correct to see a “curious mirroring” in this phenomenon of event and intervention, and he is right to crystallise this mirroring into an irreducible “between-two”, but the way he goes about this is fruitless and misleading. To begin with, *how* does a previous event establish the interventional capacity? Are events simply presentations going on behind our backs in a nether realm accessible only to those already descended into the evental underworld? If this suggestion (or any other equivalent mythologeme) is to be taken seriously, then there would have to be two stratified regimes of presentation, one exoteric (situation) and one esoteric (event). This would be an odd move, and one for which I cannot see any possible legitimation. Oddity is not itself grounds for serious objection, of course, but there is also the more significant problem that this tactic can only lead us to either an eternal and infinite string of events, each legitimating the intervention for the next, or else to a prehistoric, absolute and original event, in which the circularity of event and intervention would be consummated. Both options seem to dodge the resolution of the problem of circularity in favour of a quasi-mystical deferral, and one of them simply produces the exact same problem behind our backs.

I have already mentioned that Badiou sees, in the event, the invocation of the void itself. As his theory of the event stands, however, this powerful intuition admits of only the most trivial and insignificant interpretations. For instance, Badiou at one point says that the evental invocation of the void is discernible in the fact that the name of the event is a “pure proper name”, and can be attributed to nothing counted as one in the situation, except, perhaps, the ephemeral nomination of the event itself (BE, m17, 182/203). If this were the case, we could say

the same for *any* nonsense word; '*filbitsiwaky*,' for example, names nothing but its own enunciation (and even that is decided by fiat). To say that '*filbitsiwaky*' "invokes the void," sounds at least a little melodramatic. If this is what the event does, then it is difficult to see its metaphysical importance.

If we are to make any progress here, it is necessary to follow Badiou's guiding intuition that there is, in event and intervention, a sort of circularity that does not resolve itself into identity (making the event a pure and arbitrary decision), and a duality that does not dissolve into unity. It is especially necessary to follow this intuition in a way that avoids the traps to which Badiou's own account has succumbed: the endless recursion of events and intervention gives us nothing, and the mere affirmation of a reality independent of nomination says nothing. What is required is a theory of evental individuation by which a multiplicity comes into being both contingently (rather than by fiat) and *for itself*, and by which it maintains an original and non-decomposable duality within its structure.

The concepts necessary for such a theory are already at hand, having been forged by Sartre in his analysis of the Immediate Structures of the For-itself in *Being and Nothingness*. In order to employ these concepts, we must first recognise clearly the structural congruencies between event and consciousness that are already in place, and which we have struggled to bring to light thus far in the present study. Impersonal consciousness and presentation have been shown to be fundamentally similar forms, being determinations of the 'there is' insofar as the 'there is' is not reducible to *what* there is. Both exhibit a form which we have identified as being that of multiplicity, in that they are extensionally determined by what they present, and from which they nevertheless differ or withdraw. Consciousness markedly distinguishes itself from presentation by virtue of its reflexivity, its non-identity and its purely 'interior' mode of individuation. Presentation, in its normal state, can always be assigned an identity (at least in

principle), which is stable and fixed in 'exteriority' by the axiomatic regime of identity. This regime requires, however, that presentation not be reflexive. The formal distinction between the structure of consciousness and the structure of presentation breaks down, however, when presentation is affected by a "dysfunction of the count", and "immanentizes its own multiplicity" in presenting itself. This dysfunction is what takes place in the event. Accordingly, the event cannot be assigned an ontological identity, and can individuate itself (and thus have some semblance of identity, however unstable) only in the "curious mirroring" of intervention. This mirroring, at first glance, is indissociable from the reflexive structure of the event itself, even if Badiou goes on to dissociate the two by means of a bizarre and implausible myth of evental historicity. What I wish to emphasise here is that during this brief moment where Badiou's concept of the event is in alignment with the Sartrean concept of the for-itself, it is possible to discern a different solution to Badiou's problem. This solution requires that we take seriously the differentiating 'gap' between *any* multiplicity and its elements, and therefore the gap that the event establishes within its own existence. Operating outside the field of mathematical ontology, this gap cannot be closed by appeal to the ontological identity of e_x and e_x (qua event and qua element of the event), since this identity can no longer be secured. The intervening subject, who enters into being at the same moment, can alone establish the identity between the event and the event that the event presents. The curious mirroring between intervention and event, properly understood, is strictly analogous to the duality of *consciousness of x* and *consciousness of consciousness of x* that we uncovered in Sartre's analysis of non-thetic reflexivity.



One of the immediate boons that this analogy bequeaths us is that it allows us to finally make sense of Badiou's statement that

for the void to become localisable at the level of presentation, and thus for a certain type of intrasituational assumption of being qua being to occur, there must be a dysfunction of the count,

resulting from an excess-of-one. The event will be this ultra-one of chance, on the basis of which the void of a situation may be retroactively discerned (BE, m4, 56/69*).

In order to do this, we must return to the Sartrean analysis of non-thetic reflexivity, while bearing in mind that the structure under analysis here is precisely the structure exhibited by the event-intervention dyad (properly understood).

The unitary duality of consciousness marks one of the most difficult points in Sartre's doctrine, and a great deal of caution must be taken here. This duality must be understood in such a way that no recourse is made to intensional, substantial or qualitative difference. Under no circumstances is it legitimate to posit intrinsic differentia by which the dual aspects of consciousness (*e.g.* belief and consciousness (of) belief) would positively differ. To do so here would be to fall back on the notion of the immanent contents of consciousness, which Sartre absolutely, and to my mind rightfully rejects. Neither aspect of the reflection-reflecting pair is *in and of itself* different than the other; the difference emerges only in the unity of the two, the unity which can only form by making itself two. To secure the duality of self-consciousness by introducing positive differentia into the immanence of consciousness would be in contradiction with the axiom of intentionality, and manifestly incompatible with a strictly extensional conception of consciousness. As Sartre has it,

To introduce into the unity of a pre-reflective *cogito* a qualified element external to this *cogito* would be to shatter its unity, to destroy its translucency; there would then be in consciousness something of which it would not be conscious and which would not exist in itself [*en soi-même*] as consciousness. The separation which separates belief from itself can not be grasped or even conceived in isolation. If we seek to reveal it, it vanishes. (BN, II.I.I, 78/114)

If there is nothing positive by which the "instantaneous nucleus" (BN, I.II.III, 70/106) of consciousness differs from itself, and if this difference is nevertheless real, then only one option remains:

The fissure then is the pure negative. Distance, lapse of time, psychological difference can be apprehended in themselves and include such elements of positivity; they have a simple negative *function*. But the fissure within consciousness [*la fissure intraconscientielle*] is nothing except for the fact that it denies and that it can have being only as we do not see it.

This negative which is the nothingness of being and the nihilating power both together, is *nothingness*. Nowhere else can we grasp it in such purity. [...]he nothingness which arises in the heart of consciousness *is not*. It is *made-to-be* [*Il est été*]. (BN, II.I.I, 78/114)

This last qualification is crucial. If Sartre were to say that consciousness differs from itself by a “nothingness” that *is*, then he would be doing nothing less than violating the principle of intentionality with an absurdly chimerical entity, dishonestly pretending to sidestep the problem of positive differentiation by dressing up a mysterious ‘something’ in the sheepskin of nothingness. Fortunately, this is not the case. Nothingness is neither ‘in’ consciousness nor is it a being-in-itself. It is a mere *effect* of the negative dimension of intentionality, the dimension whereby a consciousness differs from its extension by virtue of the act of intentionality alone. When consciousness enters into its own extension, as it must necessarily do if it is to attain any degree of consistency, this negative dimension takes the form of a gap between consciousness and itself. “Nothingness” is no more than the negative dimension of consciousness—its “nihilating power”—bent back upon itself. By means of its position, the negative dimension takes on a certain form, which can be sensed only by its effects in the field of consciousness, such as the irreducibly dualistic (even if integral) structure of non-thetic reflexivity. It cannot be seized as an object; any attempt to do so would grasp only a mock-nothingness, or a symbol of nothingness, and relocate the original gap between the consciousness of this symbol and the consciousness (of) the consciousness of this symbol. Hence,

[o]ne does not *find*, one does not *disclose* nothingness in the manner in which one can find, disclose a being. Nothingness is always *elsewhere*. It is the obligation of the for-itself never to exist except in the form of an elsewhere in relation to itself, to exist as a being which perpetually effects in itself an inconsistency of being [*inconsistance d’être*]. (BN, II.I.I, 78/114*)

Insofar as it is inseparable from the act of consciousness, the production of a nothingness is not the appearance of an independent entity but “an absolute event which comes to being by means of being and which without having being, is perpetually sustained by being” (BN, II.I.I, 79/115). This is the sense in which one should understand Sartre’s assertion that “nothingness is not,

nothingness ‘is made-to-be,’¹¹⁶ nothingness does not nihilate itself; nothingness ‘is nihilated’” (BN, I.I.V, 22/57*).¹¹⁷

It is precisely in this sense that I say that the void is disclosed in the Badiouian event. The void thus produced is a localisation of the immanent negativity proper to multiplicity or presentation in general. It is “the imperceptible *gap*, cancelled then renewed, between presentation as structure and presentation as structured-presentation, between the one as result and the one as operation, between presented consistency and inconsistency as what-will-have-been-presented” (BE, m4, 54/67*). It becomes manifest in the event by virtue of its reflexivity, its act of placing, *on the same level*, presentation and what is presented (*ce qui se presente, et ce qui se presente*),¹¹⁸ and thus maintaining the dimension of presentational withdrawal within the sphere of the presented, in the form of the event’s differentiation from itself—or, to put it another way, in the form of the difference between event and intervention.

This hypothesis presents us with certain problems, however. For one, the nothingness that infiltrates Sartrean consciousness is conceived in a manner quite different from the nothingness of the Badiouian void. The chief difficulty here bears on how we are to understand a figure that is equated with two antithetical forms of Sartrean ontology.¹¹⁹ How can we identify the void with both nothingness and being-in-itself? To ask this question, however, supposes that we have already solved the equation between the void as being-in-itself, designated by letter Ø, and the void as the gap immanent to presentation. As I have mentioned earlier, this remains an open problem.



One promising avenue of approach to this problem does reveal itself, however. It requires us to reverse a judgment that, as we have seen, there is already reason to doubt: in the last analysis, the nomination of the void is *not adequate*. The relation between void and Ø is not one

of perfect coincidence, even if the act of nomination is, as Badiou insists in his Parmenidean way, inseparable from the determination of the void itself. This is to say, the inadequacy that haunts the relation between \emptyset and void is not a question of inexact reference—it is not that \emptyset fails to fully *describe* the void. The void in itself is perfectly nondescript, and ‘ \emptyset ’ does a splendid job of not-describing. It is, moreover, not a flaw in description that troubles us; even if we detect an equivocity in Badiou’s discourse on the void, this equivocity has to do first and foremost with the void’s place and function, not with what it is (in each case, it *is nothing*). The inadequacy characteristic of the void’s nomination, as Badiou formulates it, is inherent to the relation of nomination itself. In “Custos, quid noctis?” (the text where Badiou initially prefigures his mathematical ontology) Badiou fastens onto this constitutive inadequacy explicitly. Rebutting Lyotard’s critique of both dialectical and mathematical thought, Badiou, defending an idiosyncratic synthesis of the two, rejoins:

The critique of the speculative genre, exclusively centred on the theme of the result, misses the essence of the dialectical project, which is the non-arithmetical primacy of the Two over the One, the logic of scission as the form of the occurrence itself. One could establish this within the mathematical paradigm, given its necessity to name pure being and make it consist as the existential scission of the nothing and the name (for example: ‘the empty (nothing) set (name) exists’ [*« l’ensemble (nom) vide (rien) existe »*]).¹²⁰

It is remarkable that, in this text, Badiou describes the nomination of the void in the exact same terms that he uses to describe the eventual intervention, which he describes a few lines later as “the dysfunction of the capacity of the genre [*i.e.* the ‘language game’] to count the Two as One, to anticipate the cost [*le solde*] of the generic scission.”¹²¹ What this suggests is that the metaontological concept of the void, in which pure being is thought in tandem with its mathematical nomination, has a structure similar to that of the event. But how does this allow us to understand the ephemeral relation that Badiou seems to establish between the void qua \emptyset and what, for lack of a better name, we could call the void qua nothingness (where ‘nothingness’ is understood in its Sartrean sense)? Our guiding thread will be the observation that insofar as the

metaontological concept of the void is scissional, it already implicates the concept of nothingness as the insubstantial withdrawal immanent to the nominational act.

Now, we must nevertheless make it clear that this dimension of scission and internal differentiation is discernible only on the metaontological level; it is not transitive to ontology itself. Within ontology, the identity of \emptyset to itself is absolute, and nothingness (in its Sartrean sense) is inoperative. The place that the metaontological discourse constructed in *Being and Event* nevertheless leaves for this scission (which, it must be said, receives scant explicit attention) is legible in Badiou's use of the term 'suture.' There are at least three slightly distinct contexts in which this term appears in *Being and Event*. The first designates the void itself: in the context of a given concrete situation under the effect of the count-as-one, nothing of inconsistency remains; insofar as this 'nothing' is nonetheless thinkable as a remainder, it lets the situation be thought as rooted in an inconsistency anterior to structure: "I term *void* of a situation this suture to its being," writes Badiou (BE, m4, 55/68). The second sense or context appears, rather briefly, in Badiou's meditation on the evental intervention. In language reminiscent of "Custos, quid noctis?" he writes: "The event, pinned to multiple-being by the interventional capacity, remains sutured to the unrepresentable. This is because the essence of the ultra-one is the Two" (BE, m20, 206/228). The unrepresentable invoked here is, again, the void. The third manner in which 'suture' is employed concerns the relation between the void and its name, \emptyset . Discussing the singleton of \emptyset , the set whose only member is \emptyset , Badiou writes:

It is not the 'void' which belongs to the set $\{\emptyset\}$, because the void belongs to no presented multiple, being *the very being of multiple-presentation*. What belongs to this set is the *proper name which constitutes the suture-to-being* of the axiomatic presentation of the pure multiple; that is, the presentation of presentation. (BE, m7, 89/104*; e.a.)

This is one of the rare moments in *Being and Event* where Badiou avoids the shortcut of conflating "the void as the nothing that is, as the unrepresentable" and "the *name* of the void, the existent mark of the unrepresentable" (BE, m7, 88/103). Here, where the two are pried apart just

enough for us to glimpse the relation that unites them, we again find the concept of suture.

‘Suture’ names the peculiar relation of disjoint unity that ties together “the nothing and the name”, inconsistency and its unpresentation, and the void and the event. What is *suture*?

The concept of suture has a long and obscure history in Badiou’s work. The notion originates in Lacanian psychoanalysis, and was explicitly formulated for the first time by Jacques-Alain Miller in an essay entitled “La suture (elements de la logique du signifiant)”. In its original context,

Suture names the relation of the subject to the chain of its discourse; we shall see that it figures there as the element that is lacking, in the form of a placeholder [*enanti-lieu*]. For, while lacking, it is not purely and simply absent. Suture, by extension, [is] the general relation of lack to the structure of which it is an element, insofar as it implies the position of a placeholder!²²

In “Suture,” Miller elaborates the concept by way of a case study of Frege’s *Foundations of Arithmetic* through the lens of Lacanian theory. There is no need to repeat the details of his analysis here, but it is worth reiterating Miller’s conclusion: Miller argues that the Fregean definition of zero as the number belonging to the extension of the concept ‘not identical with itself’ effects a suturation of the subject, understood as a fundamentally non-self-identical void (in conformity with Lacanian theory, which is on this point fortuitously analogous to Sartrean ontology, at least in its grosser outlines). The non-self-identical is necessarily excluded from the symbolic order of mathematics, Miller claims, because admitting it would mean the ruin of mathematics’ symbolic intelligibility insofar as it would spoil the tacit principle that every mathematical symbol is substitutable for itself *salva veritate* (substitutability *salva veritate* is the definition that Frege uses for identity). Miller goes on to argue that the subject, which is invoked and then excluded in the definition of zero, nevertheless ‘insists’ between the numbers as that which effects the operation of arithmetical succession.

This essay has long been of particular significance to Badiou, who, in the 1967 essay, “Marque et manque,” mounted a thorough and scathing critique of Miller’s argument. As Badiou

observes, at no point is the non-self-identity of mathematical symbols at stake in Frege's argument, but only a formal expression of the type $(x \neq x)$ inscribed in perfectly legible and self-identical signs. The 'nothing' that Fregean arithmetic invokes is only the non-validity of the proposition $(\exists x)(x \neq x)$, a 'nothing' that is produced by mathematics and which remains entirely immanent to mathematics. Mathematics, Badiou argues, "never lacks anything that it does not produce elsewhere."¹²³ The psychoanalytic notion of suture, and the auxiliary concepts of lacuna, exteriority, and subject are all foreign to the mathematical signifier.

The force of Badiou's argument and invective in "Marque et manque" make it all the more surprising that the concept of suture becomes so prevalent some twenty years later in *Being and Event*, and does so with neither explanation nor rebuttal of his previous position. Of course, much has changed between the more or less Althusserian position that Badiou takes in the late nineteen sixties and the position articulated in *Being and Event* and afterwards. The formalist-materialist vision of mathematics propounded in those earlier text has mutated to incorporate a Platonistic identification of mathematics and ontology (though without ceding ground on either formalism or materialism), and the thesis that "there is no Subject of science"¹²⁴ is overtaken by both a new theory of the subject, which breaks significantly with Lacan, and a new conception of science as essentially subject-driven.¹²⁵ Though the precise relation between the time-worn concept of suture and his new theory of the subject is never explicitly elaborated in his later work, when asked directly as to whether or not ontologies suture to being in the nomination of the void is related to its subjective, and therefore post-evental status, Badiou does say that "in *Being and Event*, the concept of suture does in fact concern the subjective status of the procedure [of mathematical ontology], and finally the philosophical relation, as a singular relation, to this procedure."¹²⁶ At the very least, we may take this a signal that we have not strayed too far from our course. If the nomination of the void is intrinsically evental, then the complete and

philosophical concept of the void already implicates something akin to Sartrean nothingness. If the void's nihilational dimension does not appear in the strictly ontological determination of the void as \emptyset , this is, against the grain of "Marque et manque", reason for understanding the nomination of \emptyset as a suture. Now, if Badiou is using this term in the general sense that Miller gives to it, then we should expect that the nothingness excluded from mathematical ontology will come to be determined as lack. (Recall that suture is "the general relation of lack to the structure of which it is an element, insofar as it implies the position of a placeholder.") We should also expect that this determination as lack has something to do with the formation of the subject in the wake of the event.¹²⁷



That both Badiou and Sartre do, in fact, ground the subject in the presentational structure of the event (or the absolute event of the for-itself, for Sartre) presents us with a remarkable symmetry between the two philosophies. Whatever difficulties accompany our efforts to draw the correct relation between the event, the nothingness it discloses, and the void which, for Badiou, it invokes, this symmetry remains legible. Our best hope of understanding it is to move forward and ask: Why do both Sartre and Badiou find it necessary to position, at the origin of the subject, a manifestation of the void brought forth by a dysfunction of the count, or degeneration of being?¹²⁸ In order to answer this question, we need to ask what it is that the two philosophers are trying to do with their theories of the subject. What is the subject supposed by each to be, such that its inauguration must take place in an anomalous production of the void?

What is at stake for both thinkers is a conception of agency, and therefore possibility, without intensional determination, be it understood in terms of essence, virtuality or potentiality. The void is invoked in order to open a dimension of possibility that is defined strictly in terms of lack. In order to see how this might be understood, we will return to Sartre's analysis of the

immediate structures of the for-itself. We will then turn to Badiou's text, in order to isolate what role the event plays in the theory of subjective truth-procedures. What will be shown is that the event is to be understood as that which permits a synthesis of the existential structures disclosed in Sartre's analysis of the for-itself with the set-theoretic infrastructure with which Badiou subtends his theory of the subject, and which is drawn primarily from Paul J. Cohen's work on the independence of the continuum hypothesis. We will also find that the structure of lack, which was evoked by the concept of suture, plays a significant role in the formation of the post-evental subject.

VII. The Structure of Lack And the Theory of Subjective Transcendence (Sartre on Value and Possibility)

Between the motion
And the act
Falls the Shadow

— T.S. Eliot, *The Hollow Men*.

The 'extensionalist' analysis performed on the Sartrean theory of consciousness in Chapter I presents an image of Sartrean ontology that is quite foreign to what many find most memorable in Sartre's philosophy. It may seem to have left us with a picture of consciousness as a barren set of 'states', seized thetically but not 'existed' or 'lived' in the non-thetic temporality characteristic of consciousness. To picture consciousness, on the basis of the analysis performed in Chapter I, as a mere 'set' of thetically beheld objects, however, is to ignore precisely those non-setlike aspects of consciousness that the 'extensionalist' analysis reveals, and the immanent negativity to which the analysis inexorably leads. To go on to imagine that what we arrive at is an 'atemporal' (or even 'spatial') figure of consciousness is simply to stop too soon. We have yet reached neither staticity nor temporality nor spatiality in our analysis of the Sartrean concept of consciousness. I cannot offer a fullblooded analysis of the Sartrean theory of time within this confines of this essay, despite the immense importance of this theory for a thorough understanding of Sartrean ontology. It must be emphasized, however, that the crucial *importance* that Sartre grants to his the theory of time does not translate into its conceptual priority with respect to the rest of his system. Time, though inseparable from the existence of the for-itself, is a *consequence* and not the ground of the structures we have elucidated so far. Time, for Sartre, is *derived* from intentionality, via the nothingness laid bare in the 'phantom duality' of non-

positional self-consciousness. “The nothingness which separates human reality from itself,” he declares, “is *at the origin of time*” (BN, II.I.IV, 102/138; emphasis mine).

It is by way of this nothingness that I will attempt to elucidate the conceptual apparatus that stands as a sort of intermediary between the theory of intentionality and the theory of time deployed in *Being and Event*. While the description of this apparatus—which we may call the theory of lack—does not at all exhaust the Sartrean theory of temporality, it does adumbrate it in part, as it takes us from the event of nothingness to the temporal structure of subjectivity and the possibility of the act. Sartre begins his articulation of this theory by inquiring as to how consciousness may apprehend the fundamental non-self-identity that confronts it in non-thetic (self)-consciousness. How does consciousness apprehend its manner of *not being itself*? “There are many ways of not being,” Sartre notes,

and some of them do not touch the inner nature of the being which is not what it is not. If, for example, I say of an inkwell that it is not a bird, the inkwell and the bird remain untouched by the negation. This is an external relation which can be established only by a human reality acting as a witness. (BN, II.I.III, 86/122)

In cases such as these, both terms ‘are what they are’ independently of the relation. This is, of course, a consequence of his axiomatic of the in-itself, which assumes at the outset that negation, by its very definition, is contingent on the presence of a consciousness and cannot subsist in the absolute neutrality of unconscious things (there is certainly room for debate regarding the bird in the above example, but this need not detain us here). The point that Sartre is driving at here is that the kind of negation by which consciousness is not itself cannot be understood as an external relation of this sort. Consciousness has neither identity nor consistency prior to its autonegation. This negation makes consciousness what it is, insofar as it is immanent to the reflexive share of consciousness’ extensional determination—immanent, that is, to (self)-consciousness. In contrast to the external negation that is posited between the bird and the inkwell, Sartre terms a negation of this sort “an internal relation” (ibid.).

Insofar as this negativity is immanent to the very event of consciousness, “the pure event by which human reality rises as a presence in the world is apprehended by itself as *its own lack*” (BN, II.I.III, 89/125). Lack, as Sartre understands it, is,

[o]f all internal negations, the one which penetrates most deeply into being, the one which constitutes *in its being* the being concerning which it makes the denial along with the being which it denies... (BN, II.I.III, 86/122)

Consciousness, insofar as it differs from itself, and is not itself, is determined as *lacking something*. More precisely: “Every consciousness lacks *something for something*” (BN, II.I.IV, 101/137), for lack has an essentially triadic structure. Sartre elucidates this structure at length, and supplies us with some useful terminology. “A lack,” Sartre writes,

presupposes a trinity: that which is missing or ‘the lacking,’ [*e manquant*] that which misses what is lacking or ‘the existing,’ [*’existant*] and a totality which has been broken by the lacking and which would be restored by the synthesis of ‘the lacking’ and ‘the existing’—this is ‘the lacked’ [*le manqué*]. (BN, II.I.III, 86/122)

The classic example that Sartre employs to illustrate this trinity is the crescent moon,¹²⁹ where the *existing* is the visible crescent, the *lacked* is the full moon, which we cannot yet see, but which would be given to us in perception if what was *lacking* were to be revealed—the gibbous, the complement of the crescent. Sartre stresses the complementarity that holds between the *lacking* and the *existing* in the trinity: insofar as they can be unified in the *lacked*, they are of the same nature, and it is always possible to reverse our perspective on the trinity in order to apprehend, for example, the dark gibbous as the *existing*, the illuminated crescent as the *lacking* and the new moon as the *lacked*. Now, as Sartre recognizes, an example such as this can only serve as a guiding thread in the complete construction of the concept of lack. So long as the three terms of the trinity are taken as simple beings-in-themselves, or regions of the in-itself, the triadic structure of lack in which they participate remains ultimately unintelligible. Each term, in itself, coincides perfectly with itself and asks nothing of the others. Being-in-itself must be

supplemented if the structure of lack is to emerge. We must transcend the positivity of the terms towards the very Idea of Lack that sustains them in the triad, for

what is released to intuition is an in-itself which by itself is neither complete nor incomplete but which simply is what it *is*, without relation with other beings. In order for this in-itself to be grasped as the crescent moon, it is necessary that a human reality surpass the given toward the project of the realized totality—here the disk of the full moon—and return toward the given to constitute it as the crescent moon; that is, in order to realize it in its being in terms of the totality which becomes its foundation. (BN, II.I.III, 86/122)

The reason why this is possible is that human reality determines itself *absolutely* as Lack. Like a Platonic Idea, it is both lack's complete expression and the condition of possibility of its phenomenal manifestations. The ontological deficiency of the latter can be expressed by saying that whereas every 'merely phenomenal' triad of lack necessarily appeals to a fourth term—'a witness'—the lack that is being-for-itself is genuinely triadic; no fourth term is necessary to sustain its configuration as lack.

If consciousness, in its non-thetic reflexivity and self-differentiation, determines itself as the *existing*—as that which is missing something—then we must ask, first, what it is missing (what is the lacking?), and, second, what consciousness is missing the lacking *for* (what is the lacked?). In setting up the problematic of lack, we have already provided ourselves with the answers to these questions, at least in their schematic form. For, if consciousness determines itself as lack insofar as it is subject to an internal negation generated by its own difference from itself, such as is apprehended in non-thetic (self)-consciousness, then it is clear that what consciousness is *lacking* is identity with itself, "or itself as in-itself" (BN, II.I.III, 89/125). Once we have the *existing* and the *lacking* terms of the trinity, the *lacked* is given as well: that for which non-identical consciousness lacks itself-as-in-itself is "[t]he impossible synthesis of the for-itself and the in-itself" that "would preserve within it the necessary translucency of consciousness along with the coincidence with itself of being-in-itself" (BN, II.I.III, 90/126). Let

us proceed to a more complete exposition of these structures, and of the forms by which they manifest themselves in a concrete situation.

I will begin with the *lacked*.



Insofar as consciousness exists, it exists under the shadow and impetus of an impossible entity, which would be consciousness-in-itself, consciousness intrinsically defined and self-identical. Such an entity cannot *be*, since consciousness exists, in the event of intentionality that is the entirety of its being, only so long as it is purely empty (hence Sartre's rejection of the transcendental ego) and insofar as it unifies itself only while differing from itself. Likewise, the in-itself, for Sartre, tolerates no negativity or withdrawal within itself. Substance cannot be consciousness, nor can consciousness be substance. The figure of their synthesis outlines an indissoluble impossibility.

And yet the for-itself-in-itself, although ontologically inconsistent, is not altogether *unreal*. The existence of the lacked is a strict impossibility, but this very impossibility has a reality that bears directly on the ontological structure of the existent. It is important, here, to clearly determine what is meant by speaking of the lacked as a *real impossibility*. What I wish to emphasise here is not merely that the coming into existence of the lacked is truly impossible, in the sense that its existence would imply an insurmountable antinomy; this is true of a great many things which are not 'real impossibilities' in the sense aimed at here: square circles and the like, for instance. The mode of impossibility specific to the lacked is not exhausted by merely logical impossibility. It is rather a question of an unoccupiable *ontological place*, whose uninhabitability is a necessary condition for the existence of that which delineates it and gives it a semblance of being.

Concretely speaking, the figure of the lacked is the ‘shape’ of the gap that non-thetic reflexivity establishes between consciousness and itself, insofar as this gap is codetermined with the possibility (the lacking) towards which that consciousness projects itself. This is what Sartre indicates by writing that the lacked “would be exactly the *self* which we have shown can exist only as a perpetually evanescent relation,¹³⁰ but it would be this self as substantial being” (BN, II.I.III, 90/126). If the emptiness of this place were not maintained, nothing would prevent the reflexivity of the for-itself from collapsing into the identity of the in-itself and precluding intentionality, thereby extinguishing itself (or else maintaining its intentionality and inciting an ontological paradox). By the same token, it is by virtue of consciousness’ ontological non-coincidence with itself that the unoccupiable figure of the lacked is outlined. The rigid *inexistence* of the for-itself-in-itself is therefore coextensive with the existence of the for-itself, for

without this being, which it is in the form of not being it, consciousness would not be consciousness—*i.e.*, lack. [...] It has no priority over consciousness, and consciousness has no priority over it. They *form a dyad*.¹³¹ Of course this being could not exist without the for-itself, but neither could the for-itself exist without it. (BN, II.I.III, 90-1/127)

In sum, by the very fact that consciousness exists, it is “haunted” by an impossible entity that is the correlate of the nothingness that separates it from itself. Sartre characterises this relation of “haunting” precisely, and opposes it to phenomenal presence and objectivity. Like the nothingness that marks its place, and which it inverts, the for-itself-in-itself is not an *object* for consciousness. This is to say that it is not given in thetic intentionality, but structurally affects intentionality itself, without appearing before it, or inhabiting it as an immanent content (its mode of *inexistence* is neither extensional nor intensional with respect to consciousness). In ways which we will later sketch out in greater detail, this haunting reality indicates itself as a cavity around which the situation comes to be structured.

In contrast to the absolute contingency of the in-itself, and the relative contingency of the consciousness that depends on the in-itself for its existence, the lacked is that which would found its own necessity, thereby producing itself as an *ens causa sui*. One formulation of the *ens causa sui* that Sartre often employs is that it is the for-itself *existing without facticity*. The lacked would be both the act of its own unification and the identity of the unified, an entity for which “form must be to itself—and totally—its own matter. And conversely the matter must be produced as absolute form” (BN, II.III.III, 194/231). Sartre insists that it is only in that ideal figure of the for-itself-in-itself that an “unconditioned unity” (BN, II.I.III, 93/129) of being can be thought. In this light, it is not difficult to recognize the Parmenidean figure of the One in the *lacked*, insofar as we have seen the former to be the fusion of Necessity and Being, Limit and Limited, or Form and Matter (see Chapters II and IV, above). This is essentially the figure of the One that we have already encountered in Chapters II and IV,¹³² but it is shot through with an additional impossibility, insofar as the *ens causa sui* would preserve the dimension of immanent withdrawal by which the for-itself is a consciousness and a multiplicity, while cancelling out this same dimension in an absolute identity with itself. The agreement between the *ens causa sui* and the Parmenidean One is more than an accidental formal similarity. Sartre methodically connects the form of the lacked to that of the Parmenidean One, and confronts the problems of the unity and the univocity of being in the light of the manifest impossibility of this figure. In the concluding synopsis of the “metaphysical implications” of his ontology, for instance, Sartre argues that neither the concept of being-for-itself, nor of being-in-itself, nor any consistent coupling of the two concepts, provide us with the necessary ground to postulate a metaphysical monism. This is not to say that we are left with an absolutely “insurmountable dualism” (BN, c.I, 617/665), analogous to the Cartesian disjunction between the *res extensa* and the *res cogitans*. So far as substances are concerned, Sartre recognises only one, which is simply being-in-itself. The for-

itself, by contrast, is a insubstantial 'event', and it is not possible to conceive it in isolation from the in-itself. Because being-for-itself is defined as intentionality, it is only insofar as it exists as a nihilating relation to the in-itself. It is thus that "the For-itself and the In-itself are reunited by a synthetic connection which is nothing other than the For-itself itself" (BN, c.I, 617/665). The essential question, here, is not whether or not it is possible for us to determine a synthesis or form of communication between the two modes, but whether the synthesis which we already know must exist allows us to conceive of being as One.

Sartre initially formulates the question of the One in terms of the *univocity* of being: that is, as the question as to whether or not the word 'being' has, or can be given, a single *sense*. The univocity of sense is debarred from the outset by the original disjunction of being into its two modes, and so Sartre pins the hopes of univocity on the question of concrete monism. Here, it is a question as to whether or not we are able to conceive of the logical extension of 'being' as something that is concretely unified. The disjunction in the concept of being may, in retrospect, be eliminated if it is materially rejoined in the concept's extension. The univocity of 'being' would thereby be secured, not in the *sense* of the word, but through its dimension of *reference*.

Sartre interprets the hypothetical referential unity of the word 'being' in terms of *totality*. The notion of 'being' will have a monistic reference if its extension forms a totality. As Sartre has it, something may be said to be a totality when "the diversity of its structures [can] be held within a unitary synthesis in such a way that each of them considered apart is only an abstraction" (BN, c.I, 622/670). Sartre recognizes that we already have laid hold of half of the desired solution, for "consciousness considered apart is only an abstraction," but, on the other hand, "the in-itself has no need of the for-itself in order to be. [...] The *phenomenon* of in-itself is an abstraction without consciousness but its *being* is not an abstraction" (BN, c.I, 622/670). To conceive of being-in-totality would be to conceive of it in such a way that each aspect of it only

is insofar as it is part of the Whole. Hence, “we must conceive of this synthesis in such a way that the in-itself would receive its existence from the nihilation which caused there to be consciousness of it. What does this mean,” Sartre asks, “if not that the indissoluble totality of in-itself and for-itself is conceivable only in the form of being which is its own ‘self-cause’?” Being-in-totality, if it were to exist, could only be realised as the for-itself-in-itself. “Of course this *ens causa sui* is impossible, and the concept of it, as we have seen, includes a contradiction” (ibid.). Being-in-totality thus remains an impossible entity, and the sense of ‘being’ remains “cleft by a hiatus” (BN, c.I, 623/671) which cannot be effectively resolved. Univocity is cleft twice: once by the structural disjunction of modes, and once by the non-being of the One that would synthesise them in their concrete extensions.



Insofar as it produces a nothingness at its core, the event of consciousness immediately takes place in light of a figure of pure *being*, a figure which is, paradoxically, ontologically impossible, and outlines a region of unoccupiable non-being. In order to understand its structural role in the theory of the subject, we must turn to that which is in each case apprehended as lacking in the light of the lacked. It can immediately be seen that this question opens up onto a difficult problem: how can we be asked to find the missing term that must be added to consciousness in order to reach an impossible being? Is not this equation by definition insoluble? At face value, this is certainly the case. While we know from our prior definitions that the lacking must be an aspect of the in-itself, there is no way to ontologically determine what this aspect might be, whether in general or in a particular case. There is nothing in the logical structure of being that will permit us to chart the move from the existing to the lacking, if the mandate and measure of this move is to be an impossible entity. The relation between the existing and the lacked can be supplied with “no common measure” (BN, c.II, 626/674), and so

nothing in it allows us to unequivocally say whether “man makes himself man in order to be God,” or that “man loses himself in order that the self-cause may exist” (BN, c.II, 626/674). We have come to a theoretical fissure that absolutely cannot be rejoined by any strictly ontological considerations. Sartre, however, seizes upon this impasse as an opening onto the true field of subjective action, and the incalculability of the step from the existing to the lacking is a cornerstone of both his ethics and his theory of the subject.

This impasse conditions his ethics insofar as the concept that deems adequate to the figure of the lacked is that of *value*. By insisting that “the being of value qua value is the being of what does not have being” (BN, II.I.III, 93/129),¹³³ Sartre removes the figure of value from the field of any factual inquiry, setting it at an irrecoverable distance from ethical psychologism and sociologism. “To take it as being,” Sartre writes, “is to risk totally misunderstanding its unreality and to make of it, as sociologists do, a requirement of fact among other facts. In this case the contingency of being destroys value” (BN, II.I.III, 93/129). Value, as lacked, is positioned in such a way that nothing in what is given to a particular subject permits us to calculate the values that it is capable of supporting. To absorb value in the flat neutrality of facticity, on the other hand, would not only render it in principle calculable, but strip it of every imperative force. The imperative dimension proper to value can only be understood insofar as it is given to it freely by its subject, under which circumstances it becomes tautological to say that this subject is compelled or haunted by this value. And to say ‘freely’ here presupposes the incalculable gulf that separates the subject and its lacked.

There is a second face to moral realism which Sartre attacks, however, and one of the interesting vicissitudes of Sartre’s text has to do with how he brings his critique of the metaphysics of the One in line with his critique of the variety of moral realism which, rather than trying to demote value to the status of a psycho-social fact, attempts to elevate being to the

dignity of value. In his *Notebook for an Ethics*, Sartre mounts a polemic with a strange amalgam of Nietzschean and Parmenidean moral ontology.¹³⁴ At the heart of this polemic is an indictment of the general will to identify being and value, a will which manifests itself in both philosophers as a moral affirmation of what is, insofar as it is, and a radical fidelity to metaphysical holism. This will, Sartre argues, is ethically disastrous. The terse summary that Sartre grants to this doctrine in his notebooks runs as follows:

Being is perfection. Value and Being are one and the same thing. The more Being there is, the more perfection. The highest Being is the highest value. This Being is not necessarily God, it is the totality of the given. However, imperfection, being a lesser form of being, every determination is negation, therefore less being. Absolute being, being wholly positivity, abolishes all distinctions in itself. It is pure being. Immobile, inexorable, atemporal, unqualified. The Parmenidean sphere. Every destruction is positivity as a nihilation of the particular, of the determined that leads back to the undifferentiated state of Being. Everything that is *good* as *being*, bad insofar as limited, therefore not being. Hence the totality of Being is worth more than every particular, the particular is worth more than what is not yet. That is, what is is always worth more than what is *to come*, than *the ideal*.¹³⁵

From this moral ontology, Sartre derives the following, vaguely (but by no means purely)

Nietzschean, list of principles, which constitute an “ethics of force”:

1st, the victor is always right; 2nd, the principle of harshness: it is better to be unpitiful than to give way to acts of goodness which are signs of weakness; 3^d, love for struggle: the shortest route from one heart to another is the sword; 4th, the value of evil that cleanses and purifies like a fire; 5th, one has no right to resist force unless one is strong enough to hold it back; 6th, aristocracy; 7th, the vital values: nobility, ferocity, the refusal to subordinate the body to the spirit; 8th, the ethics of the weak. The refusal of slave morality; 9th, risking one's life. Acceptance of death (the master and the slave). The idea of hierarchy; 10th, the principle of ethics: the identification of force, value, and being; 11th, if every means is good it is because none of them is *essential* to the end. Incommensurability of end and means. For an absolute end, inessential means. Violence itself, violent acts, and the violent man are all inessential to the end. Whence, at the same time, abnegation and contempt for men. Abnegation because as a diversity man is bad. He is good only through participation in the whole (being/value) that justifies violence. The violence has *always already begun*. Therefore human nature is bad. The anti-individualism of the violent man (hiding a shameful individualism); 12th, the value of purity (that of the cleansing fire); 13th, the beauty of *pessimism*. Violence and aesthetics; 14th, realism, in the name of efficacy. Idealism is the end posited without force. However, realism is itself a form of idealism: it is the idea of the *value* of Being. (NE, 186)

There is certainly something at least a little perverse in fusing Parmenides and Nietzsche into a single straw-man, but this need not trouble us inasmuch as Sartre's purpose here is not to mount a critique against the two philosophers themselves, but to illustrate a general conceptual scheme,

whose subtle variations are essentially unimportant for his project. A consideration of the sheer symmetry between ‘Nietzsche’ and ‘Parmenides’ on the points of interest is enough for Sartre to position himself: one determines the formal properties of being in such a way as to produce an abstract image of value, the other remoulds value in the image of being, or rather, in the form of what there is, the world of appearance; here, the vicious structures of the phenomenal world take on an imperative force—hence that line of reasoning so common in Nietzsche’s texts, which roughly runs: the world *is*, in such and such a way, and so, if we are not to nihilistically negate life, we must affirm, even mimic, the way of the world.

Sartre argues that we must reject both the socio-psychologistic reduction of value to fact and the Nietzscheo-parmenidean elevation of fact to value. As Sartre has it, not only are both ways of confusing being and value metaphysically untenable, but both are ethically dangerous, the first by giving way to a cynical utilitarianism or even nihilism, the second by licensing a terroristic “ethics of force,” if not a complacent quietism.¹³⁶

In passing, it is worth noting that Badiou takes a position almost identical to Sartre’s on this point. Near the end of his *Deleuze*, Badiou takes issue with Deleuze’s “refined Bergsonism, for which, in the final instance, it is always what is that is right.”¹³⁷ Badiou willingly acknowledges that, in Deleuze’s case, this ethico-ontological doctrine of the One succeeds in generating an “admirable creative Stoicism” (ibid.), from which it is possible to say: “What does it matter? All is grace.”¹³⁸ Badiou’s contribution to the debate here, is that even for Deleuze, this makes grace contingent on the consistency, the existence, of the All. But for both Badiou and Sartre, “who rule out that Being can be thought as All, to say that all is grace means precisely that we are never accorded any grace.”¹³⁹ For both philosophers, however, “this is not correct,” grace, or value qua value, “does *occur*...” (ibid.).

Yet, without entertaining an antinomy it is difficult to see how this occurrence can be countenanced. What alternative exists to the trilemma of collapsing value into being, christening being as value, or denying the existence of value altogether? One is reminded of Artaud's (rather Sartrean) argument in *To Have Done with the Judgement of God*: "Is God a being? / If he is one, he is shit. / If he is not one / he does not exist."¹⁴⁰ In order to save value from the indifferent contingency of the in-itself, we deny it *being*; but it becomes difficult to see how the concept of value is any better off for this denial. It is therefore essential that we do not *simply* dismiss it into non-being, and, without reneging on its inexistence, continue with Artaud in saying that it "does not exist, / *except* as the void that approaches with all its forms" (ibid., 562; e.a.). The inexistence of value, as lacked, will be that which compels the void produced through the event of consciousness (which is, after all, value's actual and purely negative image) to approach with all its forms; or, it will be that which compels the void to take form. In order to flesh out the details of this operation, however, we need to take up Sartre's analysis of what is determined as *lacking*. Whereas the figure of the *lacked* gives us the structure of value, the *lacking* carries with it the ontological structure of possibility. The formation of the subject effectively begins here. We will begin with a careful examination of the structure of the lacking, before turning to the modal anthropology that it conditions.



The lacking is that which, were it finally united with the existing, would give us the lacked. In the present context, it is that which would produce the *ens causa sui* if only it could be seamlessly united with consciousness. Now, we know that for Sartre, this unification is impossible; nevertheless,

the *ens causa sui* remains as the *lacked*, the indication of an impossible *vertical* surpassing which by its very non-existence conditions the flat movement of consciousness; in the same way the vertical attraction which the moon exercises on the ocean has for its result the horizontal displacement which is the tide. (BN, c.I, 620/668)¹⁴¹

There is no contesting the beauty of this metaphor, but unlike the relation between moon and tide, the relation between the impossible lacked and modal tide below it is rigorously incalculable. As Badiou will do after him, Sartre positions the figure of the subject at this threshold of incalculability. The subject, at this point, is no longer understood in the sense of an immanent, substantial or transcendental *ego cogito*; it is not the subject that “absolute consciousness, when it is purified of the *I*, no longer has anything of” (TE, 106), as Sartre argued in *The Transcendence of the Ego*.¹⁴² In *Being and Nothingness* and the works which follow, the subject has the structure of a project, a configuration of the possible. To understand it, we must first clarify the structure of the possible itself as it figures in Sartre’s ontology as a consequence of the triadic structure of lack: possibility is the nature of *the lacking*.



What is *lacking* to consciousness, in its apprehension of its own non-identity, is itself. More precisely, what is lacking is a sheer being-in-itself that consciousness would be if it were coincident (*i.e.*, identical) with what it is. Insofar as the for-itself constitutes itself as *non-identical* to itself, the lacking remains at a *distance* from the existing. It seems at first that this distance must be distinct from the distance between consciousness and itself that is disclosed in non-thetic reflexivity. The two must nevertheless be intimately related, since the for-itself apprehends itself as a lack (as the *existing* term of a lack) as a consequence of the first distancing (the duality of reflexivity), and for this reason projects itself in a second distancing towards the lacking. While the first distancing is the way in which the for-itself is *one* with itself at a distance from itself, the second distancing, irreducible to the first, is “a way of being what one is—at a distance from oneself.” (BN, II.I.IV, 96/133*). What prevents us from immediately conflating these two dimensions of existential distance is that while the first is founded on a *pure* difference, a difference without a differentia, the second is founded on a *determined* difference,

or better, a difference in the process or act of determination, in which the specification of a differentia becomes obligatory. This is a point of extreme difficulty in Sartre's ontology. What we find here is a passage from one dyadic structure, which is the immediate and 'nuclear' dyad of reflected and reflecting, which are differentiated emptily and purely, to a second dyadic form, which is differentiated through a projective act which covers positive ground, the ground which separates what is now the case from what is not yet so. The formation of the second dyad is compelled by the instability of the first, but to conflate the two would be to sever the reflexive duality of consciousness into positively differentiable terms, and undermine the pure production of nothingness that is the effect of the first distancing and the condition of possibility of the second distancing, insofar as reflexively determined non-identity forces the for-itself to determine itself as lack.

And yet it seems that this conflation is actually required of us by Sartre's text. Sartre himself often writes as if the two dyads formed a single, continuous structure, for instance:

The determining relation of the for-itself to its possibilities is a nihilating relaxation of the bond of presence-to-itself; this relaxation extends to transcendence since the presence-to-itself which the For-itself lacks is a presence-to-itself which *is not*. Thus the For-itself in so far as it is not *itself* is a presence-to-itself which lacks a certain presence-to-itself, and it is as a lack of this presence that it is presence-to-itself. (BN, II.I.IV, 100-1/137)

Elsewhere this continuity appears even stronger, with Sartre writing that "[t]he nihilation by which we achieve a withdrawal in relation to the situation is the same as the ekstasis by which we project ourselves toward a modification of this situation" (BN, IV.I.I, 437/482). Or again: in his mediation on the structure of the future as the lacking, Sartre writes that the this dimension of the for-itself

is because the For-itself makes itself be by perpetually apprehending itself for itself as unachieved in relation to it. It is this which at a distance haunts the dyad reflection-reflecting and which causes the reflection to be apprehended by the reflecting (and conversely) as a Not-yet. (BN, II.II.C, 126/161)

These statements cannot be written off as a mere lapses in rigour, for the notion that consciousness gives shape to the nothingness and the value that haunt it is at the heart of Sartre's theory of the subject. The pure and unstable duality of consciousness and itself gives itself positive qualification by projecting itself towards a second extension, which is the *lacking*. This projection distends and qualifies the void, hollowing out the figure of the lacked while—to all appearances—undermining the ontological precondition of there being a pure triad of lack in the first place. Yet, to speak of the 'shape' of the void, such that it can be relaxed or distended, undermines the very structures through which this void is given shape: If consciousness is to be seized as purely different from itself, and for that reason harbouring a pure nothingness through which it determines itself as lack, this difference must be established in the absence of any differentia, otherwise the fissure that is opened between consciousness and itself could not be a pure nothingness, and consciousness would not be forced to apprehend this nothingness as a deficit of sheer *being*. In the absence of a baseless self-differentiation (an absence obtainable by qualifying the latter), there would be no reason for the structure of lack to emerge.

These difficulties can be resolved only if we examine very closely what Sartre is saying here. These apparent contradictions all rest on a subtle misinterpretation of the relation between the existing and the lacking, which has been treated here as a relation between identities. Insofar as the gap between them takes shape, we have reasoned, then the pure nothingness that we previously inferred has no *raison d'être*, for it is resolved into a purely positive difference between clearly demarcated entities. This interpretation fails because it does not pay attention to all the consequences that follow from saying that the two gaps—between reflecting and reflected, and between existing and lacking—are the *same*. This means that the play of mirrors, in which the reflecting loses itself in the reflected and vice versa, affects the relation between existing and lacking in exactly the same way that it affects non-thetic reflexivity. The haunting

affection of lack “causes the reflection to be apprehended by the reflecting (*and conversely*) as a Not-yet” (BN, II.II.I.C, 126/161; *e.a.*). Everything hinges on this “and conversely”: the reason why the distension of the pure fissure of immediate reflexivity does not fix the ephemeral duality of consciousness into two discrete and positively differentiable terms is that these two terms, whatever positive gap we wish to install between them, *cannot tell themselves apart from one another*. Their mutual indiscernibility does not abrogate the difference between them, which is emptily established on *a priori* grounds, and which can be distended to span the breadth of the concrete situation into which the consciousness is thrown. The reason why the distension and qualification of this difference does not undo its purity is that even when it is possible to supply it with positive differentia, it is impossible to assign these differentia to one term or the other. They pertain equally to both terms—a fact that would result in their equivalence were it not for the immediate and ‘nuclear’ differentiation that prevents this. Just as the mirroring of reflected and reflecting never allows the ‘two’ consciousnesses to fuse into self-identity, this same mirroring principle prevents the existing from fusing into the lacking: rather than imagining their differentia as being at any moment equal (and thus cancelling each other out), we ought to imagine them as perpetually changing hands, such that no instant can fix them in their places. It is because the structure that supports the gap between existing and lacking exists as such a play of mirrors that the relation to the lacking as possibility exists as a relation of the for-itself *to itself as other*.



Moving steadily from the ontological nucleus of lack, through its first distensions, we arrive at a point where projected lines of possibilities crystallize into more complex structures. First, we must take into account the necessary interaction of the dynamic illustrated above with the concrete situation in which it takes place. At no point throughout these processes does

consciousness escape the primacy of extensional determination or its irreducible facticity, and the *lacking* is always determined as lacking with respect to the situation in which the *existing* consciousness is embedded. Since consciousness, being extensionally determined, is qualified only with respect to the phenomena that it discloses, the deficiency that it exhibits through its participation in the structure of lack cannot be grasped as a deficiency in consciousness-in-itself (there is no such thing), but as a deficiency in the disclosed phenomena. Consciousness' lack is disclosed as a lack or a possibility belonging to its situation. In general, this is accomplished by way of a second *existing* term; this will be the existing term of a derivative triad, like those with which we began our analysis of Sartre's theory of lack (the 'crescent moon—full moon—missing gibbous' complex, for example).

Driven and encompassed by the structure of an original triad (which is nonetheless subject to change at any moment), derivative triads of lack determine the transformations that the for-itself may undergo in the situation, which itself comes to be determined as an instrumental complex or 'hodological space'. In what has come to be one of his most famous examples, the description of a peeping tom prior to his disruption by the sound of footsteps in the hall, Sartre illustrates the peculiar transcendental consistency that characterises the situation. As a reminder of the essential incompleteness of the current study, in which no account of the relation to the other is undertaken, let us look only at the first part of this celebrated description:

Let us imagine that moved by jealousy, curiosity, or vice I have just glued my ear to the door and looked through the keyhole. I am alone and on the level of a non-thetic self-consciousness. This means first of all that there is no self to inhabit my consciousness, nothing therefore to which I can refer my acts in order to qualify them. They are in no way *known*; I *am my acts* and hence they carry in themselves their whole justification. I am a pure consciousness of things, and things, caught up in the circuit of my ispeity, offer to me their potentialities as the proof of my non-thetic consciousness (of) my own possibilities. This means that behind that door a spectacle is presented as 'to be seen,' a conversation as 'to be heard'. The door, the keyhole are at once both instruments and obstacles; they are presented as 'to be handled with care;' the keyhole is given as 'to be looked through close by and a little to one side,' etc. [...] My attitude, for example, has no 'outside'; it is a pure process of relating the instrument (the keyhole) to the end to be attained (the spectacle to be seen), a pure mode of losing myself in the world, of causing myself to be drunk in by things as ink is by a blotter in order that an instrumental-complex oriented toward an end may

be synthetically detached on the ground of the world. The order is the reverse of the causal order. It is the end to be attained which organizes all the moments which precede it. [...] This ensemble in the world with its double and inverted determination (there is a spectacle to be seen behind the door only because I am jealous, but my jealousy is nothing except the simple objective fact that *there is a sight to be seen* behind the door)—this we shall call *situation*. (BN, III.I.IV, 259/298-9)

The consistency of a situation is given only through the ephemeral and reversible structures of negation and lack, which manifest themselves in complexes of possibility and potentiality. It is important to recognise that, for Sartre, these reversible structures are part of the very definition of the situation, and are a necessary condition for the consistency of the multiplicity that the situation is. According to a principle which can be illustrated fairly suggestively in our graphs (in Chapters IV and VI), the consistency of a multiplicity is, in a sense, structurally external to the elements of that multiplicity. When the multiplicity is thought as a set, or Badiouian situation, its point of consistency is self-identical and unitary, and may enter into the composition of other multiplicities, subject to the axioms of set theory. When the multiplicity is a consciousness, or Sartrean situation, its point of consistency is internally fissured, held together in an ephemeral reflexivity, and consequently subject to the structures of lack. To understand the consequences of this principle for Sartre's understanding of the constitution of the situation, we must avoid the temptation of treating the point of consistency of a situation (or multiplicity) as a self-sufficient term, even as we recognise its necessary difference from the elements of the situation itself. A modification in the point of consistency of a situation implies a modification in the immanence of the situation itself, for it is the very condition of simultaneously thinking of all the elements of the situation. In the Sartrean theory of situations, we do not have a perfectly consistent set-like multiple to which the vacillations of consciousness are added afterwards (as the notion of facticity may lead us to believe, if we forget that this notion is entirely *retrospective*); rather the situation, *qua situation*, is constituted through these very vacillations—even though the being of this situation must be understood as having already been in place, in mode of inconsistency proper to the in-itself. Consequently, “the *situation*, the common product of the contingency of

the in-itself and of freedom, is an ambiguous phenomenon in which it is impossible for the for-itself to distinguish the contribution of freedom from that of the brute existent. [...T]here is freedom only in a situation, and there is a situation only through freedom” (BN, IV.I.II, 488-9/533).¹⁴³

Within the context of a situation perfused with lack, it is possible for the for-itself *to act*. Now, when Sartre speaks of ‘acts’, he is speaking actions that are performed intentionally—in the sense of being done on purpose (motion alone does not constitute an act), but also in the sense of aiming at set of transcendent points. The structure of lack organises the configuration and subjective function of these points, and the modality of the intentions that bring them into relation. In general, this organisation is going to be such that the intentionality of the act will be “a surpassing of the given toward a result to be obtained” (BN, IV.I.I, 477/522).

Derivative triads of lack are different in certain respects from the primary triad. For one, their point of synthesis—the lacked—is not *a priori* determined as impossible. The for-itself-in-itself cannot *be*, but it is entirely possible that the moon be full, or that the shed be built. In the context of an act, the lacked is to be understood as the *end* or goal of the act; the lacking is the organized project apprehended as *means*; the *existing* is constituted as motive or *cause* for the act. One of the most significant consequences of this arrangement is how it reconfigures the terms of the perennial debates on the question of free will.

“In those tedious discussions between determinists and proponents of free will,” Sartre writes, “the latter are concerned to find cases of decision for which there exists no prior cause, or deliberations concerning two opposed acts which are equally possible and possess causes (and motives) of exactly the same weight. To which the determinists may easily reply that there is no action without a *cause*...” (BN, IV.I.I, 436/480). Sartre’s move to define subjective causality strictly within the configuration of a lack constitutes one of his most significant interventions in

this debate, which he correctly judges to be misguided in its foundations. The triadic structure of the for-itself in lack necessitates that *something* occupy the position of the existing and so act as a cause; accordingly, “[t]o speak of an act without cause is to speak of an act which would lack the intentional structure of every act” (BN, IV.II, 436-7/481). At the same time, it is impossible to grasp *how* a phenomenon can function as a subjective cause if one ignores its participation in negativity. The existing is not *merely existing*; it is missing something, and only through this negative capacity can it incite one, or be incited, to act. To say that a phenomenon is positioned as cause only when it is constituted as an *existing* is also to say that its causal efficacy can only be constituted once its complementary *lacking* has been determined. “It is on the day that we can conceive of a different state of affairs that a new light falls on our troubles and our suffering and that we *decide* that these are unbearable,” Sartre writes. “It is after [someone] has formed the project of changing the situation that it will appear intolerable to him” (BN, IV.I.I, 435/479).

The intolerability of the current state of affairs in the light of such a project results from the participation of these affairs in a structure of lack. Considered in its facticity—a consideration which is only possible by means of a retrospective fiction, since every situation is, *a priori*, structured in terms of lack insofar as it is disclosed by a consciousness—the situation offers neither intolerability nor the incitement to act. At this point, Sartre lays down two crucial principles for his theory of the subject, both of which will find a distinct echo in Badiou’s thought. They are:

1. No factual state whatever it may be (the political and economic structure of society, the psychological ‘state’, etc.) is capable by itself of motivating any act whatsoever. For an act is a projection of the for-itself toward what is not, and what is can in no way determine by itself what is not.
2. No factual state can determine consciousness to apprehend it as *anégité* or as a lack. Better yet no factual state can determine consciousness to define it and to circumscribe it since, as we have seen, Spinoza’s statement, ‘Omnis determinatio est negatio,’ remains profoundly true. [...] There is a factual state—satisfying or not—only by means of the nihilating power of the for-itself. (BN, IV.I.I, 436/479-80)

At first there appears to be something of an aporia or even a contradiction between these two principles. On the one hand, a given factual state cannot compel an act, the reason being that, qua factual state, it is devoid of negativity. On the other hand, *no* factual state is devoid of negativity, insofar as its very individuation is accomplished negatively. This is obvious enough in the very examples given in the first principle—what is an economic structure if not a vast network of debts and interests, and what are these if not negativities? The apparent contradiction can be dispelled by a closer attention to Sartre’s syntax: we must, again, distinguish between the *being* of the factual state and its *there is*. It is the latter to which Sartre assigns the dimension of negativity: “*There is* a factual state [...] only by means of the nihilating power of the for-itself.” But, if this is the case, it would seem to be a perfectly empty statement that no factual state can determine consciousness to act, since this determination is equiprimordial with the consistency of the factual state itself. If every situation is perfused with the negative, insofar as it *appears*, then we are faced with the difficulty of discerning the sort of negativity that can break with the situation from the sort of negativity that constitutes ‘business as usual’ for the situation (*e.g.* the network of wants that compose an economy). Sartre, of course, recognises this difficulty, and goes on to further specify the mode of negativity that must be engaged in order for an act to be possible. The difference, in a nutshell, between the sort of nihilation that is commensurate with the situation and the sort of negation that can incite one to transform the situation, is one of scope or radicality. Sartre gives the example of a proletarian factory worker on the threshold of revolt. Here,

It is the organized form—worker-finding-his-suffering-natural—which must be surmounted and denied in order for it to be able to form the object of a revealing contemplation. This means evidently that it is by a pure wrenching away from himself and the world that the worker can posit his suffering as unbearable suffering and consequently *canmake of it the motive* for his revolutionary action. (BN, IV.I.I, 436/480)

Every act—from tying one’s shoes to escaping from prison—is by its very nature implicated in what likely amounts to a great number of lack-structures, or instrumental complexes. Indeed,

Sartre suggests at times that we should, as a rule, consider the dense weave of the situation to consist of an infinite number of lacks, the for-itself being “an infinity of possibilities,” which “cannot be contained in one formula” (BN, II.II.I.C, 129/164). Nevertheless, the triadic structures of lack from which a situation is composed admit of a certain ordering relation, as is evident here. There exist subordinate and superordinate triads of lack, and the transformation of a subordinate triad will generally leave its superordinate structures unchanged.¹⁴⁴ That said, *nothing* prevents the complete upheaval of the entire hierarchy, which would manifest itself in the worker’s “pure wrenching away from himself” and projection towards a new possible. There is no guarantee, of course, that this would be either easy or painless. The interwoven structure of projects and triads of lack ensures that the contrary will generally be the case. Sartre gives a simple example of the form and structure of the difficulty involved in such a break in his description of the experience of collapsing from fatigue during a mountain hike:

I have yielded to fatigue, we said, and doubtless *I could* have done otherwise but *at what price?* At present we are in a position to answer this. Our analysis, in fact, has just shown us that this act was not *gratuitous*. To be sure, it was not explained by a motive or a cause conceived as the content of a prior state of consciousness, but it has to be interpreted in terms of an original project of which it formed an integral part. Hence it becomes evident that we can not suppose that the act could have been modified without at the same time supposing a fundamental modification of my original choice of myself. This way of yielding to fatigue and letting myself fall down at the side of the road expresses a certain initial stiffening against my body and the inanimate in-itself. It is placed within the compass of a certain view of the world in which difficulties can appear ‘not worth the trouble of being tolerated’; or, to be exact, since the motive is a pure non-thetic consciousness and consequently an initial project of itself toward an absolute end (a certain aspect of the in-itself-for-itself), it is an apprehension of the world (warmth, distance from the city, uselessness of the effort, *etc.*) as the cause of my ceasing to walk. Thus this *possible—to stop—theoretically* takes on its meaning only in and through the hierarchy of the possibles which I am in terms of the ultimate and initial possible. This does not imply that *I must necessarily* stop but merely that I can refuse to stop only by a radical conversion of my being-in-the-world; that is, by an abrupt metamorphosis of my initial project—*i.e.* by another choice of myself and of my ends. Moreover this modification is always possible. (BN, IV.I.I, 464/508-9)

Insofar as the vast complex of lacks and significations that articulate the surface of a situation are founded in a primary triad of lack that by its very nature opens an abyss of ontological indeterminacy, the modal consistency of a situation—what is ruled to be possible or otherwise—is fundamentally unstable.

Throughout *Being and Nothingness*, Sartre ties the affect of anxiety to the recognition of this instability, whose responsibility is born exclusively by freedom.¹⁴⁵ In *Existentialism and Humanism*, Sartre elaborates this thesis, connecting this same affect to the opposite and complementary pole of this absolute instability, which is the instantiation of *value*. There, the meaning of anxiety is understood as the fact that

When a man commits himself to anything, full realising that he is not only choosing what he will be, but is thereby at the same time a legislator deciding for the whole of mankind—in such a moment a man cannot escape from the sense of complete and profound responsibility.¹⁴⁶ (EH, 30)

Insofar as the absoluteness of value is instantiated at a level that radically precedes the particularity of egos and personalities, a dimension of universality is immediate to it. This universality is not, Sartre argues, charged against the postulation of a universal human essence, but against the very absence of essence—essence, in the last analysis, being the handmaid of particularism and an obstacle to universality insofar as it is taken to mediate the latter. A universalism dependent on the commonality of essence can only be sustained insofar as one posits a guarantor for this commonality—be it God, or Man, or some other idol. This tack loses all ground once we grasp the subject as a figure whose existence precedes its essence, and God as a form that exists only in the mode of a radical impossibility consubstantial with the nullity of the self. Every subject is, in principle, compatible with every choice, whatever the difficulty a subject may have in readjusting itself—which is to say, its *project*—to a choice that conflicts with it. A positive universality of essence is replaced by a fundamentally subtractive universality. If the subject is nothing other than its projects or purposes, then there is no difficulty in claiming, as Sartre does, that “In every purpose there is universality, in this sense that every purpose is comprehensible to every man” (EH, 46). Badiou’s seizes directly on this point in his reading of Sartre, writing:

This motif of the impossible but necessary (or real) occupation of the place left vacant by the gods—I believe that we can call this a radical humanism. Man is to itself its own absolute, or,

more exactly, it is the endless becoming of the absolute that it is. [...] Man is what man must invent. (LS, 240)

As we will see, Badiou seeks to maintain the same ideal that Sartre advances in his radical humanism, and, like his predecessor, roots the universality of the Idea for which a subject lives in its ontological anteriority to every determination of essence. Yet Badiou will break with Sartre on two significant points in this endeavour: For one, he will sacrifice the radical, if impossible, transcendence of value in favour of a fully immanent doctrine of *truths*. Second, he will contest that there exists

for consciousness the permanent possibility of effecting a rupture with its own past of wrenching itself away from the past so as to be able to consider it in the light of a non-being and so as to be able to confer on it the meaning which *it has* in terms of the project of meaning which it *does not have*. (BN, IV.I.I, 436/480; e.a.)

In a crucial departure from his forbearer, Badiou advances as one of the main tenets of his philosophy of the subject that the moments of such possibilities are incredibly *rare*, and befall us only in those strange ontological dysfunctions that he calls *events*. Though structurally similar to the Sartrean event of the for-itself, these events differ in *not* being the ordinary mode of situational consistency. They are strictly anomalous. Where Sartre insists that

there is no privileged situation. [...] here is no situation in which the *given* would crush beneath its weight the freedom which constitutes it as such—and conversely there is no situation in which the for-itself would be *more free* than in others (BN, IV.I.II, 549/594),

Badiou's rarity thesis prompts him to advance a typology of situations and a theory of event sites, as a means of outlining the necessary conditions of subjective rupture. The exceptional nature of subjective freedom is articulated a second time over in the ontology of the subjective project; here, Badiou pursues the anteriority of the subject to essence further than Sartre, and rules that subjects are only truly possible when their trajectory does not only precede essence, but diagonally evades it. We now turn to the ontology that grounds this thesis.

VIII. The Impasse of Mathematical Ontology And the Theory of Subjective Facticity (Badiou on Forcing and the Generic)

The notion of set seems to have never been understood in a unique way by mathematicians. We find in Becker an instructive account of a conversation which took place between Cantor and Dedekind. Whereas Dedekind compared sets to bags which contain unknown things, Cantor took a much more metaphysical position: he said that he imagined a set as an abyss.

- Andrzej Motowski, "Recent Results in Set Theory"¹⁴⁷

What Sartre says of the subject of a project ensemble—that it is “an infinity of possibilities [...] and cannot be contained in one formula” (BN, II.II.I.C, 129/164)—can equally be said of the truth procedure in which the Badiouian subject is embedded. It is in fact a quite remarkable feature of Badiou’s set-theoretic ontology that it is able to give precise formulation to this characteristic of the subject, at least insofar as it leaves its mark in the facticity of the subjective procedure. This proviso is significant, given that set theory, as ontology, is “beneath the distinction of the real and the possible,”¹⁴⁸ and is incapable of expressing the existential structure of possibility as such. What it can offer us is a schematisation of the ontological points of support from which possibilities may be strung or from which they may depart; it can give us the ontological ground that will come to be animated through the existential structures of the post-evental subject.

Badiou’s formalism takes up both parts of Sartre’s expression: It shows in what sense the subject’s field of possibility is infinite, and shows in what sense it cannot be captured by a single formula. We will consider these two parts in sequence, beginning with the infinite, but first let us elucidate what it means to speak of a field of possibility in the context of Badiou’s ontology.

Two distinct kinds of possibility must be taken into consideration here. On the one hand, Badiou claims that the plethora of sets deployed by the Zermelo-Fraenkel axioms express being at a level that is indifferent to modality. The inconsistent universe of sets is therefore something like the divine understanding in Leibnizian metaphysics, in which all possible worlds subsist. There is therefore, for Badiou, a sense in which possibility is understood as being that which remains once the real is subtracted from being; the possible is what is not, at a given moment, realised, but to whose realisation no strictly ontological barriers exist. Within the situation, we see this kind of possibility invoked again, given that the elements of a situation can be considered to be what is recognisably possible according its structure.

However customary it has become to speak of the possible as a superset of the real, there is something that is rather odd about this approach to modality in the context of Badiou's thought. He seems to be verging on utter contradiction, for example, when he says in the *Deleuze* that "the One is not, there are only actual multiplicities, and the ground is void,"¹⁴⁹ and "the univocity of Being requires its integral *actuality*" (ibid., 58). If this is not an outright contradiction, it can only be because for Badiou the being of the possible is actual, where actual is understood not as the manifest, but as the determinate; this, at least, would be enough to restore a basic semantic consistency to Badiou's statements while preserving the demarcation that he wishes to draw between the definite multiplicities of set theory and the indefinite and quasi-definite multiplicities of the Bergsonian virtual in which Deleuze roots his ontology of the multiple. Unsurprisingly, this acceptation of the word 'actual' is essentially the one that we encounter in mathematical discourse. Georg Cantor, founder of mathematical set theory, will say, for instance, "When the infinite appears in [...] a definite form I call it the actual-infinite."¹⁵⁰

'Possibility' has a second sense in Badiou's metaphysics, however, which does not coincide with the first. This is the sense to which he must appeal when he says that, in virtue of

an event (which is in this respect comparable to the theological notions of miracle and grace), the impossible can happen. That is, there is a dimension of possibility which is contingent on the event and which is not directly circumscribed by what is counted as possible by the structure of a situation. Whereas the support for the first sense of possibility is factual, this mode of possibility is supported by the freedom of the post-evental subject, and we will show in the next chapter how this freedom is to be understood. It would be wrong to treat these two orders of possibility as being entirely disconnected, however. As we will see, while the total assemblage of what will have compose the possibilities of the subject is not counted as possible in the situation, or even within its ontological horizon (its model), it nonetheless consists of multiplicities whose constituents are only ever elements of the situation itself. Possibilities of the second order do not *belong* to the being of the situation, but they are *included* there, at least with respect to their factual support (*qua* possibility, they remain structurally contingent on the event).

It is important to bear in mind that when Badiou employs the metaontological concept of ‘situation’ in *Being and Event*, it is *only* the facticity of the situation that is under consideration, even when the scope of a situation is understood both in terms of its manifest reality and its field of immanently structured possibility. This field gives us only the scope of *what* is possible or actual, and does involve the specific dimensionality of the possible. It is especially important to keep this point in mind as we move from the Sartrean theory of situations to its Badiouian counterpart. In an endnote to *Being and Event*, appended to the aforementioned definition of ‘situation’ given in the first Meditation (see Chapter IV above), Badiou writes that “[t]he word ‘situation’ has a Sartrean connotation for us. It must be neutralized here. A situation is purely and simply a space of structured multiple-presentation” (BE, 484/523). From a Sartrean perspective, Badiou’s notion of situation is somewhat anomalous, insofar as it decrees the existence of purely *factual* situations; situations existing in unadulterated immanence. What is “neutralized” in the

Badiou's inheritance of the concept of situation is the dimension of freedom or transcendence that Sartre understands to be coextensive with situationality as such. For, since the occurrence of 'presentation'—the 'there is' by which being-in-itself is attained a local consistency—invariably takes the form of absolute consciousness in Sartre's early ontology, the situation only exists insofar as it is unified through a network of lacks and projections of possibility in which freedom is engendered. Since the activity of freedom is inseparable from the very consistency of the Sartrean situation, "there is freedom only in a *situation*, and there is a situation only through freedom" (BN, IV.I.II, 489/534). The dimension of freedom that is integral to the situation as such is deeply implicated with the peculiar character of the mode of individuation of the situation-consciousness complex, which individuates itself extensionally, but in a way that is only accessible in interiority (see Chapter I). Only once the complex structure of interiorisation and immanent differentiation by which the Sartrean situation acquires its specific consistency is "neutralized" does it become possible to bypass Sartre's injunction that "[i]t is impossible to consider a situation from the outside" (BN, IV.I.IIE, 548/593) and to submit the situation, by and large, to a set-theoretical discourse that is exterior to it (though exterior in a somewhat problematical sense, as we have seen in Chapter III). The Badiouian situation is seized in its identity on the basis of the set-theoretic description that formalises its presentational structure. This description is considered valid in all cases where there is no question of a dimension of transcendence and 'interiority'; Badiou holds these cases to be the norm from which the largest share of human life is woven. Where such a dimension is in question—in *events* and the subjectivating truth procedures to which they give rise—metaontological description serves only to furnish a sort of mathematical infrastructure, which must be supplemented by a theory of the subject extraneous to ontology. How this supplementation takes place will be the topic of the next chapter.

We now turn to the infinite.

One of the most remarkable and historically significant aspects of Cantor's invention of set theory is that it provided mathematics, for the first time in its entire history, with the means necessary for investigating the structure of infinite multiplicities understood as both actual and complete, rather than as ineffable transcendences or ever-increasing but never-determinate quantities. What is most astonishing about this revolution in mathematics is that not only did Cantor supply the means to speak of the actually infinite in general—which in itself would mean very little—but he went on to show that there exist *different orders of actual infinity*, among which definite arithmetical relationships can be established and which can be arranged in an absolutely infinite hierarchy. The first member of the transfinite hierarchy, which Cantor named the series of alephs, is \aleph_0 , the quantity of the set of natural numbers. \aleph_0 is also the quantitative measure—the *cardinal*—of several other well-known sets, including the set of odd and the set of even numbers, the set of all primes, the set of rational numbers, and the set of algebraic numbers. When we say that two transfinite sets A and B have the same cardinal number, we mean simply that it is possible to establish a one-to-one correspondence between the elements of A and the elements of B ; when this is possible, we say that A and B are *equipollent*. For example, it is obvious enough that we may line up the totality of even numbers in such a way that each is assigned to one and only one natural number, and such that no natural number is without a mate; all that needs to be done to do this is to consider the function $(2x): \mathbb{N} \rightarrow \mathbb{E}$, and its inverse, $(x/2): \mathbb{E} \rightarrow \mathbb{N}$, where \mathbb{E} and \mathbb{N} stand for the set of evens and the set of naturals respectively.

The matter is altogether different when it comes to the set of 'real' numbers: the numbers corresponding to the points on a line and among which we find such 'transcendental', or non-algebraic numbers, as π and e . Cantor proved that it is strictly impossible to put the set of reals

into a one-to-one correspondence with any set of cardinality \aleph_0 (such as the natural numbers). The proof (which, legend has it, adorns his tombstone) runs as follows: Suppose that it is possible to enumerate the reals between 0 and 1, and placing them into a one-to-one correspondence with the natural numbers, and suppose that this enumeration is complete. If we were to write out the reals thus enumerated in their binary decimal expansion, we would have the (infinite) array:

	r_1	r_2	r_3	r_4	r_5	...
1:	0	1	1	0	1	...
2:	0	0	1	1	0	...
3:	1	1	0	1	1	...
4:	0	1	1	1	1	...
5:	1	0	0	0	1	...
:	:	:	:	:	:	...

We would then be able to define a new real number through the following process: First, we trace down the diagonal of the array, going from $(r_1, 1)$, through (r_n, n) for every n . As we trace this diagonal, we write down a zero for every one we encounter and vice versa. With the current array, this would give us the real number .11100... The number thus produced will differ from each enumerated real r_n in the n^{th} decimal place. The conclusion to be drawn is that given any supposed enumeration of the reals, it will always be possible to produce a real that has been missed by that enumeration. The cardinality of the reals is thus greater than the cardinality of the naturals; it is greater than \aleph_0 . Now, it turns out that it *is* possible to place the reals in an exact one-to-one correspondence with the set of *subsets* of the natural numbers (which is why it is possible to imagine each real as an infinitely long sequence of digits in a decimal expansion in the first place). The theorem which now bears Cantor's name in fact tells us that what is true of the relation between the naturals and the reals is in fact true for *any* set and the set of its subsets, or 'power set' as it is called. I will provide the proof shortly, but we ought first to return to the

notion of *subset*, which is of fundamental importance for both Badiou's metaontology and set theory in general.

A subset of a given set α is defined as a set whose elements are also elements of α . $\{a, b, c\}$, for example, is a *subset* of $\{a, b, c, d\}$. When β is a subset of α , we write $\beta \subset \alpha$. Note that this relation, which we will call *inclusion* or *inherence*, like the relation of belonging (\in) describes the 'contents' of a set. They are nevertheless quite distinct. $\beta \subset \alpha$ does not mean that $\beta \in \alpha$, but that $(\forall \gamma)(\gamma \in \beta \rightarrow \gamma \in \alpha)$. As a general rule, the presence of all the elements of β in α does not imply the presence of β itself in α . Inclusion does not imply belonging, nor inherence existence.

In a certain sense, the difference between a multiplicity and its elements, enshrined in the Second Criterion of the definition of multiplicity, is at stake in another form in the distinction between \in and \subset . It is in virtue of this distinction that we are able to say, within set theory, that the presentation of all the elements of a multiplicity is not the same as the presentation of the multiplicity itself. It may even be possible to rephrase our second principle of multiplicity as saying, not simply $(\forall x)(x \neq \{x\})$, but $(\forall x)(\exists y)(y \subset x \text{ \& } y \notin x)$ —that is: not simply that no multiplicity is identical to its own elements, but that every multiplicity includes a subset which is not its element. This is a way of reproducing the distinction between a multiplicity and its elements on the interior of any given multiplicity. Of course, if we do this, then it is no longer enough to define the One as the x such that $x = \{x\}$. Even here, the void slips in, giving us $\emptyset \subset x = \{x\}$.¹⁵¹ This is always possible so long as we allow that the empty set exists, and so long as we define inclusion by the formula $(\forall \gamma)(\gamma \in \alpha \rightarrow \gamma \in \beta) \rightarrow \alpha \subset \beta$. We can see that this formula is always valid when we put \emptyset for α , regardless of what set we choose for β . To secure the existence of the One in the light of this strengthened criterion of multiplicity, we would have to

assert a second Parmenidean axiom, and declare that the void does not exist, “For never shall this be proved: that things that are not are; but do thou hold back thy thought from this Way of inquiry...”¹⁵²

This law, $(\forall x)(\exists y)(y \subset x \ \& \ y \not\subset x)$, is, in fact, a theorem of set theory, whose proof can be easily supplied. Badiou offers a proof for it in Meditation 7. Here we will give a proof for a much stronger theorem—Cantor’s theorem, as it is now called—from which the above law follows as an immediate consequence. The proof is as follows:¹⁵³

Take an arbitrarily chosen set Z . Suppose that it were possible to put the elements of Z into a one-to-one correspondence with the set of all subsets ζ of Z , which we will call $\wp(Z)$ (read “ Z ’s power set”). Let f be the function that organizes this correspondence, so that for any $\zeta \in \wp(Z)$, $f(\zeta) \in Z$. Since every ζ is a subset of Z , it makes sense to ask whether or not, for a given ζ , $f(\zeta)$ belongs to ζ . Let \mathfrak{R} be the subset of Z which consists of the elements $f(\zeta)$ which *do not belong to their corresponding ζ* (i.e., $\mathfrak{R} = \{f(\zeta) \mid f(\zeta) \notin \zeta\} \subset Z$). Since \mathfrak{R} is a subset of Z , it must be an element of $\wp(Z)$, and if, as we have supposed, f is a one-to-one correspondence between Z and $\wp(Z)$, then there must be an element $f(\mathfrak{R})$ of Z . The question is, what ‘region’ of Z does $f(\mathfrak{R})$ fall into? Is $f(\mathfrak{R})$ an element of \mathfrak{R} or not?

Since for a set $f(\alpha)$ to belong to \mathfrak{R} means for it *not* to belong to α , to say that $f(\mathfrak{R})$ belongs to \mathfrak{R} means that it *does not belong to \mathfrak{R}* ! We have a contradiction.

Conversely, if $f(\mathfrak{R})$ is *not* an element of \mathfrak{R} , then it does not belong to its correspondent and so, by that very token, is an element of \mathfrak{R} ! Again, a contradiction.

In short, $f(\mathfrak{R})$ belongs to \mathfrak{R} if and only if it does not! This contradiction forces us to abandon our original hypothesis, and conclude that no set, infinite or otherwise, can be placed into a one-to-one correspondence with its power set. \square

Since to say that two sets are of the same quantity is to say that a one to one correspondence (like the impossible f in the above example) can be established between them, a consequence of this theorem is that given any infinite (or transfinite, to use Cantor’s vocabulary) set, another set of a greater power exists whose elements are the subsets of the first set (we know the power set is not of a lesser cardinality, since to every element x of the original set, there

corresponds the element $\{x\}$ of the power set). There is therefore no highest infinity \aleph_x such that $(\forall y)(\aleph_x \geq \aleph_y)$, since $|\wp(\aleph_x)|$ will always be greater than \aleph_x . Since the highest aleph would contain all the alephs below it as elements (a consequence of the definition of cardinal numbers in set theory), to say that there can be no highest aleph means that there can be no set of alephs. This point struck Cantor as one of utmost importance. In a 1897 letter to David Hilbert, he wrote:

Theorem: ‘The totality of all alephs cannot be conceived as a determinate, a well-defined, and also a finished set’. This is the *punctum saliens*, and I venture to say that this is a *completely certain theorem, provable rigorously from the definition of the totality of all alephs* is the most important and noblest theorem of set theory.¹⁵⁴

If there is no set of all alephs, then *a fortiori* there is no set of all sets (otherwise we could simply extract the set of all alephs by the axiom of separation). And so, for set theory, there is no All of being, no Being-in-Totality. Insofar as it can be thought at all, the All is an “inconsistent multiplicity.”¹⁵⁵

Upon Cantor’s discovery of his celebrated theorem (and its corollary that there is no totality of all alephs), the clearest and most urgent question that presented itself was: If the cardinality of a set is always inferior to the cardinality of its power set, what quantitative measure can be established between the two when the sets are transfinite? In other words, if the cardinal of α is \aleph_n , what cardinal can be assigned to $\wp(\alpha)$? Cantor’s initial hypothesis—now called the generalised continuum hypothesis (or GCH)—was that $\wp(\alpha)$ should have a cardinal of \aleph_{n+1} , the next largest aleph.

Georg Polya once observed that the most difficult problems in mathematics are often those which can be expressed most simply. There is perhaps no clearer example of this adage than Cantor’s continuum problem, whose solution (which Cantor pursued with singular commitment and fidelity until his eventual descent into madness) would forever elude mathematics. Nevertheless, the search for a proof or disproof of the continuum hypothesis was

not uneventful: It saw two shining moments in 1939 and again in 1963 when Kurt Gödel demonstrated its consistency with the axioms of set theory, and Paul Cohen demonstrated its independence. Both of these proofs are of extraordinary depth and philosophical interest, bringing to light both the abyssal and mysterious nature of set theory's fundamental concepts, and the power and opacity of mathematical formalism, which Gödel's work has never failed to illuminate and which Cohen's reasonings bring into even sharper focus.

If we take the mathematical formalism behind these developments seriously, as Badiou does with conviction, what is revealed to us here is none other than an *ontological impasse* that Badiou, adopting a Lacanian idiom, does not hesitate to identify as the 'real' of being (BE, 5/11). Between any infinite set and the set of its subsets, there exists an irreducible and immeasurable ontological gap, where no objective and stable relation can be assigned. This field of underdetermination that mathematical ontology reveals between set and power set is, for Badiou, the very field of subjective liberty. It is a state of affairs comparable to Kant's theory of the noumenal subject of practical reason. For Kant, the indeterminacy of the field of subjective agency is safeguarded by the transcendental difference which prevents the noumenal subject from falling under the jurisdiction of phenomenal causality; only by maintaining this difference may Kant consistently postulate the freedom of the subject as demanded by practical reason. As Badiou reads Cohen, the independence of the continuum hypothesis opens a space where freedom is possible, insofar as it renders the combinatory articulation of subsets relatively free from the laws of ontology.

Significantly, that this state of affairs also recalls the situation we earlier examined with respect to the Sartrean theory of the subject as conditioned by the structure of lack. There too, we find the subject positioned precisely where one encounters an incalculable ontological abyss; for Sartre, this is the abyss between consciousness and the impossible *ens causa sui*. The essential

difference between Sartre and Badiou on this point is that, for Badiou, this abyss is not directly contingent upon the existence of the subject,¹⁵⁶ but is an immanent feature of the ontological landscape itself. Moreover, for Badiou, it is not a question of an abyss between the existent, the possible and the impossible, but between two actualities: \aleph_n and $\wp(\aleph_n)$.

Also, unlike the immeasurable disparity between the Sartrean subject and its ontologically impossible *ens causa sui*, the gap between set and power set is not such that every attempt to cross it is doomed to failure or, at best, Quixotic paralogism (“Man is a useless passion” (BN, IV.II.III, 615/661).). In a certain sense, measure *can* be established between set and power set, or between the situation and what Badiou calls the ‘state’ of the situation, but this measure, radically underdetermined by the axioms of ontology, can be put in place only through the work of a singular subjective procedure, which, ontologically, corresponds to the construction of a new model structure produced through the exercise of the forcing relation over a generic subset.



The notion of genericity brings us to the second part of the Sartrean maxim with which we began this chapter: the infinite field of subjective possibility *cannot be contained in a formula*. This thesis is, in some ways, a restatement of Sartre’s famous axiom that, for the subject, *existence precedes essence*. ‘Essence’, here, should be understood as it is defined in the Introduction to *Being and Nothingness*: what is meant by this word is simply *the principle of a series*, the intension of a class, the rule of a sequence.

In the context of a set-theoretical ontology, it is possible to make this notion far more precise. In this context, I propose that we understand ‘essence’ just as we would customarily understand ‘intension’: as a set-theoretic formula possessing one free variable. Such a formula can be used as a predicate to isolate the set of all elements of a certain set that make the formula

true when they are substituted for the free variable. For example, the ‘formula’ “ x has fur,” when applied to the set $\Gamma = \{\text{wolf, stone, cat, cube, the American dollar}\}$, is satisfied when we replace the x with ‘wolf’ or ‘cat’. It is then possible to abstract from Γ the subset which satisfies our formula, giving us: $\{\text{wolf, cat}\}$. The axiom that permits us to do this is called the axiom of separation (which is in fact a theorem, once we have the axiom of replacement; details are provided in Appendix 2).

As I have already mentioned, one of the consequences of establishing set theory as ontology is that the situations of which it speaks are to be understood, more or less, as being model structures (or parts of model structures) for the axioms. We will say that a formula is *restricted* to a model \mathbf{M} if and only if its quantifiers range only over the elements of \mathbf{M} , and all of the constants it employs exist in \mathbf{M} . Note that in the above example, the formula we used is not *restricted* to Γ . If we take, instead, the set $\Gamma^* = \{\text{wolf, stone, cat, cube, the American dollar, fur}\}$, then we have a set to which we may say that our formula is restricted.

Now, the point of producing a model for a theory is to provide a structure in which virtually everything pronounceable according to the syntactical laws of the theory can be veridically interpreted. In order for essences to be semantically intelligible, we need to nuance our definition, and say that for an ‘inhabitant’ of a model structure, an essence is a formula, restricted to the model, with one free variable. If there exist, within this model structure (or global situation), multiplicities whose existence outstrips essence, then these must be multiplicities whose composition cannot be given in an essential formula, multiplicities which are, to use Cohen’s word, *generic*. In order for this to be possible, certain conditions clearly have to be met: First of all, no multiplicity a can outstrip the essences proper to the situation if it *belongs* to the situation, for in that case we could capture it through the essential formula ($x \in a$). The mode in which a generic multiplicity exists in a situation must be that of *inclusion* and not

membership. Second, a cannot be finite, otherwise it could be exhaustively classified by a formula such as $(x = \alpha_1 \text{ or } \dots \text{ or } x = \alpha_n)$, where n is the number of elements in a ; this restriction is imposed by what may be called a mere syntactic accident: namely, that the classical logic in which ZF is written allows only the inscription of finitely long formulae (an exigency grounded in the simple fact that we have neither the time nor the space to write down infinitely long sentences). While these conditions are certainly necessary for the articulation of such a multiplicity, they are by no means sufficient. In order to outline the sufficient conditions for the existence of a generic subset, we must be far more precise than we have been so far, and specify the exact mathematical structure of the model structure in which such sets may be articulated.

The models that we are about to produce are, a few minor details aside, the same ones that Gödel and Cohen used to prove the independence of the continuum hypothesis. The mathematical constructions that are used in Cohen's proof—generic subsets and the forcing relation—are the same constructions which provide Badiou's theory of the subject with its ontological backbone. The concept of constructibility that we find in Gödel's proof, on the other hand, provides Badiou with the ontological schema for *knowledge*, and gives us the backdrop against which a truth-procedure will be negatively defined as that which “punches a hole in knowledge”, or, in Sartrean terms, as that by which a subject's existence outstrips its essence. For the sake of concision, my treatment of these proofs will be somewhat abbreviated, and will aim only to provide enough details necessary for the metaontological component of Badiou's theory of the subject and its truth-procedures. A correlative goal will be to demonstrate why Badiou has need of structures akin to those developed in the Sartrean theory of projects in order to establish a genuine connection between the subject, truth and the event.

We begin by postulating that a model for ZF exists. There is no way of placing ourselves on firmer ground here than what can be supplied by postulation, for it is strictly impossible,

using the resources available to us in ZF, to *prove* that ZF has a model. This is a consequence of two of Gödel's most well-known theorems, which go by the name of the completeness theorem and the incompleteness theorem, respectively. The first tells us that a logico-mathematical theory is consistent if and only if it has a model. The second tells us that it is impossible for a theory to establish its own consistency, if it is powerful enough to express elementary arithmetic (which ZF is more than capable of doing).

GÖDEL: THE CONSISTENCY OF THE CONTINUUM HYPOTHESIS

In the beginning was the Word...

- John 1:1

The first model that we will consider is quite remarkable. It gives us a universe in which essence precedes and commands existence, in which every multiplicity is subordinated to intensional determination.¹⁵⁷ Gödel, who invented this model, called it the “constructible universe.”¹⁵⁸ Its construction consists in generating a cumulative hierarchy which we will designate by the sign \mathcal{L} . Its first stratum, \mathcal{L}_0 , is defined as the empty set \emptyset . Each subsequent stratum \mathcal{L}_n consists of all the subsets of \mathcal{L}_{n-1} that can be separated by formulae *restricted* to \mathcal{L}_{n-1} alone. This procedure is then repeated infinitely many times, and we define the first ‘limit’ stratum \mathcal{L}_{ω_0} as the *union* of all preceding strata—that is, as the set whose elements consist of all the elements of every preceding stratum. The procedure then repeats itself, beginning with \mathcal{L}_{ω_0} and following the entire hierarchy of the transfinite ordinals up to the first inaccessible transfinite ordinal (the smallest number that cannot be reached either by the power set operation or by union). The net result is a universe in which every set is constructible, in which being is preordained by algorithmic language. The supplementary axiom that this is the proper universe in which set theory operates is commonly abbreviated “ $\mathbf{V} = \mathcal{L}$ ”, where \mathbf{V} is shorthand for the proper class of all sets and \mathcal{L} designates the proper class of all sets belonging to some stratum of the \mathcal{L} -hierarchy, extended indefinitely. Here, the reader should note that a “proper class” is emphatically *not* a set, but what Cantor would call an inconsistent multiplicity.

Gödel has shown that it is possible to take the \mathcal{L} -hierarchy as a *model* for set theory. This is done by showing that it is possible to interpret all of the axioms of the theory in terms of constructible sets alone. The domain or ‘structure’ of this model is the constructible hierarchy \mathcal{L} ,

and the interpretation function is configured in such a way that the axioms of ZF are ‘relativised’ to the \mathcal{L} -hierarchy. The relativisation of axioms is simply the restriction of the formulae composing the axiom to some stratum or other in the \mathcal{L} -hierarchy, such that the constants employed in the formulae are all drawn from \mathcal{L} , and the formulae’s quantifiers range only over \mathcal{L} . Once all of ZF’s axioms are suitably relativised, they can all be shown to be veridical in the constructible universe on the assumption that ZF is consistent. \mathcal{L} is therefore a *model* for ZF (if ZF has a model).

In the model furnished by the constructible hierarchy, GCH is demonstrably true. The kernel of the demonstration hinges on two important lemmata, which I will present here without proof:

LEMMA 1: The constructible cardinal that numbers the quantity of each stratum \mathcal{L}_n is equal to n itself when n is infinite (when $n \geq \omega_0$).

LEMMA 2: All the constructible subsets of ω_n are elements of the stratum $\mathcal{L}_{\omega_{n+1}}$.

From these two lemmata we can derive the following:

- (i) on the basis of lemma 1, we know that $|\wp(\omega_n)| = |\wp(\mathcal{L}_{\omega_n})|$;
- (ii) on the basis of lemma 2, $\wp(\omega_n) \subset \mathcal{L}_{\omega_{n+1}}$ and therefore $|\wp(\omega_n)| \leq |\mathcal{L}_{\omega_{n+1}}|$;
- (iii) since $|\mathcal{L}_{\omega_{n+1}}| = \omega_{n+1}$ (lemma 1 again), we know that $|\wp(\omega_n)| \leq \omega_{n+1}$;
- (iv) since ω_{n+1} , by definition, is the smallest cardinal greater than ω_n , $|\wp(\omega_n)|$ can only be either ω_n or ω_{n+1} ;
- (v) by Cantor’s theorem (proven above), we know that $|\wp(\omega_n)|$ cannot be ω_n ; it is therefore ω_{n+1} . The generalized continuum hypothesis is therefore valid in the constructible universe: if every set is constructible, then $|\wp(\omega_n)| = \omega_{n+1}$. In aleph notation: $|\wp(\aleph_n)| = \aleph_{n+1}$.

It must nevertheless be observed that this does *not* constitute a proof for the universal validity of GCH: it is entirely relativised to the constructible universe \mathcal{L} . What is shown here is that GCH is *consistent* with set theory. Since \mathcal{L} is a *model* for set theory (if set theory has any models at all), meaning that every axiom and theorem of set theory can be relativised to \mathcal{L} and shown to be valid there. Accordingly, \sim GCH (the negation of the generalized continuum hypothesis) *cannot* be a theorem of set theory, or else we would have a contradiction when \sim GCH is relativised to \mathcal{L} .

COHEN: THE CONSISTENCY OF THE NEGATION OF THE CONTINUUM HYPOTHESIS

The Sacred Writers celebrate It by every Name while yet they call It Nameless.

- Dionysius the Areopagite, *The Divine Names*, §1.6.

We now turn to Cohen's work. Again, we begin by postulating the existence of a model. The next step is somewhat surprising: we decide that the model structure with which we will be working will be *countable*, that is, it will contain only as many elements as there are natural numbers. At first, this sounds absurd—how can we hope to express a science of transfinite multiplicities within a model structure restricted to merely countable infinities? Yet one of the great discoveries in mathematical logic in the last century assures us that this is possible, and, more generally, that *any* theory possessing a model also possesses a countable model, so long as the logical language in which the theory is written is countable (this result is called the Löwenheim-Skolem theorem).¹⁵⁹ What must be borne in mind here is that the countability of the model structure is strictly *external* to the model itself. What this means is that if we were to embed our model structure within a sufficiently large superstructure, it would be possible to discover a one-to-one correspondence between the first model structure and the set of natural numbers ω_0 . Within our model, which we will call a *minimal model*, no such correspondence

exists, so, in a sense, the internal vastness of the model is owing to its poverty rather than to its richness. Nevertheless, despite the heuristic value of understanding this model as being uncountable in appearance but countable in reality, we must resist the temptation of taking this view in any absolute sense. Instead, we should adhere quite strictly what we said earlier about ontological interpretation: that to think a situation or structure in its being is nothing other than to interpret it as a model for the axioms of ontology (ZF). Insofar as it is possible to interpret an intuitively countable structure as a model for statements concerning uncountable infinities, then it must be said to be uncountable in its being; there is no highest court of appeal.¹⁶⁰

Besides being externally countable, this model, which we will call S , must also satisfy a few other criteria: it must be transitive, so that if α belongs to our model then so do all the elements of α ; it must also satisfy all the axioms of ZF that can be expressed in a single formula (the axiom of the empty set, the power set axiom, the axiom of extensionality, the axiom of foundation, the axiom of infinity, and the union axiom), as well as a significant but finite number of the formulae composing the axiom of replacement (which is actually an axiom-schema, organizing an infinite sequence of similar axioms). In practice, we will consider all the axioms to be satisfied, since it is simple enough to stipulate that all the versions of replacement that we will in fact need are satisfied (after all, this will only every be a finite number, since all our demonstrations will be of finite length). At this point, an important proviso must be added: when we say that all of these axioms are satisfied in the model, we mean that all of these axioms are true when *restricted* to the model, which, again, means that all of their quantifiers range only over elements of the model structure, from which all of their constants must also be drawn. For example,

The veracity of the power set axiom in S signifies only that when $\alpha \in S$, the set of parts of α *which belong to S* is counted as one in S . But from the outside, the ontologist can quite easily distinguish a part of α which, not existing *in S* (because it *does not belong to S*), makes up part of

$\wp(\alpha)$ in the sense of general ontology without making up part of $\wp(\alpha)$ in the sense given to it by an inhabitant of S . (BE, m33, 361/397)

It is important to recognise that when we speak of the ontologist standing ‘outside’ the model or situation, we are speaking of the ontologist situating herself within another model;¹⁶¹ here this is one that subsumes S .

Cohen has shown that in such a model, GCH can be satisfied.¹⁶² The reason for this is that all of the elements of S turn out to have the character of *constructibility*, which is to say that S satisfies the Gödelian axiom $V = L$, which is a sufficient (but not necessary) condition for GCH. If we wish to produce a model in which GCH fails, the first order of business will therefore be to produce a model structure containing at least one non-constructible set. Following Badiou’s notation, we will call this set \wp , and stipulate that it must be a subset of S . The model in which \wp will exist as an element will be called $S(\wp)$. This supplementation cannot be done haphazardly, for, as Cohen observes,

\wp must have certain special properties if $S(\wp)$ is to be a model. Rather than describe it directly, it is better to examine the various properties of \wp and determine which are desirable and which are not. The chief point is that we do not wish \wp to contain ‘special’ information about S , which can only be seen from the outside, such as the countability of α_0 [sc. the smallest ordinal not in S], and will imply that any model containing \wp must contain more ordinals than those in S . The \wp which we construct will be referred to as a ‘generic’ set relative to S . The idea is that all the properties of \wp must be ‘forced’ to hold merely on the basis that \wp behaves like a ‘generic’ set in S . This concept of deciding when a statement about \wp is ‘forced’ to hold is the key point of the construction. (Cohen, 111)⁶³

Without getting bogged down in technicalities, the general shape of Cohen’s procedure is as follows:

Based on what we wish \wp to be capable of, we begin by circumscribing the region of S of which \wp will be a generic subset. We will call this region the set of *conditions* for the truth-procedure, and designate it by the sign \odot . For the current demonstration, we wish \wp to give us the information necessary for establishing a one-to-one correspondence between the continuum (the power set of ω_0) and an aleph greater than \aleph_1 . We will call this greater aleph, \aleph_τ , where τ

may stand for any ordinal we wish other than ω_0 (a stipulation due to König's lemma, which tells us that $|\wp(\alpha)|$ is never cofinal with α ; i.e., $|\wp(\alpha)|$ cannot be $\aleph_{|\alpha|}$). For this reason, we will determine \mathbb{C} as a set of sets π_n whose elements consist of ordered triples of the type $\langle \alpha, n, \varphi \rangle$, where $\alpha \in \aleph_\tau$, n is a natural number, and φ is either 1 or 0 (i.e., either $\{\emptyset\}$ or \emptyset). We will also require that \emptyset be a condition. In order to instil a measure of coherence into the raw information that is gathered in \mathbb{C} , we must further require that “none of these sets $[\pi_n]$ can simultaneously contain, for a fixed α and a fixed n , the triplets $\langle \alpha, n, 0 \rangle$ and $\langle \alpha, n, 1 \rangle$. For example, $\{\langle \alpha, 5, 1 \rangle, \langle \beta, 4, 0 \rangle\}$ is a condition, but $\{\langle \alpha, 5, 1 \rangle, \langle \alpha, 5, 0 \rangle\}$ is not” (BE, m34, 420-1/460).

Naturally, it is possible to articulate a wide variety of subsets on the basis of \mathbb{C} . In light of the ultimate aims of this procedure, we will consider a certain, coherent genus of subsets on \mathbb{C} , which, following Badiou, we will call *correct subsets* δ of \mathbb{C} . A correct subset adheres to two rules, noted Rd_1 and Rd_2 . Rd_1 instils a modicum of order in any correct δ . It states that if π_1 is an element of δ , then any condition π_2 that is a subset of π_1 is also an element of δ (in such a case, we will say that π_1 *dominates* π_2). Rd_2 formalises the character of conditional coherence that will be required of correct subsets. It does this by stating that for any two conditions π_1 and π_2 belonging to δ , there must exist a third $\pi_3 \in \delta$ such that π_3 *dominates* both π_1 and π_2 . This requirement transfers the demand of coherence that we place upon conditions onto sets of these conditions. Since the definition of *condition* does not, of itself, apply to sets of conditions, a given subset of \mathbb{C} may, for example, be of the form $\{\langle \alpha, 5, 1 \rangle, \langle \beta, 4, 0 \rangle\}$, $\{\langle \alpha, 5, 0 \rangle, \langle \beta, 4, 1 \rangle\}$. But Rd_2 prevents us from calling such a subset ‘correct’. An example of a correct subset of \mathbb{C} , by contrast, would be $\{\langle \alpha, 5, 1 \rangle, \langle \beta, 4, 0 \rangle\}$, $\{\langle \alpha, 5, 1 \rangle\}$, \emptyset .

Now that we have the concept of *correct subsets* of \mathbb{C} at our disposal, we may distinguish between correct subsets discernible by the encyclopaedia of S and correct subsets that are

indiscernible or generic. Discernibility, here, more or less amounts to local or ‘weak’ constructibility. To say that a correct subset δ is discernible means that there exists a formula, restricted to S , that is a necessary and sufficient condition for membership in δ (such that $(\forall \pi)(\pi \in \delta \leftrightarrow \lambda^S(\pi))$). By contrast, δ is *indiscernible* or *generic* if no such formula exists. In order to articulate this concept in the language of sets, we will associate every formula λ^S with a set D_λ (which Badiou calls a *domination*) such that $D_\lambda \in S$ & $D_\lambda \subset \mathbb{C}$ and

$$(\forall \pi_2)[\pi_2 \in D_\lambda \leftrightarrow (\sim \lambda^S(\pi_2) \& (\exists \pi_1)(\lambda^S(\pi_1) \& \pi_1 \subset \pi_2))].$$

In other words, the necessary and sufficient conditions for the membership of a condition π_2 in any D_λ is that π_2 not satisfy λ^S and that π_2 dominate a π_1 satisfying λ^S . Every condition is dominated by another condition in some D ; this is guaranteed by the fact that every condition is immediately dominated by at two other conditions that are incompatible amongst one another. For instance, $\{\langle \alpha, 5, 1 \rangle, \langle \beta, 4, 0 \rangle\}$ is dominated by both $\{\langle \alpha, 5, 1 \rangle, \langle \beta, 4, 0 \rangle, \langle \gamma, 22, 0 \rangle\}$ and $\{\langle \alpha, 5, 1 \rangle, \langle \beta, 4, 0 \rangle, \langle \gamma, 22, 1 \rangle\}$. If a discerning formula λ separates $\{\langle \alpha, 5, 1 \rangle, \langle \beta, 4, 0 \rangle, \langle \gamma, 22, 0 \rangle\}$, then it does not separate the $\{\langle \alpha, 5, 1 \rangle, \langle \beta, 4, 0 \rangle, \langle \gamma, 22, 1 \rangle\}$, which is for that reason counted as an element of D_λ .

A definition for a generic subset of \mathbb{C} , intuitively understood as a subset that cannot be isolated by any encyclopaedic determinant λ , is now obvious: “a correct set φ will be generic for S if, for any domination D which belongs to S , we have $D \cap \varphi \neq \emptyset$ (the intersection of D and φ is not empty)” (BE, m33, 370/406). Insofar as φ intersects with every domination, there can be no formula which discerns it (which acts as a necessary and sufficient condition for membership in φ), since for every restricted formula λ , there exists an element x in φ such that $\sim \lambda(x)$. φ therefore evades every intensional determinant of the model, and cannot be considered a

possible item of knowledge in the sense in which Badiou understands the term. By contrast, if δ is discernible in S by any λ^S then there will exist a domination D_λ exhibiting a void intersection with δ —this, of course, will be the domination whose elements satisfy $\sim\lambda^S(x)$. Every such δ is an item of knowledge in virtue of λ , which is its essence.

To say that φ is generic in S (genericity is always situation specific) is literally to say of it what Sartre says of the for-itself: that φ “is always something other than what can be *said* of it” in S (BN, IV.I.I, 439/483). But unlike the Sartrean for-itself, the generic vouchsafes its indiscernibility on the level of facticity, irrespective of transcendence. Even at the infinitely distant termination of the generic sequence, it escapes precise predication. (Here, one may be tempted to object: to say φ is generic is to predicate it! True, but this predication radically underdetermines the composition of φ ; it does not let φ be discerned. To say that φ evades precise predication is to say that no predicate can serve as a *sufficient* condition of membership in φ ; necessary conditions are easy to come by—we have used several in our definition of φ —but it is impossible on these alone to determine whether an arbitrary x is or is not in φ .)

The next phase in Cohen’s procedure involves producing an entirely new model—a new semantic universe—on the basis of φ , but only with the resources that are already available in S , in which φ only inheres as a sort of phantom subset that cannot be said to ‘exist’ in S (since it is not an element, the statement $\varphi \subset \mathbb{C}$ fails when restricted to S). The mechanism by which this is accomplished is extremely complex. Roughly speaking, it involves taking into consideration a class of ‘names’ in S , which are composed of conditions (elements of \mathbb{C}) and other names. These names are built up in such a way that their referential value is contingent on the composition of φ . The class of referential values is concocted so that every element of S finds a place within it, thus making S itself is a proper part of this class. We also ensure that φ itself is a referential value of some name in S , as is the entire gamut of sets constructible on the basis of φ by

application of the axioms. The key point is that when we take the entire class of referential values, we have a set which becomes the model $S(\mathcal{Q})$, in which virtually all of the axioms of ZF are satisfied (same proviso here as before), and in which GCH fails.

What is remarkable about this construction, of which only the roughest sketch is given here, is that it permits one to make true statements about $S(\mathcal{Q})$, *from within S itself*. The relation that enables such statements is called *forcing*, which Gödel once called “a method to make true statements about something of which we know nothing.”¹⁶⁴ It is defined in such a way that:

- if a condition π forces a statement on the names, then, for any generic part \mathcal{Q} such that $\pi \in \mathcal{Q}$, the same statement, this time bearing on the referential value of the names, is veridical in the generic extension $S(\mathcal{Q})$;
- reciprocally, if a statement is veridical in a generic extension $S(\mathcal{Q})$, there exists a condition π such that $\pi \in \mathcal{Q}$ and π forces the statement applied to the names whose values appear in the veridical statement in question. (BE, m34, 412/451)

Putting ‘ $\pi \equiv \lambda$ ’ for ‘ π forces λ ’, μ_i for the names in S , and $R_{x\mathcal{Q}}(\mu_i)$ for the referential values of these names, we have the formula:

$$\lambda(R_{x\mathcal{Q}}(\mu_1), \dots, R_{x\mathcal{Q}}(\mu_n)) \leftrightarrow (\exists \pi)[\pi \in \mathcal{Q} \ \& \ (\pi \equiv \lambda(\mu_1, \dots, \mu_n))]$$

Forcing is thus a certain kind of logical entailment, and it generally behaves as such.¹⁶⁵ For Badiou, it is the logic of subjective inquiry itself.

Recall that we have defined genericity by saying that \mathcal{Q} is generic if and only if it intersects with every domination $D \subset \mathbb{C}$. A consequence of this definition is that \mathcal{Q} contains¹⁶⁶ at least one condition presenting every element of ω_0 (every possible value of n) and every element of \aleph_τ (every possible value of α)—regardless of the choice of \aleph_τ (so long as $\tau \neq \omega_0$). Badiou provides a clear explanation of this point, which is worth repeating here. “It is quite suggestive,” he writes,

to ‘visualize’ what a domination is in the proposed example. Thus, ‘contain a condition of the type $\langle \alpha, 5, 0 \rangle$ or $\langle \alpha, 5, 1 \rangle$ ’¹⁶⁷ (in which we have fixed the number 5) defines a subset of conditions which is a domination, for if a condition π does not contain either of these, they can be added to it without contradiction. In the same manner, ‘contain a condition of the type \aleph_1, n ,

$1>, <\alpha_1, n, 0>$ in which α_1 is a fixed element of the cardinal δ [*sc.* \aleph_τ], also defines a domination, and so on. It is thus evident that φ is obliged to contain, in the conditions from which it is composed, ‘all the n ’ and ‘all the α ’, in that, due to its intersection with the dominations which correspond to a fixed α or a fixed n , for example, 5 and ω_0 (because δ is an infinite cardinal superior to ω_0 , or $\omega_0 \in \delta$), there is always amongst its elements at least one triplet of the type $<\beta, 5, 0>$ or $<\beta, 5, 1>$, and also always one triplet of the type $<\omega_0, n, 0>$ or $<\omega_0, n, 1>$. This indicates to us both the genericity of φ , its indeterminate nature, and signals that in $S(\varphi)$ there will be a type of correspondence between ‘all the elements of n of ω_0 ’ and ‘all the elements α of δ ’. This is where the quantitative arbitrariness of excess will anchor itself. (BE, m34, 421-2/461)

The *coup de grâce* is delivered by considering, in the generic extension $S(\varphi)$ produced on the basis of this φ , the collection of sets of the form

$$\gamma(n) = \{n \mid \{<\gamma, n, 1>\} \in \varphi\}$$

for each $\gamma \in \aleph_\tau$. Since $\varphi \in S(\varphi)$, and $S(\varphi)$ contains every set that can be constructed from φ ,¹⁶⁸ all of these $\gamma(n)$ exists as elements of $S(\varphi)$. What the definition of the $\gamma(n)$ sets does is put the elements of \aleph_τ (the gammas) into a one to one correspondence with subsets of natural numbers (all the n ’s paired with each γ in the generic set φ). The consequence of this is that, in $S(\varphi)$, the cardinality of the set of subsets of \aleph_0 is at least equal to \aleph_τ , *for any τ whatsoever*.¹⁶⁹ The generalized continuum hypothesis, that $|\wp(\aleph_n)| = \aleph_{n+1}$ is therefore false in $S(\varphi)$, for we have assumed τ to be greater than 1 and shown that $S(\varphi)$ entails the statement $|\wp(\aleph_0)| = \aleph_\tau$.

It is thus that the objectively indeterminate field of subjective activity can come to be determined through a course of subjective action, at least from the point of view of the projected completion of this course. What remains to be seen is how these structures come to be concretely instantiated according to Badiou’s doctrine of post-evental subjectivity.

IX. The Aftermath of the Event: On Fidelity, and the Negative Impetus of Subtraction (Lack and Genericity)

Les gerbes refusent mes liens. Dans cette infinie dissonance unanime, chaque épi,
chaque goutte de sang parle sa langue et va son chemin. La torche, qui éclaire et ferme le
gouffre, est elle-même un gouffre.

— Jacques Dupin, *Lichens*, verse 4.¹⁷⁰

We have caught a glimpse of the mathematics underlying Badiou's theory of the subjective truth-procedure. Their effect is to carve out the channels in being through which it is possible that truth may come to pass. The concept of the generic gives us the structure of truths' multiple-presentation, and the concept of forcing gives us the structure of the logical operations through which a subject of truth is capable of transforming the essential structure of the situation. These channels mark out the operational domain of the subject, which, as Sartre once said, "lives in the truth like a fish in the water" (VE, 3). For Sartre, however, this assertion is grounded in the thesis that "the essence of truth is the 'there is' [*il y a*] of 'there is being' (VE, 4), an assertion which makes truth the medium of *every* situation. For Sartre, truth is ultimately indistinct from presentation, or, more exactly, from consciousness; of course, truth may afterwards be suppressed in 'bad faith', but it is precisely the original identity of consciousness and truth that lets us see the suppression of the latter as self-deception. For Badiou, on the contrary, truth is an anomalous occurrence with which we are all too rarely graced, and there must be an intrinsic breakdown in the normal form of presentation in order for a new truth to become possible.

One aspect of the exceptional character of truths has already been made apparent in the exclusive disjunction that Badiou establishes between truths and knowledge, the latter being the subordination of multiplicity to predication, the former being the process of subtraction from predication through the tracing of a diagonal across the totality of legible predicates (the 'encyclopaedia of the situation'), leaving no domination untouched. But genericity is not, in

itself, a sufficient condition for the existence of a truth. To elevate the generic to the true, a supplement is needed. In order for a subjective truth procedure to begin, there must be a modification of presentation that effectively reproduces the structure of impersonal consciousness disclosed in Sartre's analyses. This is, of course, the event, in which presentation subtracts itself from the regime of ontological identity in the play of mirrors that constitutes an intervention. The effect of the intervention-event is a reflexive presentational multiplicity to which identity cannot be assigned, but which whose existence can be averred only in the immanence of its own occurrence, in the "phantom dyad" of event and intervening subject.

Following its initial irruption, it becomes necessary for the interventional subject to inscribe the event in the material reality of the situation in which it has occurred. The recognition of this necessity cannot be separated from the recognition of the event itself, for "the event is only possible if special procedures conserve the evental nature of its consequences" (BE, m20, 211/231). The subjectivating moment thus includes two aspects, one being the act of intervention, the other being the emergence of an *operator of fidelity*; this operator is what controls the inscription of the event's having-taken-place into the surface of the situation, by means of linking together a subset of the latter's elements in truth-procedure.¹⁷¹ In Badiou's notation, this operator is designated by the sign \sqsubset , such that an expression of the form $(\alpha \sqsubset e_x)$ indicates that the element α has been found to be positively connected to the event, and conversely, $\sim(\alpha \sqsubset e_x)$ indicates that α has been found to be unconnected to the event. At the limiting point of the completed truth procedure, $(\alpha \sqsubset e_x)$ translates more or less directly into $(\alpha \in \mathfrak{E})$. For the time being, however, the "the enquiries in which the provisional result of a fidelity is inscribed form a finite set" (BE, m23, 235/260); this set is a subset of \mathfrak{E} , which is by no means generic and which is perfectly discernible in S insofar as it is finite. Being a discernible subset of S , this finite assemblage is, in its being, an element of the 'state' of S (i.e., $\wp(S)$). Badiou

nevertheless insists that “the instantaneous state projection—which groups multiples *already* discerned as connected to the event into a part of the situation—is only a gross approximation of what the fidelity is capable of; in truth it is quite useless” (BE, m23, 235/260).

It is the subject’s relationship to the event and its impetus to inscription that causes the fidelity operator to have a subtly but fundamentally different structure than the set-theoretic epsilon. As Badiou points out, the finite set in which the provisional results of a fidelity are arrested by the state is insufficient to grasp its specific temporality—the procedural and futural dimension of the fidelity, by which it is compelled to range over the entire expanse of the situation, which, as a rule, is infinite (BE, m23, 235/260).¹⁷² This infinite expanse constitutes the factual support of the subject’s field of possibility, and is a necessary condition for the future genericity and truthfulness of the subject’s procedure. Nothing in this expanse’s *being*, however, entitles us to recognise it as anything other than the neutral facticity of the situation. Nothing on the ontological level establishes a significant connection between the subject and its uncertain future. “By consequence,” Badiou writes, “this infinite capacity is not effective, since at any moment its result allows itself to be projected by the state as a finite part” (BE, m23, 235/261). This gap between what the subject is (insofar as it *is*) and what it *has to be* (insofar as it is the subject of a truth) leads Badiou to characterise the subject’s fidelity in terms that are dramatically reminiscent of *Being and Nothingness*:

thought in its being—or according to being—a fidelity is a finite element of the state, a representation; thought in its non-being—as operation—a fidelity is an infinite procedure adjacent to presentation. A fidelity is thus always in non-existent excess over its being. Beneath itself, it exists; beyond itself, it inexists. (BE, m23, 235/261)

The distinctly Sartrean tenor of this passage is difficult to miss, even if it remains rather superficial at this point: the subject’s project—or procedure—necessarily outstrips its momentary facticity towards an infinite field of possibilities. This resonance becomes more compelling, however, once Badiou eliminates any recourse to intensional determination or potentiality that

could otherwise have been used to explain this excess. Echoing the Sartrean precepts that “[b]ehind the act there is neither potency nor ‘hexis’ nor virtue” (BN, in.I, xlvii/12), and that “[t]he freedom of the for-itself is always *engaged*; there is no question here of a freedom which could be undetermined and which would pre-exist its choice” (BN, IV.II, 479/524), Badiou insists that “[f]idelity must not be understood in any way as a capacity, a subjective quality, or a virtue. Fidelity is a situated operation which depends on the examination of situations. Fidelity is a functional relation to the event” (BE, m23, 233/258). Nor is it legitimate, for those fidelities which are in the service of truths, to suppose that an intensional determination comes to a fidelity *after* the event and in the midst of the situation, for as soon as a procedure submits itself to a discernible intension or essence, it abandons genericity.

At first sight, it would seem that we are driven to damn the faithful procedure to a state of perpetual chaos if its genericity is to be preserved. This would be to misunderstand the character of genericity, however. The generic is not mere noise. As we have seen, it is entirely possible to rigorously constrain the formation of a generic subset in such a way that it is rich in certain kinds of information. In example given in the previous chapter, a generic set was conditioned so as to provide all that was needed to establish a precise one-to-one correspondence between $\wp(\aleph_0)$ and \aleph_τ ; it is inconceivable that this should be possible were the generic subset in question created as a purely chaotic stream of multiplicities. It is entirely consistent to suggest that the path of a subject may be both generic and intelligible, and that it be at every individual moment capable of supporting predicative determination, but that it be, in its totality, undetermined by any predicate.

The subject of fidelity’s “non-existent excess” over its finite facticity is an effect not of predication or essence, but of an “interior transcendence” (LS, 244) immanent to the subject’s existence. There is a certain dimension of what Sartre calls *ekstasis* or *ek-sistence* in the Badiouian subject. The subject, which is a finite subset of a truth, is *outside what it is*: “The

subject is ‘between’ the terms that the procedure groups together” (BE, m35, 396/434). The outstripping of *essentia* by *existentia* is anticipated by its evasion through *ek-sistence*. It remains for us to determine this *ek-sistence* more precisely. To do this, we will return to Sartre.



Fidelity is the Badiouian name for freedom, and it is not difficult to find, in the former, the fundamental structures that Sartre’s analyses disclosed in the latter. Doing so, moreover, will finally allow us to elucidate Badiou’s theory of the event without falling into the pitfalls encountered earlier in his meditations on event and intervention. This approach also promises to shed further light on the riddle of the evental void that we raised in Chapter VI. In fact, let us begin with this. The apparent isomorphism that we found between intervention/event and the structure of non-positional (self)consciousness led us to hypothesise that the event, which falls outside of the ontological regime of identity, is individuated only by means of the intervention from which it cannot dissociate itself and into which it cannot be completely collapsed.¹⁷³ If Badiou’s statement that the event convokes the void follows from this, then it is because the difference that takes place between event and intervention, without positive extensional or intensional differentia, is perfectly analogous to the nothingness that Sartre finds at the heart of consciousness. If this analogy is rigorous, then we should expect that there would emerge from the event certain structures bearing a strong similarity to those uncovered by Sartre. We should expect, for example, that the nothingness at the heart of the event and its intervention would be apprehended as lack, and that the event would thereby exhibit an imperative force on the subject insofar as this subject has its origins in the intervention.¹⁷⁴ In other words, we should expect to find, among the metaphysical consequences of the event, both value and possibility *qua* possibility (as opposed to possibilities determined merely in terms of their factual support).

My hypothesis is that the invocation of lack is precisely what we should understand by Badiou's statement that "the event is only possible if special procedures conserve the evental nature of its consequences" (BE, m20, 211/233). The possibility of the event is referred to procedures that may be carried out only after it has occurred. One cannot claim that the event has taken place without pledging oneself to the project that will enable the event *to have taken place*. To intervene in the existence of an event is therefore to apprehend the event itself as having the form of a promise, a mandate yet to be fulfilled, or an *existing* standing in relation to a *lacking*, and under the sidereal gravity of the *lacked*. We find precisely this triadic structure invoked in Badiou's text itself—in the final meditation, for instance, where he writes that "[t]he 'there is' of the subject is the coming-to-being of the event, via the ideal occurrence of a truth, in its finite modalities" (BE, m37, 434/474). The subject is a local configuration of a triad of lack, with the event as existing ("coming-to-be"), the "ideal occurrence of a truth" as lacked, and the "finite modalities" that make up the future facticity of the subject and the adumbrated series of its *lacking*.

Pursuing this analogy, the relation of fidelity ($\alpha \sqsubset e_x$) may be read as " α is lacking to e_x ." The *existing-lacking* axis would be the character of the transcendence by which fidelity is in non-existent excess over its results. It is the *reason* why the set $\{\alpha \mid \alpha \sqsubset e_x \text{ at time } t\}$ is inadequate to the fidelity that it arrests, and why it is true that a subject exists "between the terms" that it gathers into its procedure: Insofar as fidelity is a relation of lack, the subject exists "at a distance from itself" (BN, II.IV, 96/133). This analogy also permits us to make sense of Badiou's intuition that the structure of the event ought to release to us a specific dimension temporality, insofar as the latter is "not coextensive with structure, not the *sensible form of the Law*" (BE, m20, 210/232), and to do so without having recourse to the mythical underworld of evental duplicity. The form of a unique temporality specific to the event is rendered intelligible, not by

positing a *second event* that would antedate the first—we would have to *presuppose* temporality in order to establish this—but by following the consequences of the duality that is already immanent to the single event under consideration. Insofar as the event-subject constitutes a phantom dyad, and constitutes itself as lack, it effectively ‘possibilizes’ a future: the elements of the situation seen anew as elements that may or may not be positively connected to the event. The event as lack constitutes itself in determining its generic trajectory as a project, bearing a dimension of futuricity, which is “not coextensive with structure, not the *sensible form of the Law*,” insofar as its composition escapes both law and structure in virtue of genericity.¹⁷⁵ In this way existence, in excess of being, supplements and supplants essence. The futural dimension of the subject, for its part, can then be given stricter determination through the ontological operation of forcing, by which the potential consequences of the subject’s procedure in $S(\varnothing)$ are linked to the generic trajectory that it is capable of anticipating (\varnothing).

If we ask what the lacking is lacking *for*, then we are first led to the conclusion that this could only be the being of the event (“if the event is to be possible...”), or the event *as* being—a structure which never comes to pass insofar as the event *qua* event is outside ontology: the only ontologically acceptable portion of the event is its site. On the other hand, it would seem reasonable to say that that for which the lacking is lacking is the complete generic set \varnothing . This is not enough, however. What is attested to as lacked in the event is the *truth*, which cannot be simply identified with \varnothing insofar as \varnothing is only the *being of the truth*. Truth, *qua* truth, cannot be adequately conceived on the sheer basis of its generic support, but must at the same time be seized in its post-evental character.¹⁷⁶ The truth is the (lacked) point at which being and event converge, and the subject is the bearer of this convergence.

The convergence of being and event in truth may be understood as the meaning of Badiou’s remark that the signifier ‘ \varnothing ’, “to which, in the beginning, nothing which is presented in

the fundamental situation corresponds—is the ontological transcription of the supernumerary nomination of the event” (BE, m33, 356/392), which was earlier marked by the symbol ‘ e_x ’. One of the interesting rhetorical aspects of Badiou’s discussion of the generic consists of a subtle metonymy that he draws between genericity (*i.e.* indiscernibility) and being *qua* being:

What could this ‘one’ be which—subtracted from language and constituted from the point of the eventual ultra-one—is indiscernible? Since this part has no particular expressible property, its entire being resides in this: it is a part, which is to say it is composed of multiples effectively presented in the situation. An indiscernible *inclusion*—and such, in short, is a truth—has no other ‘property’ than that of referring to *belonging*. This part is anonymously that which has no other mark apart from arising from presentation, apart from being composed of terms which have nothing in common that could be remarked, save belonging to *this* situation; which, strictly speaking, is its being, *qua* being. But as for this ‘property’—*being*; quite simply—it is clear that it is shared by *all* the terms of the situation, and that it is coexistent with every part which groups together terms. Consequently, the indiscernible part, by definition, solely possesses the ‘properties’ of any part whatsoever. It is rightfully declared *generic*, because, if one wishes to qualify it, all one can say is that its elements *are*. The part thus belongs to the supreme genre, the genre of the being of the situation as such—since *in* a situation ‘being’ and ‘being-counted-as-one-in-the-situation’ are one and the same thing. (BE, m31, 338-9/373-4)⁷⁷

This metonymy, which carries through genericity carries us to the idea of being *qua* being, in turn opens onto a metaphorical invocation of the void, as both being-in-itself and as that to which the nothingness of the event is joined in suture. The event’s lack of being *qua* being can be answered by the generic insofar as the generic as a whole expresses nothing other than the being-in-situation of the elements it assembles. By way of the generic invocation of pure multiplicity, the void indirectly answers to the nothingness produced in the event, insofar as the former is the proper name of that which finds its truth in a procedure commanded by latter. It is through this figurative detour that we can see truth taking the position of the lacked, and the subject becoming the bearer of the conjunction between being and event. In the unbinding of presentation from structure, which comes about in the generic splintering of language’s domination over thought, and which occasions the infinite task of closing the impasse of being (the indeterminate gap between \aleph_n and $|\wp(\aleph_n)|$), the event—“incandescent non-being of an existence” (BE, m17,

183/204)—becomes a symbol of the void itself. By chance or design, the same can be read in Dupin's verse, where he writes:

The sheaves refuse my bonds. In this infinite, unanimous dissonance, each ear of corn, each drop of blood, speaks its language and goes its way. The torch, which lights the abyss, which seals it up, is itself an abyss.¹⁷⁸

For Badiou, it is precisely insofar as the generic speaks only of being that it is indifferent to this dissonance; only in this way does it become a bearer of the universal. Sartre's radical humanism aimed at universality by making its incision at a transcendental moment anterior to the constitution of the situation as such—at the moment in which the primary triad of lack is instantiated, value is determined, and the ground is laid for the modal configuration of the situation's instrumental complex. By contrast, Badiou seeks to ensure universality not only through the anteriority of the subject's acts but in their posterity as well, and the entire mobilisation of genericity aims at thinking the outstripping of essence by existence—the precondition for a universalism indebted to neither God nor Man—as a material *result* and not only an *a priori* condition.



At this point our analogy begins to lose its grip, however. Truth, in the last analysis, has a structure that resists being identified with value, no matter how closely the two seem to approach one another. The vertical and impossible transcendence of value in *Being and Nothingness* is quite foreign to the idea of truth in *Being and Event*, which resists even an allegorical identification with a crossed-out God or the One. "Truth," Badiou insists, "is not of the order of something which stands above the givenness of experience; it proceeds or insists within experience as a singular figure of immanence" (TW, 121). While truths, insofar as they are transfinite, do transcend every finite instance of the fidelities in their service, this transcendence is entirely 'horizontal', as Sartre would say. The truth transcends the subject *in the direction of the lacking, not in the direction of the lacked*, whose function it nonetheless replicates; it is of the

same ontological kind as its finite fragments. As such, the integral ontological impossibility that Sartre attributes to value does not burden truth, even if the finitude of human endeavours prevent one from completing the infinitely long generic procedure in the situation where it has begun. The truth is not impossible in principle; rather, it is precisely as a matter of principle that its possible completion is postulated, much in the same sense that Kant finds it necessary to postulate the immortality of the soul. The condition of genericity, like the holiness at which the Kantian subject aims, is “a perfection of which no rational being in the sensible world is capable of at any moment in his existence. Since, nevertheless, it is required as practically necessary,” if the procedure is to be affirmed as a *truth*, “it can only be found in a *progress in infinitum* towards that perfect accordance [sc. in this case: discordance] with the Law.”¹⁷⁹ This is quite different from saying that a truth is an intrinsically paradoxical entity—in fact, this notion is so far from Badiou’s theory of truth that he responds to this Kantian exigency by conceiving of the subject as effectively *immortal*, outstripping and exceeding the inevitably mortal individuals that contribute to its composition.



We have now come to the point where Badiou’s theory of the subject begins to diverge sharply from Sartre’s. These divergences, however, are extremely nuanced and complex. In order to understand the significance of the homologies that we have so far traced between the two philosophies, we must examine these divergences attentively. Fortunately, it turns out that Badiou himself has already done no small measure of the work this examination requires. On the tenth anniversary of Sartre’s death, the editors of *Les temps modernes* broke with their usual tradition of keeping more or less silent on the subject of their founder’s work and published a double-issue tribute to his intellectual legacy. The first article in this publication is a short piece by Alain Badiou, entitled “Saissement, desaisie, fidélité.” The axis on which this piece turns is

one of Sartre's celebrated definitions of consciousness, which Badiou credits as having crystallised the entire "philosophical awakening" of his adolescence: "Consciousness is a being such that in its being, its being is in question in so far as this being implies a being other than itself" (SDF, 14).¹⁸⁰ This formula, Badiou writes,

fixes a double maxim, of which I should say that it continues to organize my thinking:

- First of all, the Ego, or interiority, is deprived of all interest, and so odious, if it does not bear an effect of sense of which the measure can only be the world in its entirety, the totality of that which is disposed when thought seizes it in its disposition. This can be stated: psychology is the enemy of thought.
- Secondly, the entire world, such as it is disposed, is of interest only insofar as it is taken up and utilized in the subjective prescription of a project whose measure shall be its extension. The world must, literally, be put into question. This can be stated: pragmatic empiricism, compromise, and the "we must cultivate our garden" [*il faut cultiver notre jardin*], are also enemies of thought. (SDF, 14)

There is no difficulty in recognising the organising force of this double maxim in both of the philosophies that we have studied in this work. No more in Badiou's work than in Sartre's is the commonsensical identification of subject and ego given any countenance, and no less than his predecessor does Badiou seek to think the situation with the sole end of asking how it can be transformed through the freedom of a subjective project. "And yet," we are told,

if this conviction remains, which makes of the subject that which detaches itself from being so that there may be a truth of being, the way in which this conviction came to articulate itself forced me renounce Sartre's formula, piece by piece. I will hence say that my trajectory of thought may be perceived as the paradoxical combination of a somewhat *energetic* fidelity to the Sartrean message, and of the formal decomposition [*mise en pièces*] of the dialectical schema that supports this message. (SDF, 15)

The remainder of the text goes on to illustrate this decomposition, before returning to an affirmation of the "energetic fidelity" that sustains this very operation. Badiou proceeds one phrase at a time. He begins by distancing his philosophy from any notion of consciousness, and then moves through the other components of Sartre's formula, reiterating his decision to declare mathematics identical to ontology when he comes to the word 'being', and marking his distance from the hermeneutic tradition when he reaches 'question'. When he reaches consciousness' implication of "a being other than itself," he offers a brief and condensed critique of

intentionality. It is with this critique that we will begin, reversing the order of Badiou's own exposition. Afterwards, we will turn to Badiou's remarks on the concept of consciousness itself, where his distance from Sartrean thought is most radical.



Quite correctly, Badiou reads Sartre's adjunction to Heidegger's formula—the adjunction which subordinates the question of the being of consciousness to the a being other that is other than it—as an invocation of the principle of intentionality. In a dense and significant paragraph bearing on this concept, Badiou writes the following:

What grounds my reticence with regard to the theme of intentionality is that it continues to demand the category of the object, as a correlate of conscious aims, and more generally it demands the dialectic of subject and object, of which the Sartrean motif of the in-itself and the for-itself is a fine projection. I defend a doctrine of the subject *without object*, of the subject as an evanescent point of a procedure originating in an eventual supplement without a motive.

There is not, in my eyes, an other-being of the subject, unless it is the situation of which a truth is a truth. I have doubtlessly paid my debt by taking from Sartre the theme, which he diversified with a confounding virtuosity, of the 'situation'. But this apparent Other of the subject is, for me but also for Sartre although in an altogether different way, the Same, because truth realises in an immanent fashion the generic-being, the commonplace, the indiscernible of the situation itself. (SDF, 20)

This passage can be broken down into two main components (as I have already done typographically), one addressing the place of the *object* in the two philosophies of the subject, and the other addressing the relation between the subject and the situation. Let us approach this passage one piece at a time.

To begin, I can only say that insofar as Badiou sees in the Sartrean axiom of intentionality little more than a tethering of the subject to its objects, he has missed precisely what is most interesting and important in Sartre's adoption of this principle. To beat a long-dead horse, what this axiom first and foremost accomplishes is to determine consciousness extensionally, and thus determine the 'there is' as multiplicity. The figure of the object is altogether secondary to that of the extension—this is the substance of Sartre's dispute with Husserl in the Preface to *Being and Nothingness*: In view of the celebrated reduction of the entity

to the series of appearances which manifest it, object, like essence, becomes a name for a certain kind of intensionally determined, infinite apparitional series. If objectivity is the primary mode of transcendence, the primary mode by which the data of consciousness signal beyond the immanence of consciousness itself, then we will find ourselves forever unable to step outside of idealism, since objectivity in the last analysis will rest on an intensional content of consciousness (namely, a lack). Sartre argues that the true virtue of the axiom of intentionality is that it determines the extension as transcendent *prior* to any objectification; it is in every case already outside consciousness, which is an event without immanence. Moreover, the initial modality in which the transcendence of the extension is given is that of pre-objective and transphenomenal being-in-itself, its contingent, enstatic and identical inconsistency, wherein absolute opacity and absolute pulverisation become indistinct, as Badiou elsewhere recognises.¹⁸¹ It is not clear whether or not Badiou would identify the ‘objects’ that the theory of the subject does without in a manner akin to Sartre and Husserl. Prior to *Logiques des Mondes*, where a novel and sophisticated theory of objects is developed, the term recurs frequently in Badiou’s texts without anything resembling an explicit definition.¹⁸² Badiou nevertheless declares that the task of constructing a theory of the objectless subject has as its condition of possibility the univocal determination of the existent as multiplicity. On October 24th, 1987, Badiou began a seminar at Paris University VIII with an exemplary restatement of this thesis, which had been latent in his work from the very beginning:

With Cantor, mathematics breaks with the category of object. The multiple in itself is not an object; it is the general form of the exposition of being. Mathematics [is the] anti-idealist exercise *par excellence*. The knot of idealism is the category of the object not that of subject. The support of Kantian idealism is not the transcendental subject but the category of object. For centuries, mathematics have been paradigmatic for idealism, they have nourished idealism because they provide a pure model of the object. The Cantorian revolution is materialist. A materialist, and thus objectless, theory of the subject is necessarily of the school of post-Cantorian mathematics.¹⁸³

Insofar as the same break is instituted by Sartre in his reinterpretation of the Husserlian doctrine of intentionality, upon which he formulates a theory of the subject as a multiple-configuration indifferent to any intensional determinant (and thus to objectivity as such), we might wish to extend Badiou's thesis: a materialist, and thus objectless, theory of the subject is also constructible on the basis of post-Sartrean phenomenology—a thesis to which Sartre himself subscribed, as Hazel Barnes has observed in a study devoted to this theme.¹⁸⁴ This structural kinship comes as no surprise in view of the homology earlier established between the intentionality of consciousness and the extensionality of sets.

The next part of the above-cited passage on intentionality from Badiou's article bears on the subject and its situation, and the relations of otherness or sameness that the two philosophers establish between the two. On this point, Badiou is correct on both counts: First, that the only 'other' of the subject in his philosophy is the situation of which it is the subject, if 'other' is to be understood in a sense analogous to its occurrence in Sartre's formula. Second, that this situation is, in another sense, 'the Same' as the subject; a thesis which also rings true in the context of Sartre's philosophy, but in a quite different sense. The similarities and differences evoked here can be expressed very simply. For Sartre, the being of the situation is radically exterior to the subject, which, insofar as it is a modality of consciousness, is nevertheless the principle of consistency of the situation, insofar as the latter is a multiplicity ($x \in situation \leftrightarrow x \in subject$). For Badiou, the subject inheres in the being of the situation, such that in its own being, it is a multiplicity that is properly *included* in the situation ($subject \subset situation$). If the Sartrean subject can be said to be a modality of the situation's ephemeral consistency, the Badiouian subject emerges as a configuration of its latent *inconsistency*.

So far as the concept of consciousness is concerned, Badiou claims that he “no longer support[s] its philosophical pertinence” (SDF, 18). While recognising the crucial and “glorious” role that the notion of consciousness has enjoyed through centuries of philosophical thought, Badiou now insists that if it has a role to play at all, this role is not philosophical, but strictly political, if not also psychoanalytical (SDF, 18).¹⁸⁵ The subject, for Badiou, is not a consciousness, but neither is it a structure of the Freudian *unconscious*. It is something else: the structural support of the passage of a truth, the transient *hupokeimenon* of a truth procedure.

Let us take note of a certain curiosity, however, and ask what it might mean for Badiou to support the pertinence of the concept of consciousness in politics and psychoanalysis while discharging it from the field of philosophy. Both of these non-philosophical domains are, in Badiou’s eyes, possible modalities of truth—politics in itself is explicitly recognised as a genus of generic procedure, or truth condition, while psychoanalysis is said to be a particular species of the ‘amorous’ genus of truths (love is a truth procedure). In addition to these genera, there exist two others, namely art and science. Beyond the general formal account sketched out in *Being and Event* and recapitulated above, each of these truth conditions has a configuration that is particular to it; there is no need to enter into the details here. The point that interests me here is that consciousness is recognised to be at least a *dimension* of two of the four procedures, two of the four generic configurations that a subject may take. If Badiou maintains that the concept of consciousness is operative within the vicissitudes of these truth procedures while being impertinent in philosophy, this is possible only insofar as he “no longer believe[s] [...] in the happy transitivity between philosophy and politics” (SDF, 18), or between philosophy and any other truth procedure for that matter. What this means is that the modality of thinking that is proper to each truth procedure cannot be absorbed into philosophical discourse. Philosophical thought, insofar as it is philosophical, cannot produce truths—it yields neither art, nor science,

nor political action, nor love. Its task is only to produce a conceptual space in which the disparate truths of its epoch can be thought together, not with respect to their interior vicissitudes, but with respect to their intelligible effects. Philosophy must remain in a relation of intimate exteriority to these conditions, avoiding both conflation and neglect.¹⁸⁶

In contrast to ‘consciousness’, Badiou affirms that ‘subject’ names a properly philosophical concept. It functions as a point through which the activity of an extra-philosophical truth procedure may offer something of philosophical intelligibility, without altogether fusing with this donation. In view of this, it is sensible to ask to what extent the apparatus that we have constructed here, which bridges the concepts of subject and consciousness, is capable of mediating, or in danger of suturing, philosophy and its conditions. It seems clear enough at present that our apparatus is as of yet in no shape to perform either function, though whether has the potential to be elaborated accordingly is yet unknown. We will leave this question open for now.

Lest we lose sight of the heterogeneity of Badiou’s concept of the subject, however, let us note also its distance from the concept of the individual. Not every individual is a subject, nor is every subject an individual. The first of these assertions proceeds from Badiou’s iconoclastic thesis on the *rarity* of subjects. Once we have hinged the subject’s emergence on an aleatory ontological dysfunction, we no longer have any grounds for assuming that there will always be some subjects—much less that every individual is, or even supports, a subject. The second assertion—that not every subject is an individual—is no less unusual or interesting a feature of Badiou’s metaphysics. Badiou nicely illustrates this thesis in the *Ethics* with a series of examples drawn from three of the four truth conditions, all of which bear repeating:

the subject induced by fidelity to an amorous encounter, the subject of love, *is not* the ‘loving’ subject described by the classical moralists. For this kind of psychological subject falls within the province of human nature, within the logic of passion, whereas what I am talking about has not ‘natural’ pre-existence. The lovers as such enter into the composition of *one* loving subject, who *exceeds* them both.¹⁸⁷

In the same way, the subject of a revolutionary politics is not the individual militant—any more, by the way, than it is chimera of the class-subject. It is a singular production, which has taken different names (sometimes ‘Party’, sometimes not). To be sure, the militant enters into the composition of this subject, but once again it exceeds him (it is precisely this excess that makes it come to pass as immortal).

Or again, the subject of an artistic production is not the artist (the ‘genius’, etc.). In fact, the subject-points of art are the works of art. And the artist enters into the composition of these subjects (these works are ‘his’), without our being able in any sense to reduce them to ‘him’ (and besides, which ‘him’ would this be?).¹⁸⁸

What emerges in these brief sketches is a vision of the subject that is as removed from psychologism as it is from phenomenology, and, consequently, from Sartrean existentialism.

Badiou’s separation of the subject from individual consciousness divides him from Sartre more than anything else, and does so at the precise point where the two philosophies are most kindred—namely, where one encounters their shared insight that to speak of a subject only has meaning insofar as the subject transcends *what there is*. It is in this capacity that both maintain, with axiomatic fortitude, the truth of Sartre’s maxim: “Insofar as human reality is act, it can be conceived only as a rupture with the given, in its being” (BN, IV.I.I, 478/523*).

For Sartre, however, this transcendence is internal to the givenness of the given itself. Presentation is uniquely thought as consciousness, which, owing to structural features that make it the perfect analogue of the Badiouian *event*, perpetually instigates the formation of the subject. The structures of event, presentation and subject are, by contrast, altogether separated in Badiou’s thought, as if by centrifuge. It is in this initial, metaphysical separation of these three structures from one another, that Badiou’s break with Sartre is founded. To no small measure, this separation begins with Badiou’s baptism of Zermelo-Fraenkel set theory as ontology. The absolute illumination of the in-itself that this baptism initiates is not what marks its most significant departure from Sartre; this is marked most significantly by the fact that this decision purges from ordinary presentation the immanent non-identity of presentation that is constitutive of the subject—the Badiouian subject no less than its Sartrean counterpart. This purge is reflected in Badiou’s decision to place the event outside of mathematical ontology—a placement

that is determined in the same movement of thought that identifies ontology and set theory. It is important that we grasp this point fully: what *Being and Event* proposes is a theory of the subject supplementary to ontology, a theory of the exceptional subject. This stands in an essential contrast with the ambit of *Being and Nothingness*, which is to sow the structural possibility of subjectivisation into the very fabric of presentation, and to make freedom an immanent dimension of the 'there is', thereby determining "for consciousness the permanent possibility of effecting a rupture with its own past" (BN, IV.I.I, 436/480). It is this permanence that Badiou contests, arguing instead for the genuine *rarity* of this possibility. The philosophical concept that supports this rarity is the disjunction between being and event.

Parenthetically, we may note that this reconstruction is borne out by some of Badiou's remarks in *Logiques des mondes*, in a passage where he positions both his work and Deleuze's with respect to Sartre's. "Deleuze," Badiou writes,

always paid tribute to Sartre as the figure who, during the thirties and forties, woke French philosophy from its academic slumbers. He considered the 1937 article, 'The Transcendence of the Ego', the origin of everything. Why? Because, in this text, Sartre proposes the idea—I am citing Deleuze—of 'an impersonal transcendental field, having the form neither of a personal synthetic consciousness nor subjective identity—the subject, to the contrary, always being constituted.' [...]

Deleuze remarked also that Sartre had been prevented from thinking all of the consequences of this idea because he had attached the impersonal field to a (self)-consciousness. This is altogether exact. We could also say: Sartre continued to believe in an auto-unification of the transcendental. He did not expose the subject to the hazard [l'*aléa*] of a pure Outside. Now, one of the names of the Outside is 'event'.¹⁸⁹

Considered extrinsically (with respect to its place), 'event' is the name of the exteriority of the principle of auto-unification from the transcendental field, which is in turn another name of the 'there is'. Considered intrinsically (with respect to its structure), 'event' names the structures on which subjectivisation is contingent; in this light, "to expose the subject to the hazard of a pure Outside" means to de-eventuate the transcendental field. The centrifugal operation we spoke of earlier is encapsulated here. The effect of this operation is, of course, to untangle the subject

from the 'there is', the result being that "the idea that there have always been subjects," is "a debt [to Descartes] whose account must be closed" (BE, m37, 434/474*).¹⁹⁰

**X. In Conclusion:
What I have done,
And what I have failed to do.**

The general approach taken in this essay could broadly be called ‘structural’, to use a term that seems so broad as to be trivial in this context. Here, I mean it only to indicate that the centre of gravity of this essay consisted in the exposition of formal homologies and points of contrast, and, accordingly, was founded upon a reduction of the key terms of the two philosophies to homogeneous structures whenever possible. Only by doing something along these lines was it possible to establish any sort of commensurability between the two theories that went deeper than what is readily and superficially apparent. Yet whenever this sort of approach is taken, there is something of a trade-off. In seeking to render certain aspects of the subject matter more or less transparent, other no less significant details and particularities become lost. Consequently, accidentally or purposefully, one stifles the possibility of quite different structures coming into view. This is indeed a trade-off, in that there is a definite exchange of one field of intelligibility for another: if we failed to make certain details disappear, and render them transparent and atmospheric, then it would be difficult to see past them—the forest would be hidden by the trees. If we stayed with what is manifestly more ‘descriptive’ in Sartre’s work, for example, it would be difficult to see how we would establish any clear points of contact to the ascetic mathematics of *Being and Event*.

One of the consequences of my selective focus is that I have symmetrically omitted any accounts of what Badiou has come to call, in contrast to the ‘metaphysical’ doctrine deployed in *Being and Event*, the *physics* of the subject: the theory of corporeality, of the body as such. Accordingly, my exposition remains ultimately incomplete, if not rather hollow. In the end, it is not enough to disclose the fields of possibility and their formal structure; it is necessary to show how subjects may concretely realise these possibilities. This omission was by design. It was not

motivated by considerations of the relative importance of ‘physics’ to ‘metaphysics’, but, above all else, by considerations of my own (physical) constraints. For one, to adequately investigate this dimension of Badiou and Sartre’s thought would have obliged me to work through a great deal more material, which in all justice would have included both Sartre’s *Critique of Dialectical Reason* and Badiou’s *Logiques des mondes*: Only in the latter does Badiou offer any theory of corporeality at all, while any engagement with the theory of the body in *Being and Nothingness* would, out of respect for completeness, require me to take into account its rational development in the *Critique*. The chief difficulty here was that *Logique des mondes* was only published as I was nearing the end of this (already quite lengthy) project.

Even more dramatic—if only because it bears on one of the few well-known aspects of *Being and Nothingness*—is the total omission of Sartre’s theory of the Other from this work, accompanied by a comparable omission Badiou’s treatment of the same. Likewise absent is any account of Sartre’s painstakingly articulated theory of practical ensembles, along with a total neglect of Badiou’s ‘metapolitical’ thought. It is quite strange, to the point of appearing careless or irresponsible, to omit, from a work that foregrounds the concept of multiplicity, any mention of the problems that arise when one is confronted with a multiplicity of consciousnesses or multiple participants in a truth. The only excuse for this, again, is economical.¹⁹¹ Given time and space, I would have extended the study in this direction with great interest. Nevertheless, others have already taken up this particular avenue of research. In England, Nina Power is steadily developing a remarkable series of works bearing directly on the problem of humanism between Sartre’s *Critiques* and Badiou’s thought from *Théorie du sujet* to his most recent works. In France, Eustache Kouvelakis has also begun some intriguing inquiries into differences and kinships between the theory of multiplicities developed in the *Critique* and Badiou’s set-theoretic political metaphysics.

Rather than dwelling on omissions, let us see what can be said of the measure of work that has been undertaken here, when considered as a totality.



There are two ways of understanding what we have done here. On the one hand, it could be said that we have attempted a synthesis of Badiou and Sartre with respect to the concept of the subject, a synthesis whose conditions of possibility were arduously established throughout the preceding chapters of this essay. The upshot of this synthesis could be understood as a crucial supplement to Badiou's constructions, with the purpose of putting a bit more flesh on the bones of his formal theory of the subject (even if this 'flesh' consisted only in differentiation and lack). This project already includes another, which would be the task of underpinning Sartrean existentialism with a mathematical ontology more robust than the tripartite axiomatic of the in-itself which leaves being ultimately unintelligible. The work of synthesis can be read either way.

Yet another interpretation suggests itself just as readily. In this view, what we have done, however partially, is to establish a relation between the philosophical systems of Sartre and Badiou that is effectively akin to the relation between a theory and its model. There is no question of producing a fusion or synthesis between the two systems on this account, but of producing a mediating apparatus (a function of interpretation) that allows us to shed light on both by reconstructing the one as a support for the formalised statements of the other. On this account, Badiou's reluctance to supply any semantic depth to his formalistic and elliptical accounts of the subject, including his obscure invocation of its necessary but unexplained dimension of transcendence, is to be understood as a calculated neutrality. It is plausible that this neutrality is motivated by Badiou's ambition to give his formulations the greatest possible generality, and the most diverse class of models, that could be supplied without distorting its essential message.

Indeed, this is the intention that Badiou appears to communicate at the outset of *Being and Event* where he writes that

The categories that this book deploys, from the pure multiple to the subject, constitute the general order of a thought such that it can be *practiced* across the entirety of the contemporary system of reference. These categories are available for the service of scientific procedures just as they are for those of analysis or politics. They attempt to organize an abstract vision of the requirements of the epoch. (BE, 4/10)

If this is the intended function of *Being and Event*, then it would be pointless to attack it on the grounds of being excessively abstract, elliptical or formalistic, so long as this syntactic minimalism does not leave the theory so sparse that it offers no foothold for ‘practice’ or interpretation. This approach allows us to explain the curiously withdrawn and metatheoretical position that Badiou often takes when discussing some of his system’s most crucial concepts. In discussing the relation between the evental intervention and the operator of fidelity, for instance, Badiou writes that

Much light would be shed upon the history of philosophy if one took as one’s guiding thread such a conception of the subject, at the furthest remove from any psychology—the subject as what designates the *junction* of an intervention and a rule of faithful connection. The hypothesis that I propose is that even in the absence of an explicit concept of the subject, a philosophical system (except perhaps those of Aristotle and Hegel) will always possess, as its keystone, a theoretical proposition concerning this junction. (BE, m24, 239/264)

Upon its first encounter, the perplexing thing about Badiou’s meditation on fidelity is that, as crucial as he claims the proposition conjoining fidelity operator and event to be, no such proposition can be found in *Being and Event*. This omission alone should give us pause. Badiou does not deny that it is a question requiring an answer; he even says that “this is the problem which remains for philosophy, once the famous interrogation of being-qua-being has been removed (to be treated within mathematics)” (BE, m24, 239/264).

Much is clarified if we take the omission of such a proposition in *Being and Event* to be by design, and to be part and parcel with the general ambition of this book to serve as an axiomatic framework for philosophy that is intentionally emptied of as much material content as

is possible. From this perspective, for the purposes of *Being and Event*, it simply does not matter *how* the operator of fidelity emerges from the event, so long as it is such that, from the perspective of its infinite completion, the set $\{\alpha \mid \alpha \sqsubset e_x\}$ will not have fallen under an encyclopedic determinant; that is, so long as $\{\alpha \mid \alpha \sqsubset e_x\}$'s existence will have preceded its essence, and by that right be generic.

It is therefore clear that we should not be in any way surprised if we find, in Sartrean philosophy, “a theoretical proposition concerning [the] junction” between event and fidelity, in the form of the theory of lack, which connects the nihilating event of consciousness to the “concatenation of freedom in situation” that takes place in the articulation of the project ensemble.¹⁹² This is just what we should expect. After all, Badiou hypothesises that some such proposition may be found to be the “keystone” of virtually *any* philosophical system (with the possible exclusion of Hegel and Aristotle). It is neither mysterious nor surprising that we can verify this hypothesis in a system as genealogically close to Badiou's as Sartre's is. What is perhaps more significant is that Sartre satisfies Badiou's hypothesis in a way that is intrinsically compatible with the axiomatic framework to which Badiou formally submits it: the conjunction of event and fidelity, or consciousness and freedom, though the theory of lack determines the field of the subject as “an infinity of possibilities that cannot be contained in one formula” (BN, II.II.I, 129/164), and which cannot be derived from the factual starting point of the subject itself. This conception of the subject is as affirmation of genericity before the letter, an affirmation which is, as Badiou himself has recognised and as we have demonstrated, one of the crucial meanings of existentialism's defining principle: that the existence of the subject precedes its essence.

This principle is precisely what makes it possible to construct a Sartrean interpretation of Badiou's theory of the subject. It not only leads us to articulate the subject in such a way that it is

conceptually compatible with genericity, but prompts us to interpret the subjacent ontology of consciousness in terms of multiplicity, insofar as Sartre's insistence on the precedence of essence by existence allows itself to be understood as the precedence of intension by extension—as was obliquely suggested by Heidegger in his scathing dismissal of Sartre's project. If we had not begun by reformulating the Sartrean theory of consciousness through the concept of multiplicity, the claim that the Sartrean subject is conceptually compatible with the idea of genericity would scarcely have been intelligible.¹⁹³ Once we have decided to approach consciousness in terms of its multiple-structure, moreover, we see at once that it is formally quite different from the set—and therefore, from presentation as Badiou conceives it—yet altogether homologous with Badiou's conception of the event. This homology, finally, indicated to us the modality that the conjunction between event and fidelity would take in our Sartrean model: it would be understood as the very principle of lack that carries us from the phantom dyad of the consciousness-event to the concatenation of projects. This connection, moreover, allowed us to discern certain relations between what could be called the metaphysics and the ethics of the subject in Badiou's theory, which were not immediately evident in its formal syntax.

To view the connection that we have made between Sartre and Badiou as being roughly semantico-syntactic in structure rather than as one of supplementation or synthesis allows us to avoid obscuring one of the more ambiguous virtues of Badiou's project, which is precisely its formal minimalism. If we were to see our work as welding Sartrean structures directly onto the scaffolding of Badiou's apparatus, we would risk foreclosing in advance its range of application, and frustrating Badiou's aspiration to grant this apparatus a maximal range of deployment over the practical and theoretical configurations offered by our epoch. Nevertheless, we have not left the formal structure of Badiou's theory entirely unmodified throughout the course of this project. We noted that on at least one point, his apparatus calls for correction. The formal structure of

evental intervention that Badiou proposes is flawed, insofar as the solution that he proposes to the joint problems of the event's circularity and the subject's spontaneity results in a regress that either trails off indefinitely and solves nothing, or else terminates in a sort of *Ur-event* that reproduces the problem it was intended to solve. At this point, something of a graft from Sartrean ontology was performed. Insofar as the event showed itself to be homologous to consciousness, if only in terms of its presentational structure, it seemed reasonable to draw the same series of consequences from its immediate structure as Sartre drew in his analysis of the for-itself. It then appeared that Badiou's difficulties stemmed from insufficiently recognising the mode of individuation that the formal theory of the event implied, a mode which avoids both the pitfalls with which he was concerned—we have neither a spontaneously self-constituting subject of intervention, nor a perfectly circular and self-enclosed event, but something closer to what Sartre alternately called an ekstatic unity or phantom dyad of event and intervention, which remains too divided from itself and too anchored in its facticity to be grasped as an *ens causa sui*, or an absolute and transcendental sovereign. In many ways, this modification in the formal structure of Badiou's theory was prerequisite for the subsequent interpretation of post-evental fidelity as a reconfiguration of Sartrean freedom.

At various points in the preceding study, the distinct bearings that Badiou and Sartre's conceptions of the subject have on the ethical plane have been briefly indicated. What remains is for us to develop these in greater detail, with the crucial aim of determining the ethical implications of Badiou's rarefaction of the subject and his restoration of a Sartrean subtractive universalism to its effects. It is ultimately in the consequences that these two ontologies propose for practical reason that their full significance is felt. As Sartre promised before us, we shall devote to them a future work.

Notes

¹ Alain Badiou and Bruno Bosteels, "Can Change be Thought? An Interview with Alain Badiou," in Gabriel Riera, ed., *Alain Badiou: Philosophy and its Conditions*, (Albany: SUNY Press, 2005), 242.

² Badiou, "Saïssiement, desaiïssie, fidélité," in *Les Temps modernes*, v.531 (1990), 20. Henceforth cited as SDF.

³ "We must even admit that it explicitly contradicts certain of [set-theoretic ontology's] formal axioms, just as Lucretius's clinamen contradicts the major axioms of atomist and materialist ontology.

In fact, I have often compared the absolute hazard of the event, subtracted as it is from the general determinations of being, to this clinamen, required for the assemblage of atoms to be thought as generic truth of worlds." (Badiou, Robin Mackay and Ray Brassier, "Philosophy, Science and Mathematics: Interview with Alain Badiou," in *Collapse*, vol. 1 (2006), 17.)

⁴ Sartre gives an amusing example of this nonsense at the beginning of his lecture. "I have lately been told of a lady," Sartre writes, "who, whenever she lets slip a vulgar expression in a moment of nervousness, excuses herself by exclaiming, 'I believe I am becoming an existentialist.'" (EH, 24).

⁵ Aristotle, *Metaphysics*, trans. Hugh Tredennick, (Cambridge: Harvard University Press, 1933), Book Γ, 1003a.

⁶ One may be reminded of the ontologies of Deleuze, Heidegger and (possibly) Merleau-Ponty here. In the context of Sartre scholarship, it would be especially significant if it could be shown that the work of the latter two fall under the category of 'maximal ontology', since, if Sartre's ontology is correctly understood as a 'minimalist' (as I argue), then Desanti has provided us with a helpful matrix for taking stock of an important difference between Sartre and two of his chief interlocutors, with which he is either unjustly conflated or hyperbolically distinguished.

⁷ Jean-Paul Sartre, 'Une idée fondamentale de la phénoménologie de Husserl: l'intentionnalité,' in *Situations I*, (Paris: Gallimard, 1947), 31. The revolutionary enthusiasm that Sartre shows for this idea is manifest, as he goes continues:

To know is to "explode towards" [*« s'éclater vers »*], to wrench away from most gastric intimacy in order to draw out, down there, beyond the self, towards that which is not self, down there, next to the tree and yet outside of it, since it escapes me and drives me back and I can no more lose myself in it than it can dissolve in me: outside of it, outside of me. Is it that you do not recognize in this description your demands and your presentiments? You know well that the tree is not you, that you cannot make it enter into your sombre stomachs and that knowledge cannot be, without dishonesty, compared to possession. In the same stroke, consciousness is purified, it is as clear as a great wind, there is no longer anything in it but a movement in flight from itself, a sliding outside of itself; if, impossibly, you were to enter "into" a consciousness, you would be seized by a whirlwind and ejected outside of it, next to the tree, completely driven back, because consciousness has no "inside"; it is nothing but the outside of itself and it is this absolute flight, this refusal to be substance which constitutes it as consciousness. Now imagine a connected series [*suite liée*] of explosions wrenching us away from ourselves, which do not even permit an "ourselves" the leisure of forming itself behind them, but which on the contrary thrust us beyond them, into the dry dust of the world, onto the rough ground, amongst things; imagine that we are thus rejected, by our very nature forsaken in an indifferent, hostile and stubborn world; you will have then grasped the profound sense of the discovery that Husserl expresses in this famous phrase: "All consciousness is consciousness of something." Nothing more is needed to put an end to the delicate philosophy of immanence, where everything is done by compromise, by protoplasmic exchanges, by a tepid cellular chemistry. The philosophy of transcendence throws us onto the road, into a hostile environs [*au milieu des menaces*], under a blinding light. [...] This necessity for consciousness to exist as consciousness of something other than itself, Husserl names "intentionality". (32-3)

⁸ Likewise, the number 2 is only the presentation of 1 and 0, and the number 1 is nothing but the presentation of 0.

Employing set-theoretic notation, we may rewrite these statements: $3 = \{2, 1, 0\} = \{\{1, 0\}, 1, 0\} = \{\{\{0\}, 0\}, \{0\}, 0\}$.

Exchanging '0' for its set-theoretic equivalent, the empty set \emptyset , we have: $3 = \{\{\{0\}, \emptyset\}, \{0\}, \emptyset\}$. These formulations of the ordinal numbers are due to John Von Neuman.

⁹ Some time after preparing this syllogism, I came across Akihiro Kanamori's essay, 'The Empty Set, the Singleton, and the Ordered Pair,' in *The Bulletin of Symbolic Logic*, vol. 9, no. 3 (Sept., 2003), pp. 273-298. Kanamori points out that Richard Dedekind had produced precisely the same argument in his unpublished notebooks, in order to criticise certain confusions that he, himself, had committed between α and $\{\alpha\}$ in 1888's *Was sind und was sollen die Zahlen?* (see Kanamori, p. 278).

¹⁰ For example, take the set $\xi = \{\emptyset, \{\emptyset\}, \{\emptyset, \{\emptyset\}\}$. (For now, this is just the ordinal 3 again.) If $\{\emptyset\} = \emptyset$, then we could rewrite ξ as $\{\emptyset, \emptyset, \{\emptyset, \emptyset\}\}$, which would be the same as $\{\emptyset, \{\emptyset\}\}$. Applying this equation again, we get $\{\emptyset, \emptyset\}$, which is just $\{\emptyset\}$. One last stroke of our equation then gives us \emptyset . One can thereby see how the equation $\{\emptyset\} = \emptyset$ would imply, without any appeal to *ex falso quodlibet*, ($\forall \alpha)(\alpha = \emptyset)$, i.e., $\sim(\exists \alpha)(\alpha \neq \emptyset)$.

¹¹ Rigorously speaking, we should write $\sim(\forall x)(x = x)$, since it is possible to define *particular* sets which are identical with their singleton, but it cannot generally be the case that set-formation does not introduce difference between its material (x) and its product ($\{x\}$).

¹² Badiou, "Custos, quid noctis?" in *Critique* (November, 1984), n° 40, 862.

¹³ Louis Althusser, "Philosophy and the Spontaneous Philosophy of Scientists," in *Philosophy and the Spontaneous Philosophy of Scientists & Other Essays*, ed. Gregory Elliot, (London: Verso, 1990), 128.

¹⁴ Gilbert Varet, *L'Ontologie de Sartre*, (Paris: Presses Universitaires de France, 1948), 23.

¹⁵ Klaus Hartmann, *Sartre's Ontology; A Study of Being and Nothingness in the light of Hegel's Logic*, (Evanston, Ill.: Northwestern University Press, 1966), 34.

¹⁶ In this light, Sartre's axiom of intentionality can, perhaps, be interpreted as a dramatic example of what Hegel calls a "speculative proposition". Hegel's explanation of this concept is worth repeating here at length:

the general nature of the judgement or proposition, which involves the distinction of Subject and Predicate, is destroyed by the speculative proposition, and the proposition of identity which the former becomes contains the counter-thrust against that subject-predicate relationship. [...] To illustrate what has been said: in the proposition 'God is being', the Predicate is 'being'; it has the significance of something substantial in which the Subject is dissolved. 'Being' is here meant to be not a Predicate, but rather the essence; it seems, consequently, that God ceases to be what he is from his position in the proposition, viz. a fixed Subject. Here thinking, instead of making progress in the transition from Subject to Predicate, in reality feels itself checked by the loss of the Subject, and, missing it, is thrown back on to the thought of the Subject. (*Phenomenology of Spirit*, trans. A.V. Miller, (Oxford: Oxford University Press, 1977), ¶¶ 61-2, p. 38.)

The recognition of the dialectical structure of the axiom of intentionality—evident in the simultaneous unification and differentiation that takes place between 'subject' (consciousness) and 'predicate' (consciousness of something)—suggests another line of interpretation and another formalism that could be adopted here, either in combination or instead of the quasi-mathematical line adopted here. This other formalism may take as its starting point Badiou's formulation (in *Théorie du sujet*) of the structure of dialectical scission, wherein an entity is identified with the pair consisting of itself and its placement. In Badiou's notation this is written:

$$A = (AA_p)$$

In the case of the axiom of intentionality, we may read A as 'consciousness' and A_p as 'consciousness of something'. The 'place' in question would simply be the *extension* of the consciousness. It is notable that in *L'être et l'événement* Badiou implicitly reinvokes this structure by naming the figure of the event as an instance of 'scission', writing:

Considered, not in its multiple-being, but in its position, or its situation, an event is an *interval* rather than a term: it established itself, in the interventional retroaction, between the empty anonymity bordered on by the site, and the addition of a name. Moreover, the matheme inscribes this originary *scission*, since it only determines the one-composition of the event e_x inasmuch as it distinguishes therein between itself and the represented elements of the site—from which, besides, the name originates. (BE, m20, 228/204*)

What these indications suggest is the existence of a second conceptual thread connecting the event and being-for-itself: one which proceeds according to the concept of dialectical scission rather than that of multiplicity. If this thread can be shown to hold, it would offer further support to the thesis that there exists a significant homology between being-for-itself and the Badiouian event. I will leave this line of inquiry in suspense for the time being.

¹⁷ The French text actually avoids using the word "*interne*" [internal], having instead, "Il échappe doublement à l'être, par désagrégation *intime* et négation expresse." Barnes' translation is adequate, and avoids the awkwardness and inaccuracy that would accompany the expression "an intimate disintegration," but we should note that Sartre does not conceive of consciousness as something that has an 'inside' and an 'outside', not even in a metaphorical or analogous sense.

¹⁸ Much of the difficulty surrounding Sartre's theory of non-thetic reflexivity has its basis in our tendency to treat of objects, which, in their being, are rooted in the identity of the in-itself, and which, being in the respect homogeneous with the material elements of language, naturally lend themselves to discursive intelligibility. Both letter and object can be apprehended as self-identical; both can be apprehended in exteriority. If Sartre's analysis, however difficult, remains intelligible, it is because the analysed structures are those of consciousness itself, and their specific mode of non-identity is homogeneous with the act thought.

¹⁹ Sartre, *The Transcendence of the Ego*, trans. Forest Williams and Robert Kirkpatrick, (New York: Noonday Press, 1957), 83. Henceforth cited TE.

²⁰ The symbol ϕ is chosen to recall the split unity and internal differentiation of consciousness, a sort of aesthetic consolation for the violence done to the notion of consciousness by entering it into calculations through the proxy of a self-identical symbol. This choice is perhaps somewhat akin to Lacan's choice of the sign $\$$ for the subject of the unconscious, which is also 'split', but by the internal differentiation proper to the signifier as such. That they both happen to be monetary signs is an uncanny fluke.

²¹ The remaining possibility, that the extension of ϕ forms a 'subset' of the extension of $\tilde{\phi}$ is ultimately untenable, given that, as a result of the transcendental structure of consciousness that we will examine later, extensions are determined holistically as 'homological spaces'. The embedding of extension α in extension β would result in a transformation of the very structure of α . The result, call it α_0 , would no longer be identical to α .

²² This is, of course, something of an abstraction. Consciousness never exists in such a way as to let itself be conceived 'instantaneously', that is, without duration. It is a useful abstraction, however, insofar as it allows us to analyse the character of consciousness' non-self-identity in a way that is not immediately dependent on temporality. Consciousness is non-identical with itself even under the supposition that its extension remains unchanged, that is, its non-identity is not a consequence of the perpetual transformation of its extension in time.

²³ When something happens to the body so as to disturb the unitary synthesis of the extension—when the corpus colossus is severed, for instance—we have reason to believe that consciousness fissures as well, yielding two consciousnesses whose extensions are somewhat distinct (often, one inherits the sensory input from the right side of the body and the other inherits the input bequeathed by the left).

²⁴ Sartre himself offers a complex and acute analysis of the bodily dimension of the for-itself, which we will not explore in any depth in this essay. For the elaboration of this theory, see *Being and Nothingness*, Part III, Chapter II.

²⁵ As Sartre notes, “it is in theory possible but in practice impossible to distinguish facticity from the project which constitutes it in situation” (BN, IV.II.III, 603/649), a remark that will be clarified in Chapter VII when we come to examine the structure of projects. Using the limited conceptual apparatus we have so far constructed, we can already say that it is in theory possible but in practice impossible to distinguish facticity from the consciousness which takes it as extension.

²⁶ The reader already familiar with *L'être et l'événement* may note the following anticipated results: (1) If e_x is an event, then $F(e_x)$ is the event site X . (2) If t is a truth procedure, then $F(t)$ is the generic set φ which represents the being of that truth. (3) If σ_i is a subject of the truth t , then $F(\sigma_i)$ is a proper subset of φ (i.e. $F(\sigma_i) \subset F(t) = \varphi$). A fuller explanation of these points will be given in Chapter IX.

²⁷ “There exists something that is P ”

²⁸ Crane’s precise formulation is as follows:

A linguistic or logical context (i.e., part of some language or logical calculus) is intensional when it is not extensional. An extensional context is one of which the following principles are true:

(A) The principle of intersubstitutivity of co-referring expressions.

(B) The principle of existential generalization. (Crane, 1995, 33)

²⁹ “To the extent that men had believed in noumenal realities, they have presented appearance as a pure negative. It was ‘that which is not being’; it had no other being than that of illusion and error. But even this being was borrowed, it was itself a pretence...” (BN, in.I, xlvi/12)

³⁰ As Joseph Catalano puts it in his *Commentary*,

It is clear that, for Sartre, the being of phenomenon is not itself a phenomenon, a meaning, or something hidden beneath phenomenon. Rather, being is simply the condition of all disclosure or revelation. (28)

³¹ In the interest of fairness to Kant, of whom Sartre was not always a careful reader, it should be noted that there exist interpretations of the Kantian noumenon that are not entirely at odds with Sartrean transphenomenal being. For a nice summary of these interpretations (the “two aspect” theory), and their contrast with the more pronouncedly un-Sartrean “two world” theory, see Hoke Robinson, “Two Perspectives on Kant’s Appearances and Things in Themselves,” in *Journal of the History of Philosophy*, Vol. 32, No. 3, (July, 1994).

³² Sartre is not altogether consistent on this point. A strange theory of motion as a sort of material quasi-transcendence lurks in the background of the latter chapters of *Being and Nothingness*. According to this theory, when a being moves—even if this being is without consciousness—it minimally separates from itself. Motion is comes to be thought as a sort of brute ekstasis, “a malady of being”. This peculiar theory, however, never comes to play a significant part in the main developments of *Being and Nothingness*. In the end, he relegates it to ‘metaphysical’ speculation, as opposed to ontological analysis, and withdraws is own hands from the doctrine:

In particular the task falls to the metaphysician to decide whether movement is or is not a first ‘attempt’ on the part of the in-itself to found itself and to determine what are the relations of motion as a ‘malady of being’ with the for-itself as a more profound malady pushed to nihilation. (BN, c.I, 621/669*)

³³ Sartre, *Nausea*, trans. Lloyd Alexander (New York: New Directions, 1949), 178. Quoted in Catalano, 46.

³⁴ Cf. W.V.O. Quine, “On What There Is,” in *From a Logical Point of View: 9 Logico-Philosophical Essays*, (Cambridge: Harvard University Press, 1964), I.

³⁵ Klaus Hartmann is by no means the only commentator to make this equivocation. To offer a far from exhaustive list, we may consider the following:

- a. In their “Introduction to Sartre,” Charles Guignon & Derk Pereboom write, “In various respects, this view of being-in-itself recalls the ancient Greek philosopher Parmenides’ description of being.” (Guignon & Pereboom, eds. *Existentialism: Basic Writings*, (Indianapolis: Hackett, 2001), 261)
- b. In *The Mirror of Ideas*, Michel Tournier explicitly calls Sartre’s being-in-itself “Parmenidean Being.” (Tournier, *Mirror of Ideas*, trans. Jonathan Krell, (Nebraska: University of Nebraska Press, 1998), 122)
- c. In “Dialectic without Mediation. On Sartre’s Variety of Marxism and Dialectic,” Yirmiahu Yovel writes, “Being In-itself is in fact like Parmenides’ ‘One’: undifferentiated, immovable, self-identity, which allows of no inner negativity.” (in Shlomo Avineri, ed., *Varieties of Marxism*, (The Hague: Martinus Nijhoff, 1977), 184)
- d. In *Freedom’s Embrace*, J. Melvin Woody writes, “Sartre’s being-in-itself is not a world, but a seamless, immutable unity reminiscent of Parmenides’ monism.” (University Park, Penn.: Pennsylvania State Press, 1998).
- e. In “Specular Grammar: the Visual Rhetoric of Modernity,” Barry Sandywell writes, with regards to Sartre’s ontology, that “[i]t is the appearance of human activity that introduces nothingness into the Parmenidean Sphere of Being.” (in Barry Sandywell & Ian Heywood, eds. *Interpreting Visual Culture: Explorations in the Hermeneutics of the Visual*, (London: Routledge, 1999), 48)

f. In James P. Mackey's *Critique of Theological Reason*, we read that Sartrean "dualism now seems as dichotomous, as crude as ever. On one side is a personal absolute [...]. On the other side is a Parmenidean Being that neither comes to be nor passes away." ((Cambridge: Cambridge University Press, 2000), 66.)

Dermot Moran, in his *Introduction to Phenomenology*, offers a more measured appraisal of the relation between Sartre and Parmenides, and avoids the common error of confusing the two. He writes,

In a sense, Sartre subscribes in part to a Parmenidean vision of this being: being is; non-being is not. Moreover, being is undifferentiated, pure self-identity, *être en-soi*. He differs from Parmenides in not seeing a relation between being and reason, and in not seeing being as necessary. Being is opaque to itself, brute, inert, neither active nor passive, meaningless, resistant to consciousness, pure 'immanence' (BN, xli; 32). (London: Routledge, 2000), 356.

Let us note that Sartre himself employs the term 'Parmenidean' twice in *Being and Nothingness*: The first occurrence is to be found on (BN, III.I.IV, 300/340), in Sartre's analysis of the problem of the totality of all consciousnesses. Such a totality cannot consistently exist *qua* totality, Sartre argues, because each for-itself encounters a limiting nothingness between itself and the Other:

In fact it seems that this nothingness has slipped into this totality in order to shatter it just as in the atomism of Leucippus non-being slips into the Parmenidean totality of being and makes it explode into atoms. Therefore it represents the negation of any synthetic totality in terms of which one might claim to understand the plurality of consciousnesses.

The Parmenidean totality, here, is not being-in-itself, but the postulated figure of a totality of mind, which is found to be ontologically inconsistent. The second occurrence of the term 'Parmenidean' is even more significant with respect to the current problematic. It is to be found on (BN, IV.II.III, 613/660). Here, the predicate 'Parmenidean' serves as the attribute of an infantile phantasy. 'Parmenidean being' is the form of the value correlative of a child's fascination with filling holes, sucking his thumb, etc. Following Sartre's existential psychoanalysis, when the child

puts his fingers in his mouth, he tries to wall up the holes in his face; he expects that his finger will merge with his lips and the roof of his mouth and block up the buccal orifice as one fills the crack in a wall with cement; he seeks again the density, the uniform and spherical plenitude of Parmenidean being...

The common theme in both of these passages is, first of all, that Sartre uses the term 'Parmenidean' only to refer to idealities that have no place among being-in-itself. The second point to which we should attend is that the concept of Parmenidean being has an essential connection to Sartre's theory of value, which we shall examine in greater detail in the next chapter.

³⁶ We may leave outside of this account Sartre's more or less Platonic opposition to Parmenides on the question of nothingness. Sartrean consciousness, like the Platonic Other, makes nothingness be. Hartmann, however, has restricted the parallel to Parmenidean Being and Sartrean being-in-itself, and is plainly aware that the Sartrean theory of consciousness is radically anti-Parmenidean.

³⁷ Parmenides, *Way of Truth*, Frag. 8, lines 30-1. Quoted in F.N. Cornford, *Plato and Parmenides*, (Indianapolis: Bobbs-Merrill Co., 1939), 42.

³⁸ Aristotle took this thesis to be absurd: "If [Parmenides'] One is one in the sense of continuous, it is many, for the continuous is divisible *ad infinitum*" (*Phys.*, 185^b, 5-10).

³⁹ The *Notebook for an Ethics* contains several illuminating passages in this respect, and goes a long way to detailing the transition between the in-itself of *Being and Nothingness* and the in-itself of the *Critique of Dialectical Reason*, where being-in-itself is thought primarily in terms of its inertial inconsistency, or tendency towards infinite divisibility. Consider, for instance, his 38th note on "The Ambivalence of History and the Ambiguity of Historical Fact," where he writes: "Aspects of the event: 1st, unity to the extent that it is mental, it includes within itself a divisibility to infinity insofar as it is nature." (Sartre, *Notebook for an Ethics*, trans. David Pellauer, (Chicago: University of Chicago Press, 1992), 34).

⁴⁰ Herman Weyl, "The Current Epistemological Situation in Mathematics," trans. Paolo Mancosu, in Paolo Mancosu ed. *From Brouwer to Hilbert: The Debate in the Foundations of Mathematics in the 1920's* (Oxford: Oxford University Press, 1998), 136.

⁴¹ Heidegger, *Being and Time*, trans. John Macquarrie & Edward Robinson, (New York: Harper & Row, 1962), 60/H36.

⁴² Cf. Morris Kline, *Mathematics for the Nonmathematician*, (New York: Dover, 1967), 452-3:

Euclidean geometry, as well as developments such as arithmetic, algebra, and calculus, rests upon axioms. The Greeks, who formulated the axioms of Euclidean geometry, believed that human minds immediately recognised some truths about the geometrical properties of physical objects and of space. Thus, it seemed indubitable that two points determined one and only one line, and that equal line segments added to another pair of equal line segments gave equal sums. For two thousand years the entire intellectual world accepted the Greek doctrine that the axioms of Euclidean geometry and of mathematics in general, were truths about the physical world, truths so clear and so evident that no one in his right mind could question them. Of course, since the axioms of geometry were truths, and since the theorems were logically necessary consequences of the axioms, the entire body of Euclidean geometry constituted a collection of indubitable truths about idealized objects and phenomena of the physical world.

Much of the historical account that follows is indebted to Kline's book.

⁴³ Indeed, the original order of reasons by which Sartre proceeds in *Being and Nothingness* can be seen as a successful attempt to derive the axiom of intentionality from the cogito (or, rather, from the pre-reflective cogito).

⁴⁴ See David Hilbert, *The Foundations of Geometry*, trans. E.J. Townsend, (La Salle, Illinois: Open Court, 1965).

⁴⁵ The opening statements of Hilbert's geometry are emblematic of the whole:

Let us consider three distinct systems of things. The things comprising the first system, we will call *points* and designate them by the letters, *A, B, C, ...*; those of the second, we will call *straight lines* and designate them by the letters *a, b, c, ...*; and those of the third system, we will call *planes* and designate them by the Greek letters $\alpha, \beta, \gamma, \dots$ (3)

No 'definition' of points, lines or planes follows, only axioms describing the relations in which they stand to other terms (or, in the last instance, other sets of letters). One of the remarks that Hilbert makes in a letter to Frege following the publication of the *Foundations of Geometry* nicely encapsulates Hilbert's assessment of his own project.

every theory is only a scaffolding or schema of concepts together with their necessary relations to one another, and that the basic elements can be thought of in any way one likes. If in speaking of my points I think of some system of things, e.g. the system: love, law, chimney-sweeps... and then assume all my axioms as relations between these things, then my propositions, e.g. Pythagoras' theorem, are also valid for these things.

David Hilbert, Letter to Frege dated December 29th, 1899, trans. Hans Kaal, in Gottlob Frege, *Philosophical and Mathematical Correspondence*, ed. Gottfried Gabriel, (Chicago: University of Chicago Press, 1980), 40.

⁴⁶ For a closer look at the way this aspect of formalism manifests itself in Badiou's work, see Justin Clemens, "Letters as the Condition of Conditions for Alain Badiou," in *Communication & Cognition*, Vol. 36, N° 1-2 (2003), 73-102.

⁴⁷ See Louis Althusser, *Lenin and Philosophy*, (New York: Monthly Review Press, 1971).

⁴⁸ Alain Badiou et al., "Ontology and Politics: An interview with Alain Badiou," in *Infinite Thought*, ed. Oliver Feltham and Justin Clemens, (London: Continuum, 2003), 174. Henceforth cited 'OP'.

⁴⁹ Not yet, anyhow. He undertakes something along these lines in his most recent book, *Logiques des mondes: l'être et l'événement 2*.

⁵⁰ This is an inevitable assumption, since set theory, being a situation, is itself a presentation and cannot be internal to itself by the very axioms it proposes.

⁵¹ There do exist variations of set theory which allow for the existence of *atoms*, which can belong to sets but which are not themselves sets. But these variations are not under consideration here.

⁵² See Appendix 3 for the complete interview.

⁵³ Quine's original formulation—"to be is to be the value of a variable"—is, in fact, cited by Badiou himself, in the context of discussing set-theory's ontological univocity:

If we admit—with a grain of salt—Quine's famous formula: 'to be is to be the value of a variable', we may conclude that the system ZF postulates that there is only one type of presentation of being: the multiple. (BE, m3, 44/55*)

For the original source of Quine's aphorism, see "On What There Is," in *From a Logical Point of View*.

⁵⁴ Krivine put it wonderfully in his *Théorie axiomatique des ensembles*, where he describes set theory as "the theory of binary relations satisfying the Zermelo-Fraenkel axioms" (6). Accordingly, a set is nothing but a term of a relation satisfying these axioms. Now, it may fairly be said that this just shifts the bump in the rug, since we have only replaced the notion of 'set' with that of 'relation' (a move which Badiou is rather hostile to, incidentally—see the appended interview).

⁵⁵ Badiou, *Le Concept de modèle*, (Paris: Maspero, 1969), 52.

⁵⁶ Badiou, *Ethics*, 128.

⁵⁷ For a brilliant, though almost entirely different approach to the same problems dealt with here—those concerning the relation between presentation, sets, the ontological situation, and other situations—which arrives at more or less the same conclusion (that the distinction between discourse and world becomes unsustainable for Badiou), see Ray Brassier "Presentation as Anti-phenomenon in Alain Badiou's *Being and Event*," in *Continental Philosophy Review*, (2006), vol. 39, pp. 59-77.

⁵⁸ Badiou, *Le Concept de modèle*, 21.

⁵⁹ John Von Neumann & Oskar Morgenstern, *Theory of Games and Economic Behaviour*, (Princeton: Princeton University Press, 1953).

⁶⁰ Neumann & Morgenstern, 32.

⁶¹ Neumann & Morgenstern, 32.

⁶² Badiou, *Le Concept de modèle*, 21.

⁶³ Serres offers an example of such a defence, and one which again traces its heritage back to Plato. Cf. the discussion entitled "Style, or Mathematics Continued by Other Means," in Serres' interviews with Latour, for instance:

Bruno Latour: [...] Now, if I've understood rightly, style, in fact, is best if it imitates mathematics as precisely as possible, in a domain mathematics can't enter.

Michel Serres: At least the rigour and precision of mathematics. Does Plato himself proceed otherwise? Every time he has something somewhat difficult to say, he abandons technical vocabulary and goes to myth, telling a

story that globalizes his point even more. He is always in the process of moving obliquely, as you said. Where neither mathematics nor logic can go, let myth go! (Serres & Latour, 72).

Whether an analogous approach can be taken in the context of Badiou's work remains an open question.

⁶⁴ Ray Brassier has written an excellent article ("Presentation as Anti-phenomenon in Alain Badiou's *Being and Event*," in *Continental Philosophy Review*, (2006), N° 39, 59-77) examining the 'methodological idealism' implicit in *Being and Event*, and which deals with many of the problems examined here (though he charts out a quite different route in his proposed solution, one which rearranges the terms of the problem so drastically that I have difficulty incorporating any detailed discussion of his work into what I have here). With respect to Badiou's persistent rhetorical appeal to materialism, I think that Justin Clemens put it best. Referring to the opposition between 'democratic materialism' (bad) and 'materialist dialectic' (good) that Badiou establishes in the introduction to *Logiques des mondes*, Clemens writes:

I cannot see what's at stake in affirming a 'materialist dialectic' today against a 'democratic materialism'. These are terms from the struggles of a previous era, one that still held out hope for the Whole, even in its negation or loss [...]. Nothing intrinsic justifies this [Badiou's metaphysics] as a 'materialism' (why not just call it a 'realism'), other than old fondnesses. Idealism, materialism: this has become a distinction without difference in a world where foundational physics invents incommensurable and untestable string-theories that are nonetheless each consistent; where the legacy of political activism engages in local struggles that hardly require any kind of doctrine or praxis of 'materialism' for their organization or effects. What's in this 'dialectic' today that's worth saving? What's in this 'materialism' that's not a mere slogan. [...] The slogan one might brandish instead is *translitteral absolutism*. (Clemens, "Had we but worlds enough, and time, this absolute, philosopher..." in Paul Ashton, A.J. Bartlett & Justin Clemens, eds. *The Praxis of Alain Badiou*, (Melbourne: re.press, 2006), 141)

⁶⁵ Ray Brassier, "Presentation as Anti-phenomenon in Alain Badiou's *Being and Event*," in *Continental Philosophy Review*, (2006), N° 39, p. 69.

⁶⁶ "Πάν το μετεχόν του ενός και εν εστί και ουχ εν." Proclus' proof is worth repeating in this context:

For inasmuch as it cannot be pure unity (since participation in unity implies a distinct participant), its 'participation' means that it has unity as an affect, and has undergone a process of becoming one. Now if it be nothing else but its own unity, it is a bare 'one' and so cannot participate unity but must be pure unity. But if it has some character other than oneness, in virtue of that character it is not-one, and so not unity unqualified. Thus being one, and yet (as participating unity) in itself not-one, it is both one and not-one. It is in fact unity with something added, and is in virtue of the addition not-one, although one as affected by unity. Everything, therefore, which participates unity is both one and not one.

(Proclus, *Elements of Theology*, trans. E.R. Dodds, (Oxford: Clarendon Press, 1971), 3).

⁶⁷ The original text reads: "L'expérience dont l'ontologie, depuis sa disposition parméniennienne, fait la portique d'un temple ruiné, est la suivante : ce qui se présente est essentiellement multiple ; ce qui se présente est essentiellement un." Lost in translation is the ambiguity between the reflexive and passive voice that is carried by the French "se présente", which could be rendered "is presented" just as well as "presents itself".

⁶⁸ "To be brief, I hold as axiomatic the identical proposition which varies only in emphasis: that what is not truly *one* entity is not truly *one entity* either. It has always been thought that 'one' and 'entity' are interchangeable. Entity is one thing, entities another; but the plural presupposes the singular, and where there is no entity, still less will there be many entities." (Gottfried Wilhelm Leibniz, "Letter to Arnauld, Göttingen, April 30th, 1687," in Leibniz & Antoine Arnauld, *The Leibniz-Arnauld Correspondence*, (Manchester: Manchester University Press, 1967), 121).

⁶⁹ Graphs such as these, as far as I am aware, were first explicitly described by Thoralf Skolem. See, for instance, T. Skolem, "Some Remarks on Axiomatized Set Theory," (1922), in *From Frege to Gödel*, 290-302. For a detailed reconstruction of set theory using graph notation, see Peter Aczel, *Nonwellfounded Set Theory*, (Stanford: CSLI, 1988).

These graphs are similar in appearance to those used in category theory, but they should not be confused with the latter. The 'mechanics' of the graphs are strictly set-theoretic, and the arrows are read as inversions of the set-membership relation, not as morphisms. It is nonetheless possible to present set theory in entirely categorical terms, however. For a brilliant demonstration of how this can be done, see F. William Lawvere & Robert Rosebrough, *Sets for Mathematics*, (New York: Cambridge, 2003).

⁷⁰ In choosing this symbol for the self-singleton, I am following a convention used by Peter Aczel in his pioneering text on non-wellfounded set theory (set theory without the axiom of foundation, the axiom which prohibits the existence of sets such as Ω). The book is fittingly entitled *Non-wellfounded Set Theory*. (The theistic connotations of this symbol are not, however, irrelevant, as we will see when we come to the Sartrean theory of the *ens causa sui*.)

⁷¹ Let us note that Zermelo-Fraenkel set theory does not consider $\Omega = \{\Omega\}$ to be a set. It is even, for ZF, an impossible entity, and is debarred by the axiom of foundation, which demands that every set possess an element with which it has no elements in common. Clearly, Ω has *everything* in common with its only element (itself). There do exist other set theories, however, which allow structures such as Ω to take shape. Peter Aczel has devoted an excellent text to such matters. See *Non-well-founded Sets*, (Stanford: CSLI, 1988).

⁷² "Limited" and "finite" are English translations of the same Greek word (περαίνω).

⁷³ Ray Brassier seems to have come to a similar conclusion. In “Presentation as Anti-Phenomenon in Alain Badiou’s *Being and Event*,” Brassier remarks that the operation of the count “is not, because it cannot count itself as one” (Brassier, 2006, 8). If it could, it would yield nothing other than the figure of the One, which is precisely the count of the count that it is: $\Omega = \{\Omega\}$.

⁷⁴ In its original context in *Being and Event*, the statement that “the one is the non-being of being-multiple” is given a somewhat different sense. It appears in a discussion of Cantor’s theological attempt to identify absolutely inconsistent multiplicity—multiples that cannot, without paradox, be thought of as unified into a set—with the Absolute or the Divine. As Badiou recounts,

Cantor’s ontological thesis is evidently that inconsistency, mathematical impasse of the one-of-the-multiple, orients thought towards the Infinite as supreme-being, or absolute. [...] Cantor, essentially a theologian, therein ties the absoluteness of being *not* to the (consistent) presentation of the multiple, but to the transcendence through which a divine infinity in-consists, as one, gathering together and numbering any multiple whatsoever. [...] Cantor’s thought thus wavers between onto-theology—for which the absolute is thought as a supreme infinite being, thus as trans-mathematical, in-numerable, as a form of the one so radical that no multiple can consist therein—and mathematical ontology, in which consistency provides a theory of inconsistency, in that what proves an obstacle to it (paradoxical multiplicity) is its point of impossibility, and thus, quite simply, *is not*. Consequently, it fixes the point of non-being from whence it can be established that there is a presentation of being. (BE, m3, 42/53)

⁷⁵ Alain Badiou et al., “Ontology and Politics: An interview with Alain Badiou,” in *Infinite Thought*, ed. Oliver Feltham and Justin Clemens, (London: Continuum, 2003), 170.

⁷⁶ These temporal metaphors (retroaction, inertia, priority, upstream, downstream, etc.) serve only as heuristic devices; the structure that they are intended to illustrate is atemporal.

⁷⁷ Georg Cantor, “Letter to Dedekind,” (1899), in *From Frege to Gödel*, 114. Badiou quotes part of this same passage in (BE, m3, 41-2/52-3).

⁷⁸ Badiou, “Custos, quid noctis?”, 861.

⁷⁹ Though it sounds a bit barbaric in English, I cannot think of a more appropriate translation of “*imrepresentable*.”

“Unrepresentable,” after all, already has a meaning of its own (however unrelated to the context of this paper).

“Imrepresentable” at least follows common enough patterns of English word-formation (after the model of “impossible,”

“implausible,” “improbable,” “imperfect,” etc.).

⁸⁰ “The method of ‘postulating’ what we want has many advantages; they are the same as the advantages of theft over honest toil.” Bertrand Russell, *Introduction to Mathematical Philosophy*, (London: Allen & Unwin, 1919), 71.

⁸¹ Lyotard does this without any mention of Sartre, however.

⁸² Roughly, for our present purposes, we may say that the relation between phrase and presentation in Lyotard parallels the relation between situation and presentation in Badiou. A phrase (which need not be linguistic) is that which is presented, grasped in its specific consistency.

⁸³ Badiou, “Custos, quid noctis?”, 853.

⁸⁴ Recall that elsewhere Lyotard calls presentation “the event of [the given’s] inapprehensible presence” (63).

⁸⁵ Jean-François Lyotard, *The Differend: Phrases in Dispute*, trans. Georges Van Den Abbeele, (Minneapolis: University of Minnesota Press, 1988), 75.

⁸⁶ Why an epsilon? Fortuitously, the symbol was introduced by Giuseppe Peano as an abbreviation for the Greek copula ‘εσ τι’ (‘is’). For details, see Kanamori’s “Empty Set, Singleton, and Ordered Pair,” p. 79.

⁸⁷ Peter Hallward takes this *pensée* as the epitaph to his monograph on Badiou (Hallward, vi).

⁸⁸ As reported to me by one time president, Peter Schotch.

⁸⁹ This last proviso—that set theory homogenises *classical* mathematics—is significant. After the invention of set theory, mathematics had gone on to develop structures outstripping the expressive capacity of the ZF axiomatic in one way or another. One example is category theory, intuitionist mathematics is another, so is the plethora of nonwellfounded set theories that Aczel addresses in his marvellous *Non-wellfounded Set Theory*. This, of course, happens so often that it could with all justice be called *essential* to mathematics.

⁹⁰ These are the signs for “...implies...”, “not...”, “there exists...” and “...is identical to...”.

⁹¹ It is interesting to observe how the formalist enterprise of transcribing all of mathematics into a universal notation, when coupled by Badiou’s axiomatic identification of mathematics and ontology, is given the retroactive interpretation as having been commanded by the univocity of being itself. In the retrospective light of metaontology,

the essence of mathematics is in no way formalization. Mathematics is a thought, a thought of being qua being. Its formal transparency is a direct consequence of the absolutely univocal character of being. Mathematical writing is the transcription or inscription of this univocity. (“Being and Appearance,” in TW, 173)

Such discursive torsions are characteristic of Badiou’s method. While they require a certain measure of acclimatisation, they are in a certain sense more or less unavoidable, insofar as mathematics is understood as the inventive production of the intelligible form of what cannot but be its own conditions of possibility—or, to put it another way, as the situation to whose contingent vicissitudes the ontological structure of situations in general is subject.

⁹² “[I]n every situation in thinking the problem of unity and of identity are indiscernible. I have to elaborate the question of identity from the question of the unity of the multiplicity—it’s the same thing. The unity of the multiplicity *is* the ontological identity.” (OP, 170)

⁹³ Cf. Thomas Forster, “Axiomatising Set Theory with a Universal Set”: “[E]xtensionality, in conjunction with the axiom of foundation, enables us to decide when $x=y$ by seeing if their members are identical. The regress we are launched on here must terminate because the ranks of the things we are looking at is reduced by the induction step. I am becoming more and more convinced that the appeal of the axiom of foundation is simply that it provides us with this elegant recursive characterisation of identity and thereby spares us the need to give the matter any further thought.” (Unpublished manuscript).

⁹⁴ In this text Cohen refers to this axiom as the ‘axiom of regularity.’ Both names for this axiom are commonly used.

In *Set Theory*, Jech makes remark similar to Cohen’s, writing that the axiom of foundation is “irrelevant for the development of ordinal and cardinal numbers, natural and real numbers, and in fact for all of ordinary mathematics” (p.59, quoted in B. Madison Mount, “The Cantorian Revolution: Alain Badiou on the Philosophy of Set Theory,” in *Polygraph*, N° 17, 2005, pp. 41-91.)

⁹⁵ ‘Working mathematician,’ is in English in the original.

⁹⁶ If he or she so desires, the reader may skip this proof without affecting the continuity of the essay. I do not recommend doing so, of course, if only because it provides a glimpse of the inner mechanisms of what Badiou takes to be ontological reasoning, and is a fairly interesting demonstration in its own right.

If the reader has any difficulty in following the proof, which I have aimed to present as clearly as possible, I refer him or her to Appendices 2 and 3, where a glossary of mathematical notation and a summary of the Zermelo-Fraenkel axioms.

⁹⁷ To speak of the *smallest infinite ordinal* no doubt sounds a bit strange to those unfamiliar with set theory. As I will explain later in this essay, set theory admits a vast hierarchy of distinct infinities. The smallest of these, written ω_0 when it is being thought of as an ordinal, and \aleph_0 when being addressed as a cardinal, is the set of natural numbers, in their set-theoretic schematisation.

⁹⁸ The remark in parentheses can easily be translated into strictly set-theoretical notation. “ $n + 1$ ” is simply shorthand for “ $n \cup \{n\}$ ” and “ $n < m$ ” is synonymous with “ $n \in m$ ” when n and m are ordinals.

⁹⁹ The proof given here is my own, though the theorem itself is well-known, and there is little doubt that proofs similar to mine have been devised in the past. I have not been able to locate any proofs for this theorem in my research, however. This is likely because the theorem itself appears sufficiently trivial that few bother publishing proofs of it.

¹⁰⁰ The axiom of foundation, of course, could not itself be brought into effect without a criterion of identity; otherwise it would be impossible to tell whether or not the two sets in question have anything in common.

¹⁰¹ “The Greek language speaks more directly than ours, which is encumbered by the incision of the Subject, legible, since Lacan, in the expletive ‘ne’. For ‘rien n’est’ (‘nothing is’) is actually said ‘οὐδέν ἐστιν’, that is, as: ‘rien est’ (‘(the) nothing is;’). Therefore, what should be thought here is rather that ‘nothing’ is the name of the void: Plato’s statement should be transcribed in the following manner; if the one is not, what occurs in the place of the ‘many’ is the pure name of the void, insofar as it alone subsists *as being*” (BE, m2, 35/45).

¹⁰² Badiou, “L’entretien de Bruxelles,” in *Les Temps Modernes*, n° 526 (1990), 24.

¹⁰³ In Meditation 3, Badiou writes that

being-nothing is as distinct from non-being as the ‘there is’ is distinct from being.

Just as the status of the one is decided between the (true) thesis ‘there is oneness’ and the (false) thesis of the ontologies of presence that ‘the one is’, so is the status of the pure multiple decided, in the immanence of a non-ontological situation: between the (true) thesis ‘inconsistency is nothing’, and the (false) structuralist or legalist thesis ‘inconsistency is not.’ (BE, m4, 53-4/66-7)

¹⁰⁴ See the next page.

¹⁰⁵ In Meditation 4 of *L’être et l’événement*, in which Badiou introduces his concept of the void, we read:

For an indeterminate situation, there is thus an equivalent to what Plato named, with respect to the great cosmological construction of the *Timaeus*—an almost carnivalesque metaphor of universal presentation—the ‘errant cause’, recognizing its extreme difficulty for thought. What is at stake is an unrepresentable yet necessary figure which designates the gap between the result-one of presentation and that ‘on the basis of which’ there is presentation; that is, the non-term of any totality, the non-one of any count-as-one, the nothing particular to the situation, the unlocalizable void point in which it is manifest both that the situation is sutured to being and that the *that-which-presents-itself* wanders in the presentation in the form of a subtraction from the count. (BE, m4, 55/68)

The expression “errant cause” in the *Timaeus* is the name of Necessity (ἀναγκή) insofar as it is other than, and irreducible to, Reason (νοῦς). As A.E. Taylor and F.M. Cornford have observed (see A.E. Taylor, *Commentary on Plato’s Timaeus* (Oxford: Clarendon Press, 1962), and F.M. Cornford, *Plato’s Cosmology* (London: Routledge, 1956)), ‘Necessity’ in this sense has little or nothing to do with the mechanistic order of nature discerned by post-Galilean science; if it were to be situated at all with regard to modern science, which is a risky situation to say the least, since the notion of Reason therein is

¹¹¹ In the meditation entitled “The Matheme of the Event,” Badiou attempts to elucidate (but not justify) his decision to characterise events as reflexive with an illustration of the French Revolution:

Take the syntagm ‘the French Revolution’. What should be understood by these words? One could certainly say that the event ‘the French Revolution’ forms a one out of everything which makes up its site; that is, France between 1789 and, let’s say, 1794. There you’ll find the electors of the General Estates, the peasants of the Great Fear, the sans-culottes of the towns, the members of the Convention, the Jacobin clubs, the soldiers of the draft, but also, the price of subsistence, the guillotine, the effects of the tribunal, the massacres, the English spies, the Vendéans, the *assignats* (banknotes), the theatre, the *Marseillaise*, etc. The historian ends up by including in the event ‘the French Revolution’ everything delivered by the epoch as traces and facts. This approach, however—which is the inventory of all the elements of the site—may well lead to the one of the event being undone to the point of being no more than the forever infinite numbering of the gestures, things and words that co-existed along with it. The halting point for this dissemination is *the mode in which the Revolution is a central term of the Revolution itself*; that is, the manner in which the conscience of the times—and the retroactive intervention of our own—filters the entire site through the one of its eventual qualification. When, for example, Saint-Just declares in 1794 ‘the Revolution is frozen’, he is certainly designating infinite signs of lassitude and general constraint, but he adds to them that *one-mark* that is the Revolution itself, as this signifier of the event which, being qualifiable (the Revolution is ‘frozen’), proves that it is itself a *term* of the event that it is. (BE, m17, 180/201)

That this example, in truth, proves nothing, can be readily seen. There are two clear reasons why this argument fails. The first is that radically subjectivating events are scarcely alone in possessing this self-referential property. Nothing prevents me from saying, on Groundhog’s Day, for example, “Today is Groundhog’s Day, and it’s cloudy.” If by making such a statement I cause the situation “Groundhog’s Day” to present its own presentation of itself, then either (1) Zermelo-Fraenkel set theory is a rather useless ontology of (ordinary) situations, or (2) I am at the heart of a genuine event. Either conclusion seems to suck the lifeblood out of Badiou’s entire theoretical edifice, rendering either his ontology useless or his notion of event trivial and mundane. It only seems charitable to assume that the *appearance* of self-reference does not itself imply the presentation of a non-wellfounded multiplicity. This means that there must be other ways of modeling ‘self-referential’ structures in ZF. This leads us to the second reason why Badiou’s empirical argument is unsatisfactory: it is by no means impossible to model apparently self-referential structures without violating the axiom of foundation, and to do this with at least enough precision to account for such empirical phenomena as are exemplified in the above illustration. Such constructions tend to be somewhat awkward and artificial compared with the resources offered by non-wellfounded sets, it is true, but awkwardness alone hardly justifies tearing the fundamental structure of being asunder. A rather trivial way of modelling the above example may be to say that the set R_0 , schematising the French Revolution, does not, in fact, contain itself as an element: Under this hypothesis, what R_0 does contain, in addition to the “gestures, things and words” belonging to its site, is a highly similar set R_1 , containing everything R_0 contains, with the exception of R_1 itself; in the place of R_1 , R_1 has R_2 as an element. The process then repeats itself, until we reach R_x , where x is an arbitrarily high but finite ordinal (if x is not finite, then we have an infinite \in -descent on our hands). For all intents and purposes, this construction would be empirically indiscernible from the situation Badiou describes.

¹¹² The acronym stands for, “Zermelo-Frankel *without* the axiom of foundation, *plus* the anti-foundation axiom.” The elaboration of this axiomatic can be found in Aczel’s *Nonwellfounded Set Theory*.

¹¹³ When I asked Badiou why he did not decide upon ZF+AFA rather than ZF as the axiomatic for ontology, he had this to say:

The question of systems irreducible to ZF or ZFC is a very interesting and important question. I have not closely studied Aczel’s system, as I have, for example, studied Quine’s (NF) or NBG [*sc.* the Neuman-Bernays-Gödel axiomatic for set theory]. Even if, after all, the event turns out to be a singular type of being (characterised by self-membership, the exposition of a site, etc.), that would not be a disaster! You will see that in *Logiques des mondes*, I maintain the ‘illegal’ dimension of the event, but you will also see that it is less important, that that is not is ‘power’. The definition of the latter depends on an entirely different register: that of the ‘sublation’ [*« relève »*] of an inexistent. That said, it is necessary to look at things very closely. Could you point me to where I may study the system AFA in detail, as well as where I could find the relative consistency proof [*sc.* the proof that ZF+AFA is consistent if and only if ZF is]? All the same, I should point out that I find the admission of classes repugnant, whatever conveniences they may offer, because as I see it, they are nothing but the projection of logical, or linguistic, categories, and have no pertinence on the plane of the intuition of multiplicities (or, if you like, on the level [*« au ras »*] of their inconsistency).

My response to the claim that if the event lets itself be thought as a type of being, there would be no disaster, is that this is true only if severe modifications are made to both the metaontology and the theory of the subject put forth in *Being and Event*, since it is precisely insofar as it operates outside the scope of being and identity that the event provides a sufficient point of departure for the subject itself. As I understand it, however, *Logiques de mondes* does institute a number of radical changes in Badiou’s doctrine, and it is too soon to say how successful these may be in accommodating an absorption of the event into ontology (an absorption that Badiou does not attempt in that book, however).

Incidentally, the best source for those interested in studying AFA is Aczel’s own book, *Nonwellfounded Set Theory*. Barwise’s writings provide a nice, philosophical accompaniment to Aczel’s mathematics. See, for instance, *Vicious Circles*.

While the metamathematical consistency proofs in this book do exploit a few proper classes, classes do not form an integral part of the axiomatic itself (such as we find in NBG, for example). Where they are employed in the metalanguage, it is probable that they could be replaced by a suitable recursive technique.

¹¹⁴ Mallarmé, “Un coup de dés...”

¹¹⁵ The apparent proximity of Badiou’s theory of the event to the Schmittian theory of sovereignty has been a popular site of critique amongst Badiou’s critics. See, for example, Giorgio Agamben, *Homo Sacer: Sovereign Power and Bare Life*, trans. Daniel Heller-Roazen, (Stanford: Stanford University Press, 1998), 24-26. Lyotard, too, noticed this proximity, and called the theory of the event given in *Being and Event* “a pure decisionism” and Badiou “a sort of new Carl Schmitt” at a colloquium addressing the publication of Badiou’s book. Badiou responds to these interpretations in (OP, 172-3).

¹¹⁶ The French text reads, “*Il est été*,” and employs *être* as a transitive verb in the passive voice (as one would say, “the goat is milked”). As Hazel Barnes points out (BN, I.I.V, 22), this expression could be translated literally as “It is been,” “an expression as meaningless in ordinary French as in English” (ibid.). Barnes’ translation works well enough to convey the intended meaning, which I hope to have elucidated here, and spares us some syntactical awkwardness. It must nevertheless be kept in mind that nothingness is not “made to be” in the same sense that a child may be “made to cry”; in the second case, the child actually is crying after the act of “making-cry”, while in the first, nothingness no more *is* after the act than before it. It is still only “been”.

¹¹⁷ For unknown reasons, Barnes has seen fit to capitalise every occurrence of “nothingness” in this passage, though Sartre nowhere capitalises *néant* in the French text. She does not consistently capitalise the word throughout the book, and I am at a loss to understand why she does so in the places that she does. I see no reason to preserve this irregularity.

¹¹⁸ The event “‘mobilizes’ the elements of its site, but it adds its own presentation to the mix.” (BE, m17, 182/203)

¹¹⁹ Peter Hallward has already noted the strange involution of Sartrean ontology that Badiou enacts with his theory of the void, writing that in Badiou’s transition from Sartrean to mathematical ontology,

No more was an empty and indeterminate freedom to be faced with the “absolute plenitude and unadulterated positivity” of being [SDF, 19]. From then on, *both* subject and be-ing would be based on that ‘nothing’ or absence of positivity that is the medium of pure thought. (Hallward, 60)

¹²⁰ Badiou, “Custos, quid noctis?”, 861-2.

¹²¹ Badiou, “Custos, quid noctis?”, 862.

¹²² Jacques-Alain Miller, “La Suture (elements de la logique du signifiant),” in *Cahiers pour l’analyse*, vol. 1 (Feb., 1968), 41.

¹²³ Badiou, “Marque et manque,” in *Cahiers pour l’analyse*, vol. 2, 151.

¹²⁴ “Marque et manque,” 163.

¹²⁵ Science, for Badiou, is now understood as being one of the four fundamental varieties of truth procedures, along with love, art and politics. See *Manifesto for Philosophy* for details.

¹²⁶ Alain Badiou, Personal communication, Friday, December 1st, 2006.

¹²⁷ Some time after the completion of this thesis, I had the opportunity to pose the ‘suture problem’ to Badiou in an interview prepared by Tzuchien Tho and myself. The question I asked, and the answer that we received from Badiou, are reprinted in Appendix 3. The complete interview will appear in Badiou, *The Concept of Model*, trans. Z.L. Fraser, (Melbourne: re.press, 2007).

¹²⁸ “...a dysfunction of the count is required, which results from an excess-of-one. The event will be this ultra-one of hazard...” (BE, m4, 56/69) “This perpetual act by which the in-itself degenerates into presence to itself we shall call an ontological act. [...] It is an absolute event...” (BN, II.I.I, 79/115)

¹²⁹ Like so many of the details, but so few of the fundamentals, in *Being and Nothingness*, this example is actually borrowed from Heidegger’s *Being and Time*. See Heidegger’s discussion of the ‘not-yet’, on page 287/H.243 of *Being and Time*.

¹³⁰ See Chapter I for an analysis of the self as perpetually evanescent relation.

¹³¹ To be more precise, we should be speaking, here, of a *triad* and not a *dyad*. This is clear from the context: ‘human reality’ and value form a dyad, but the human reality thus indicated has already been seen to be dyadic, comprising both its existing and its lacking aspects. The include the lacked aspect makes three aspects in total, not two. This triad, in turn, is founded on a more primitive dyad, which is the ‘instantaneous nucleus’ of the reflected-reflecting, and if we include the nothingness that separates the reflected from the reflecting, we have an original, ‘nuclear’ triad. The second triad—that of lack—is logically posterior to the first. We may call these triads the nuclear and the projective triads, respectively.

¹³² The definition of facticity that we concocted at the end of Chapter I is consistent with this thesis, insofar as an application of the facticity operator onto the One gives us $F(\Omega) = \emptyset$. The One is without facticity.

¹³³ To this statement Sartre cautiously adds, “Yet if we are not to be taken in by fine words, we must recognize that this being which is beyond being possesses being in some way at least” (BN, II.I.III, 93/129).

¹³⁴ It could be said that we owe it to Deleuze for having made this amalgam appear more natural than it could otherwise have seemed.

¹³⁵ NE, 186-7. Sartre mentions “the Parmenidean sphere” twice in his discussion of the ethics of force. The first mention occurs about a page before the cited passage. There, Sartre writes, “The violent man does not think he can suppress *being*.”

On the contrary, he thinks that pure Being is indestructible. He thinks he can destroy the diversity on the surface of Being. [...] Being devours Being to end up at the Parmenidean sphere. The goal and final justification of violence is always *unity*.” (NE, 185-6)

¹³⁶ To this Sartrean opposition to Nietzsche/Parmenides, Badiou offers the highest praise, affirming that “it is not man, as Nietzsche thought, that must be overcome. What must be overcome—this is Sartre’s decisive intuition—is being, such as it is being qua being.” Foreshadowing his own doctrine of the subject, Badiou adds that “man is that chance without relation to humanity, that inhuman chance, that stands out as subject in the infinite generic becoming of a truth” (SDF, 15).

¹³⁷ Badiou, *Deleuze*, 97.

¹³⁸ Georges Bernanos, *The Diary of a Country Priest*, quoted in Badiou, *Deleuze*, 97.

¹³⁹ Badiou, *Deleuze*, 97.

¹⁴⁰ Antonin Artaud, *To Have Done with the Judgement of God*, in Susan Sontag, ed. *Antonin Artaud: Selected Writings*, (Los Angeles: University of California Press, 1976), 561.

Elsewhere in *Being and Nothingness*, Sartre makes a similar remark in what is perhaps one of his most memorable aphorisms: “God, if he exists, is contingent” (BN, II.I.II, 81/117).

¹⁴¹ Interestingly, Badiou favours a metaphor similar to Sartre’s, and one which, according to a certain hermetic tradition, immediately succeeds it: taking a page from Mallarmé, he often describes the truth that follows from an event as *the star* that hangs above the infinite expanse of the ocean. For an example of this predilection, see, in BE, ‘Meditation 19: Mallarmé.’ In “Saïssissement, desaisie, fidélité,” Badiou explicitly ties the Mallarméan imagery of the constellation and the throw of dice to the Sartrean theory of the subject.

¹⁴² It is possible, however, that certain analogies may be drawn between the ego, as it is described in TE, and the figure of *value* as described in BN. My hypothesis is that a close analysis of the two figures will show them to have an essentially similar structure, so that the ego as described in TE is not so much an *object*, but a species of *value*; the ego is not transcendent, but lacked.

¹⁴³ It is alarming how often Sartre’s commentators tend to overlook this basic tenet of Sartre’s theory of situations, and tend to instead attribute this thesis to Merleau-Ponty in *opposition* to Sartre! (A crime of which Merleau-Ponty himself is not innocent.) See Simone de Beauvoir’s essay, “Merleau-Ponty and Pseudo-Sartreanisms,” in Jon Stewart (ed.), *The Debate between Sartre and Merleau-Ponty*, (Evanston: Northwestern, 1998), for a survey and a thorough corrective of some of the more prevalent misinterpretations of Sartre.

¹⁴⁴ Cf. Sartre’s discussion of *indifferents* in (BN, IV.I.I, 470/515).

¹⁴⁵ “The anxiety which, when this possibility is revealed, manifests our freedom to our consciousness is witness of this perpetual modifiability of our initial project. In anxiety we do not simply apprehend the fact that the possibles which we project are perpetually eaten away by our freedom, we apprehend that our choice, which is to say ourselves, as unjustifiable.” (BN, IV.I.I, 464/509) See also (BN, I.I.V, *passim*.) for an extended discussion of this phenomenon.

¹⁴⁶ Sartre, *Existentialism and Humanism*, trans. Philip Mairet, (London: Methuen, 1948), 29. Henceforth cited EH.

¹⁴⁷ Andrzej Mostowski, “Recent Results in Set Theory,” in Lakatos, ed. *Philosophy of Mathematics*, (Amsterdam: North-Holland Publishin Co., 1967), 82.

Motowski refers this anecdote to *Grundlagen der Mathematik in geschichtlicher Entwicklung* (München, 1954), p. 316.

^{148/100} “To the extent that we abstract the ‘that which is presented’ in the diversity of situations, to consider the presentation of presentation itself—that is to say, in the end, pure multiplicity—then the real and the possible are rendered necessarily indistinct. What I call ontology is the generic form of presentation as such, considered independently of the question as to whether what is presented is real or possible. It is the reason why people have always debated the status of mathematical idealities, the status of their reality. Are they real, do they exist somewhere, are they merely possible, are they linguistic products...? I think we have to abandon these questions, simply because it is of the essence of ontology, as I conceive it, to be beneath the distinction of the real and the possible.” (Peter Hallward and Alain Badiou, “Politics and Philosophy: An Interview with Alain Badiou,” in *Ethics: An Essay on the Understanding of Evil*, (Verso: New York, 2001), 127-128.)

¹⁴⁹ Badiou, *Deleuze*, 53.

¹⁵⁰ Georg Cantor, “On Infinite, Linear Point-Manifolds,” in *Transfinite Numbers*, trans. & ed. George A. Bingley, (Annapolis, NS: Classics of the St. John’s Program, 1941), 93.

¹⁵¹ Cf. “In fact it seems that this nothingness has slipped into this totality in order to shatter it just as in the atomism of Leucippus non-being slips into the Parmenidean totality of being and makes it explode into atoms.” (BN, III.I.IV, 300/340)

¹⁵² Parmenides, *The Way of Truth*, Frag. 7.

¹⁵³ In both the previous and the following proof, I follow closely the expositions given by Mary Tiles in *The Philosophy of Set Theory: An Introduction to Cantor’s Paradise*, (Oxford: Basil Blackwell, 1989), 64, 110. Badiou also gives an exposition of the following proof in Meditation 26 of *L’être et l’événement*. One may note the affinity between Cantor’s proof and the Russell paradox, to which it is closely related.

It is no exaggeration to call Cantor’s Theorem the ‘fundamental theorem’ of transfinite set theory. In a single stroke, it opened up the boundless field of investigation that is the transfinite hierarchy, and permitted Cantor to state the question that drove set-theoretic research for over seventy years: how many subsets inhere in a given infinite set? Or, more specifically, what is the cardinality of the continuum? Cantor’s initial hypothesis was that it should be \aleph_1 , the next

transfinite number after \aleph_0 , and that in general, the cardinality of $\wp(\aleph_n)$ for any n should be \aleph_{n+1} , with no transfinite cardinals existing between \aleph_n and \aleph_{n+1} , but a proof was not forthcoming: in the 1960s, Paul Cohen successfully demonstrated that the continuum hypothesis is *undecidable* on the basis of the axioms of set theory. Cohen's theorem is of central importance to Badiou's theory of the subject, and I will treat it in greater detail in Chapters III and IV below.

¹⁵⁴ Cantor continues:

One must only understand the expression 'finished' correctly. I say of a set that it can be thought of as *finished* (and call such a set, if it contains infinitely many elements, 'transfinite' or 'super-finite') if it is possible without contradiction (as can be done with finite sets) to think of *all its elements as existing together*, and so to think of the set itself as a *compounded thing for itself*; or (in other words) if it is *possible* to imagine the set as *actually existing* with the totality of its elements.

So the 'transfinite' coincides with what has since antiquity been called the 'actual infinite', and is to be considered as an *αφωρισμενον* ['something determined']. [...] In contrast, infinite sets such that the *totality* of their elements cannot be thought of as 'existing together' or as a 'thing for itself' or an *αφωρισμενον*, and that therefore also in *this totality* are absolutely not an object of further *mathematical* contemplation, I call '*absolutely infinite* sets', and to them belong the 'set of all alephs'.

So much for today.

Georg Cantor, "Letter to Hilbert: 2 October 1897," in William Ewald, ed., *From Kant to Hilbert: A Source Book in the Foundations of Mathematics*, vol. 2, (Oxford: Clarendon Press, 1996), 927.

Cantor would later adopt the convention of withdrawing the appellation 'set' from such inconsistent totalities. In a letter to Dedekind from the year 1899 (see next note), he presents the same basic categories, but calls the absolutely infinite 'inconsistent multiplicity'—a term which Badiou would later borrow in *L'être et l'événement*, but to which he would give a wider conceptual range, applying it to consistent sets considered *from the point of view* of the priority of their being to their consistency.

¹⁵⁵ In a now-famous letter to Dedekind, written in 1899, Cantor wrote,

If we start from the notion of a definite multiplicity (a system, a totality) of things, it is necessary, as I discovered, to distinguish two kinds of multiplicities (by this I always mean *definite* multiplicities).

For a multiplicity can be such that it is impossible to conceive of the multiplicity as a unity, as 'one finished thing'. Such multiplicities I call *absolutely infinite* or *inconsistent multiplicities*.

As we can readily see, the 'totality of everything thinkable', for example, is such a multiplicity; later still other examples will turn up.

If on the other hand the totality of elements of a multiplicity can be thought of without contradiction as 'being together', so that they can be gathered together into 'one thing', I call it a *consistent multiplicity* or a 'set'." (Georg Cantor, "Letter to Dedekind," in Jean van Heijenoort, ed. *From Cantor to Gödel: A Source Book in Mathematical Logic*, (Cambridge, Mass: Harvard University Press, 1967), 114.

¹⁵⁶ That is to say, this abyss is not contingent upon the existence of a subject, save insofar as the existence of ontology itself is the work of a subject. This proviso is the source of several fascinating and difficult problems in Badiou's work, one of which I address in "The Law of the Subject," in Paul Ashton, A.J. Bartlett & Justin Clemens, eds., *The Praxis of Alain Badiou*, (Melbourne: re.press, 2006), 23-70.

¹⁵⁷ Of course, this does not mean that these multiplicities—these sets—are not also extensionally determined. In the last instance, they certainly are, for their identity is uniquely established by the axiom of extensionality. To say that they are also intensionally determined means that the selection of elements which determine a multiplicity are, in each case, prefigured by an intensional criterion.

¹⁵⁸ Gödel gives a concise summary of his proof in a short article from 1939 entitled "The consistency of the generalized continuum hypothesis." He uses slightly different notation than that employed here (though ours conforms to the contemporary standard notation), writing Λ for \emptyset , Σ for \cup , and M for \mathcal{L} . The article is quite brief, and I quote it here in its entirety:

We use the following definitions: 1. $M_0 = \Lambda$; 2. $M_{\alpha+1}$ is the set of those subsets of M_α which can be defined by propositional functions containing only the following concepts: \sim , \vee ['or'], the \in -relation, elements of M_α , and quantifiers for variables with range M_α ; 3. $M_\beta = \sum_{\alpha < \beta} M_\alpha$ for limit numbers β . Then M_{ω_ω} or M_Ω (Ω being the first inaccessible number) is a model for the system of axioms of set theory (as formulated by A. Fraenkel, J. von Neumann, T. Skolem, P. Bernays) respectively without (or with) the axiom of substitution [sc. axiom of replacement], the generalized continuum hypothesis ($2^{\aleph_\alpha} = \aleph_{\alpha+1}$) [sc. another formulation of $|\wp(\aleph_\alpha)| = \aleph_{\alpha+1}$] being true in both models. Since the construction of the models can be formalized in the respective systems of set theory themselves, it follows that $2^{\aleph_\alpha} = \aleph_{\alpha+1}$ is consistent with the axioms of set theory, if these axioms are consistent with themselves. The proof is based on the following lemma. Any subset of M_{ω_α} which is an element of some M_β is an element of $M_{\omega_{\alpha+1}}$. This lemma is proved by a generalization of Skolem's method for constructing enumerable models. Since the axiom of choice is not used in the construction of the models, but holds in the models, the consistency of the axiom of choice is obtained as an incidental result. (Kurt Gödel, "Consistency Proof for the Generalized Continuum Hypothesis (1939)," in Gödel, *Collected Works, Vol II: Publications 1938-1974* ed.

Solomon Feferman et al., (Oxford: Clarendon Press, 1989), 27.)

Badiou provides Gödel's proof with a more lengthy exegesis than the one I present in this thesis in (BE, m29). Other insightful discussions of this demonstration can be found in Tiles' *The Philosophy of Set Theory*, and in Jean Cavailles' *Transfinité et continu*, (Paris: Hermann, 1947).

¹⁵⁹ "In 1915 Löwenheim proved that any satisfiable sentence is satisfied in some model that is at most denumerable. A few years later, Skolem extended this result to sets of sentences." (Bas C. van Fraassen, *Formal Semantics and Logic*, (New York: Macmillan Company, 1971), 122.) Van Fraassen offers a demonstration on the following pages.

¹⁶⁰ "We can speak in an absolute way of the property of being a set, but this property does not have an absolute extension. Its extension is a function of which universe of set theory we take our activity to concern." (Jon Barwise, "Situated Set Theory," in *The Situation in Logic*, (Standord: CSLI, 1989)

¹⁶¹ I refer the reader again to Barwise's "Situated Set Theory" on this point.

¹⁶² The reason for this has to do with Gödel's theorem that GCH is true in any model that can be shown to be *constructible*.

¹⁶³ I have adjusted Cohen's notation to conform to Badiou's, in order to reduce confusion in the following exegesis. Here, this involves putting 'S' for 'M', ' $S(\varphi)$ ' for 'N', and ' φ ' for 'a'. Of course, none of these changes affect the material content of Cohen's remarks.

¹⁶⁴ Hao Wang, *Kurt Gödel: A Logical Journey*, (Cambridge, Mass.: MIT Press, 1996), 252.

¹⁶⁵ To be more specific, the forcing relation is isomorphic to classical entailment when the set of forcing conditions is generic, and isomorphic to intuitionistic entailment when this is not the case (for example, when the set of forcing conditions is finite, or even a finite subset of a generic set). For a detailed study of the implications of this fact on the system laid out in *Being and Event*, see my essay, "The Law of the Subject: Alain Badiou, Luitzen Brouwer, and the Kripkean Analyses of Forcing and the Heyting Calculus," in Ashton, Bartlett & Clemens, eds., *The Praxis of Alain Badiou*, (Melbourne: re.press, 2006), pp.23-70. (Also available for download at <http://www.cosmosandhistory.org>)

¹⁶⁶ "Contains" here is a bit of an abbreviation. As an inspection of the formulae employed above will reveal, neither elements of \mathbb{N}_+ nor elements of \mathbb{N}_0 are themselves *elements* of φ . To say that 5, for example, is 'contained' by φ , in this context, abbreviates the proposition that some condition $\{<\alpha, 5, \varphi>, \dots\}$ is an element of φ . Since $<\alpha, 5, \varphi>$ is shorthand for $\{\{\alpha\}, \{\alpha, \{5\}, \{5, \{\varphi\}\}\}\}$ (the standard form by which ordered sequences are formalised in ZF), it is technically more correct to say that 5 is *an element of an element of an element of an element of an element of φ* , or that 5 is an element of $U(U(U(U(\varphi))))$. But this is a bit cumbersome.

¹⁶⁷ Here, for reasons similar to those explained in the previous endnote, Badiou ought to say, "contain conditions of the type $\{<\alpha, 5, 0>\}$ or $\{<\alpha, 5, 1>\}$," for conditions are not ordered triples, but *sets of ordered triples*.

¹⁶⁸ See Chapter IV of Paul Cohen's text for details.

¹⁶⁹ Badiou's presentation of these results is to be found in § 4 of Meditation 34 of *L'être et l'événement*.

¹⁷⁰ "The sheaves refuse my bonds. In this infinite, unanimous dissonance, each ear of corn, each drop of blood, speaks its language and goes its way. The torch, which lights the abyss, which seals it up, is itself an abyss." (Jacques Dupin, *Lichens*, trans. Paul Auster, in Auster, ed. *Jacques Dupin: Selected Poems*, (Newcastle: Bloodaxe Books, 1992), verse 4, p.5).

¹⁷¹ "I term *subjectivization* the emergence of an operator [*sc.* of fidelity], consecutive to an interventional nomination." (BE, m35, 393/430)

¹⁷² "Almost all situations are infinite" (BE, m23, 235/260).

¹⁷³ This duality is both the reason for and the effect of the event's reflexivity; that a 'causal' description of the event should be circular is of a piece with this very reflexivity, and with the 'miraculous' nature of the event (Badiou does in fact compare events to miracles (See Meditation 21 of *Being and Event*, in which Badiou reads Pascal).

¹⁷⁴ Sartre's assertion that "the pure event by which human reality rises as a presence in the world is apprehended by itself as *its own lack*" (BN, II.I.I, 89/125), would thus receive a valid interpretation in both philosophies, even if in slightly different senses.

¹⁷⁵ "Structure" here is to be taken in its technical sense, as "what prescribes, for a presented multiple, the regime of its count-as-one" (BE, m1, 24/32). The count-as-one of the generic truth is not prescribed by the situation, and does not take place within it ($\varphi \notin S$).

¹⁷⁶ "It is impossible for mathematical ontology to dispose of a concept of truth, because any truth is post-evental, and the paradoxical multiple that is the event is prohibited from being by that ontology. The process of a truth thus entirely escapes ontology." (BE, m33, 355/391)

¹⁷⁷ In the same vein, Badiou elsewhere writes:

such a subset is a truth of the situation as such, an immanent production of its pure multiple being, a truth of its being qua being—as opposed to a knowledge of this or that regional particularity of the situation. ("Truth: Forcing and the Unnameable," in TW, 124)

¹⁷⁸ Jacques Dupin, *Lichens*, trans. Paul Auster, in Auster, ed. *Jacques Dupin: Selected Poems*, (Newcastle: Bloodaxe Books, 1992), verse 4, p.5.

¹⁷⁹ Immanuel Kant, *Critique of Practical Reason*, trans. T.K. Abbott, (Amherst: Prometheus Books, 1996), 148.

¹⁸⁰ In its original context, this formula appears as follows:

Certainly we could apply to consciousness the definition which Heidegger reserves for *Dasein* and say that it is a being such that in its being, its being is in question. But it would be necessary to complete the definition and formulate it more like this: *consciousness is a being such that in its being, its being is in question in so far as this being implies a being other than itself.* (BN, in.V, lxii/29)

¹⁸¹ Badiou, 'L'entretien de Bruxelles,' 24.

¹⁸² Even in his first book, *Le Concept de modèle*, we find the now-familiar repudiation of the category of object. In Badiou's introduction to the procedures involved in the construction of a logico-mathematical model, we read:

The first idea is to secure the domain of objects where the correspondence with the marks of the system is founded. And yet nothing is more indistinct, and more empiricist, than the notion of a collection of objects, to the point that if it maintains this notion, semantics will have no chance of articulating itself scientifically: *it is solely to the measure that it deploys the mathematical concept of set, and consequently transforms the notion of domainial multiplicity [multiplicité domainiale] that the theory of interpretations of a formal system escapes this powerlessness.* (Badiou, *Le Concept de modèle*, 37)

¹⁸³ Alain Badiou, "Orientation de pensée transcendante," a seminar conducted on October 24th, 1987 at Paris University VIII. I wish to thank François Nicolas for providing me with the notes that he had compiled from this seminar.

¹⁸⁴ Hazel E. Barnes, "Sartre as Materialist," in Peter Arthur Schlipp, ed. *The Philosophy of Jean-Paul Sartre*, (La Salle, Illinois: Open Court, 1981), 661-684. The thesis of this article is, in a nutshell, given on p. 662: "Sartre's ontology, as developed in *Being and Nothingness* and anticipated as early as *The Transcendence of the Ego*, must properly be viewed as a new and unique brand of materialism."

¹⁸⁵ Notably, Badiou understands both politics and psychoanalysis to be modes in which truth procedures may come into effect.

¹⁸⁶ For a detailed discussion of Badiou's theory of conditions, see his *Manifesto for Philosophy*.

¹⁸⁷ Interestingly, this doctrine is also at the furthest remove from the Aristophanic myth of love as fusion. The subject of an amorous truth is not the molten amalgam of the two lovers, but the activation of the irreducible difference between them. For details, see Badiou's "What is Love?"

¹⁸⁸ Badiou, *Ethics: An Essay on the Understanding of Evil*, trans. Peter Hallward, (London: Verso, 2001), 43-4.

¹⁸⁹ Badiou, *Logiques des mondes: L'être et l'événement 2*, (Paris: Seuil, 2006), 403.

¹⁹⁰ In its original context, this remark is directed at Lacan, not Sartre. It reads: "What Lacan still owes to Descartes, a debt whose account must be closed, is the idea that there have always been subjects" (BE, m37, 434/474*).

¹⁹¹ The essay also betrays a dearth of concrete examples that is quite foreign to both Sartre and Badiou (especially the former, who graces us with no less than one hundred and three examples and concrete illustrations throughout *Being and Nothingness*). I have no better explanation to offer for this deficit than did Kant:

As regards *clearness*, the reader has a right to demand, in the first place, a *discursive* (logical) clearness, through *concepts*, and secondly, an *intuitive* (aesthetic) clearness, through *intuitions*, that is, through examples and other concrete illustrations. For the first I have sufficiently provided. That was essential to my purpose; but it has also been the incidental cause of my not being in a position to do justice to the second demand, which, if not so pressing, is yet still quite reasonable. I have been almost continuously at a loss, during the progress of my work, how I should proceed in this matter. Examples and illustrations seemed always to be necessary, and so took their place, as required, in my first draft. But I very soon became aware of the magnitude of my task and of the multiplicity of matters with which I should have to deal; and as I perceived that even if treated in dry, purely *scholastic* fashion, the outcome would by itself be already quite sufficiently large in bulk, I found it inadvisable to enlarge it yet further through examples and illustrations. These are necessary only from a *popular* point of view; and this work can never be made suitable for popular consumption. Such assistance is not required by genuine students of the science, and though always pleasing, might very well in this case have been self-defeating in its effects. Abbot Terrasson has remarked that if the size of a volume be measured not by the number of its pages but by the time required for mastering it, it can be said of many a book, *that it would have been much shorter if it were not so short*. On the other hand, if we have in view the comprehensibility of a whole of speculative knowledge, which, though wide-ranging, has the coherence that follows from unity of principle, we can say with equal justice *that many a book would have been much clearer if it had not made such an effort to be clear*. For the aids to clearness, though they may be of assistance in regard to details, often interfere with our grasp of the whole. The reader is not allowed to arrive sufficiently quickly at a conspectus of the whole; the bright colouring of the illustrative material intervenes to cover over and conceal the articulation and organisation of the system, which, if we are to be able to judge of its unity and solidity, are what chiefly concern us. (Kant, *Critique of Pure Reason*, trans. Norman Kemp Smith, Axviii-xix)

¹⁹² Sam Gillespie's work on Badiou can also be seen in this light. Gillespie, too, tackles the problem of the relation between the event and fidelity, searching for the source of the subject's impetus in the structures of the two phenomena. His solution is to propose a Lacanian model to verify this empty relation: he interprets the event as having the structure of Lacan's *objet petit (a)*, and the driving force as being the affect of anxiety. It would be an interesting project to see to what extent Gillespie's Lacanian interpretation is consistent with the Sartrean interpretation I offer here—the phenomenon of anxiety

offers one obvious point of contact, since the Sartrean reconstruction predicts this as well (see Chapter VII). Recently, Sara Vassallo has produced an extensive study drawing connections between the Sartrean theory of lack and Lacanian psychoanalysis, which would no doubt be fruitful in pursuing this sort of inquiry. See Gillespie's posthumous publication, "Giving Form to its Own Existence: Anxiety and the Subject of Truth," in Paul Ashton, A.J Bartlett & Justin Clemens (eds.), *The Praxis of Alain Badiou*, and Vassallo, *Sartre et Lacan: Le verbe être: entre concept et phantasme*, (Paris: L'Harmattan, 2003).

¹⁹³ I say that it is *conceptually* compatible, rather than compatible as a mere matter of fact, insofar as *any* finite subset of a minimal model can be given a generic extension. On the other hand, if a finite subset is intensionally determined not only *de facto* (as is any finite set, retrospectively considered) but *de jure*, then it is manifestly incompatible with the idea of genericity, insofar as its inherent law has already decided that its extension will be in agreement with its prior intension. For details, see my "The Law of the Subject: Alain Badiou, Luitzen Brouwer, and the Kripkean analysis of Forcing and the Heyting Calculus," in *The Praxis of Alain Badiou*, (Melbourne: re.press, 2006), pp. 23-70.

Appendix 1: Glossary of Logical and Mathematical Notation

Of the following symbols, only \exists , \sim , $\&$, $()$ and \in need be taken as *primitive*. The others can all be defined by means of these five. It is nevertheless convenient to have separate notation for them. The following table includes most of the mathematical and logical notation used in the thesis, save for a handful of signs used for more complex mathematical concepts that are clearly defined in the body of the text (signs such as φ and \perp , for instance).

In the table below, P and Q stand for formulae, while α and β stand for sets. x is a variable (which, in the current context, ranges over sets).

Sign	Name	Syntactic Context	Meaning
<u>Logical signs</u>			
\exists	Existential quantifier	$(\exists x)(P)$	there exists an x such that P
\forall	Universal quantifier	$(\forall x)(P)$	for all x , P
\sim	Negation	$\sim P$	it is not the case that P
$\&$	Conjunction	$P \& Q$	P and Q
\vee	Disjunction	$P \vee Q$	P or Q
\rightarrow	Implication	$P \rightarrow Q$	if P then Q
\leftrightarrow	Equivalence	$P \leftrightarrow Q$	P if and only if Q
$()$	Parentheses	(P)	used for punctuation, enclose formulae or subformulae
$[]$	Square Brackets	$[P]$	same as parentheses, used to reduce ambiguity
<u>Set theoretical signs</u>			
\in	Belonging	$\alpha \in \beta$	α belongs to (is an element of) β
\notin	Non-belonging	$\alpha \notin \beta$	α does not belong to β
\subset	Inclusion	$\alpha \subset \beta$	α is included in (is a subset of) β (if x belongs to α then x belongs to β , for all x)
$\not\subset$	Non-inclusion	$\alpha \not\subset \beta$	α is not included in β (there exists an element x that belongs to α , but not to β)
$=$	Identity	$\alpha = \beta$	α is identical to (has the same elements as) β
\neq	Non-identity	$\alpha \neq \beta$	α is <i>not</i> identical to β (either α has an element that β does not, or vice versa)
\cap	Intersection	$\alpha \cap \beta$	the set of all elements belonging to <i>both</i> α and β
\cup	Union	$\alpha \cup \beta$	the set of all elements belonging to

–	Set Subtraction	$\alpha - \beta$	<i>either α or β</i> the set whose element belong to α but not to β
{ }	Set brackets	$\{\alpha, \beta \dots\}$	a set containing $\alpha, \beta \dots$ (the brackets enclose the contents of a set).
< >	Sequence brackets	$\langle \alpha, \beta \dots \rangle$	the <i>sequence</i> containing $\alpha, \beta \dots$, <i>in</i>
\wp	Powerset Operation	$\wp(\alpha)$	the set of all subsets of α (the set of all x such that x is included in α)
\emptyset	Empty Set	\emptyset	the set to which nothing belongs
ω_n	n^{th} Limit Ordinal	ω_n	the n^{th} ordinal that is not a successor (that is not equal some other ordinal $m + 1$); these ordinals are <i>transfinite</i> identical to ω_n , differs only in context; used when discussing cardinality (quantity) rather than order
\aleph_n	n^{th} Cardinal	\aleph_n	
	Cardinality Operator	$ \alpha $	indicates the cardinality (the ‘size’) of α ; the cardinality of α is the smallest ordinal that can be put into a one-to-one correspondence with the elements of α

Appendix 2: The Axioms of Zermelo-Fraenkel Set Theory

The Axiom of Extensionality

$$(\forall z, y)(z = y \leftrightarrow (\forall x)(x \in z \leftrightarrow x \in y))$$

z and y are identical if and only if they possess all of the same elements. This axiom is discussed in Chapter IV of the thesis.

The Axiom of the Empty Set

$$(\exists x)(\neg(\exists y)(y \in x))$$

There exists a set to which nothing belongs. This set is unique after the axiom of extensionality, and we denote it ' \emptyset '. This axiom is discussed in Chapters V and VI.

The Axiom of Unions

$$(\forall x, y)(\exists z)(z = x \cup y)$$

For every two sets x and y , there exists another set z which contains all the elements of x and all the elements of y .

The Axiom of Power Sets

$$(\forall x)(\exists y)(\forall z)(z \in y \leftrightarrow z \subset x)$$

For every set x , there exists another set y whose elements are the subsets of x . This axiom is discussed in Chapter VIII.

The Axiom of Infinity

$$(\exists x)(\emptyset \in x \ \& \ (\forall y)(y \in x \rightarrow y \cup \{y\} \in x))$$

$y \cup \{y\}$ is the set-theoretic representation of the *successor* of y when y is an ordinal (i.e. $y \cup \{y\} = y + 1$). This axiom states that there exists a set to which \emptyset belongs, and which contains the successors of all of its elements. The successor of \emptyset is $\{\emptyset, \{\emptyset\}\} = 1$, so 1 belongs to this set; the successor of 1 is $\{\emptyset, \{\emptyset, \{\emptyset\}\}\} = 2$, so 2 belongs as well, etc. The result is a set which contains *all* the finite ordinals, and which, therefore, is *infinite* or *transfinite*. This set is uniquely determined, and we denote it ω_0 . It is the smallest transfinite ordinal.

The Axiom Schema of Replacement

$$(\forall a)[(\forall x, y, z)(x \in a \ \& \ \varphi(x, y) \ \& \ \varphi(x, z) \rightarrow y = z) \rightarrow (\exists b)(\forall y)(y \in b \leftrightarrow (\exists x)(x \in a \ \& \ \varphi(x, y)))]$$

This unweildly-looking schema is not as complicated as it appears. In essence, it states that, given a set a , there exists another set b , whose composition results from replacing the elements of a , one by one, with new elements, and that the law governing this replacement is *functional*, which means that this law (expressed by a 'propositional function' φ) relates each element of a to one and only one element of the new set. Accordingly, the new set b can be smaller than a , but not larger. Because ZF set theory is expressed in first-order predicate logic, we cannot quantify over the propositional functions φ . If we could, we would preface this formula with ' $(\forall \varphi)$ '. As it stands, we are forced to write the 'axiom' as an infinite series of particular axioms, each with a different propositional function. For this reason, the Axiom of Replacement is called an 'axiom schema'.

Separation Theorem-Schema

$$(\forall a)(\exists b)(\forall x)(x \in a \ \& \ \pi(x) \rightarrow x \in b)$$

Often, this schema is presented as an axiom in weaker versions of set theory, in lieu of the axiom of replacement. In *Being and Event*, Badiou presents both in the same axiomatic for set theory. This is unnecessary, however, since the 'Axiom (Schema) of Separation' can be easily derived from the Axiom Schema of Replacement, by fixing φ so that it is endomorphic to a (i.e., so that it maps elements of a to elements of a subset of a). In essence, the Separation Theorem allows us to abstract from an existing set according to a property expressible in the formal language. For example, given the set of natural numbers, we may separate the set of primes.

The Axiom of Foundation

$$(\forall x)(\exists y)(y \in x \text{ \& } y \cap x = \emptyset)$$

Each set contains an element with which it has no elements in common. This axiom is discussed at length in Chapter V of the thesis.

The Axiom of Choice

$$(\forall x)(\exists f)(\forall y)(y \in x \rightarrow f(y) \in y)$$

First, note that f is a *function* (i.e. a set of ordered pairs $\langle a, b \rangle$ such that if $\langle a, b \rangle$ and $\langle a, c \rangle$ both belong to f , then $b = c$. This means that f assigns a unique value to each argument). This axiom states that for every set x there exists a function that 'chooses' an element from each of the elements of x , without stating a general rule for how this choice is made.

Appendix 3: Two Interviews with Badiou

Part I: “Réponses à Luke Fraser”, 2005

The following ‘interview’ is no more than the result of an informal exchange of letters. I sent the four questions together in a single text, hence the lack of interaction between Badiou’s responses and my questions. The questions are composed a bit roughly, and are scattered with subtle misunderstandings that I would later come to correct. They nevertheless seemed to maintain enough interest to warrant their inclusion in this appendix. They also serve the useful function of illustrating some of the lines of thought that fed into this thesis. The crude and circular definition of multiplicity as a relational structure that I offer here, for example, was later refined into the definition with which the entire thesis is framed. I remain much closer to Krivine than to Badiou on this point, and find it self-evident that ZF is “the theory of binary relations satisfying the Zermelo-Fraenkel axioms.”¹ The history of foundational mathematics in the twentieth century bears witness to the robustness of the dialectic between relation and multiplicity, each concept usurping the other in turn. (The pendulum seems to have swung back to relation these days, with the ascendancy of category theory over set theory in current mathematical practice.)

*ZLF: It seems to me that the ontico-ontological difference in your work is doubled. The being of a situation is its set, while the being of the set is ultimately absolute inconsistency. What exactly is the ontological status of the set? You say in *L’être et l’événement* that you are not making the Pythagorean claim that the world is composed of mathematical objects, but you often state that “multiplicities are all there is” and that these multiplicities are, univocally, sets. At times, however, the correspondence between a being and its set seems to be a strictly lateral one, where a given set is the being of an entity as refracted through the ontological situation. If this is the case, then the difference between a given being and its set is not ontological, since the set would exist in the same sense that any other being would. If this is the case, what establishes the correspondence between an entity (a situation) and its mathematical set? What makes the set the ontological description of the entity?*

AB: The difficult point in your question is the sense of the word ‘correspondence’. In my theoretical apparatus [*dispositif*], I believe that the question, ‘what is the correspondence between a being and its set’ is deprived of signification. In effect, only the notion of set gives sense to that of being, in the context of an ontology—so that it must be posited that ‘all being’ [*« tout l’être »*] is thought in and by the set. This does not at all mean that a given multiplicity is ‘the same thing’ as the set, but only that insofar as one thinks it in its being, one thinks it as a set. One can also think this multiplicity in its appearing, or in its ‘world’. The meaning of this is exhaustively investigated in my book *Logiques des mondes*, which will appear in March. But even then, if one asks what the ‘correspondence’ between a being-there and ‘its set’ might be, the only answer one would have is that the set is exactly *what* appears, without being able to distinguish ‘what’ the set is [*sans pouvoir distinguer entre « ce qui » est l’ensemble.*]

ZLF: The opposition that you draw between multiplicity and relationality strikes me as a bit odd. The most basic notion of ‘multiplicity’ is, in my view, the empty form of relationality. X is a multiplicity if the being of X is exhausted in accounting for the extent of beings that X is in

¹ Jean-Louis Krivine, *Théorie axiomatique des ensembles*, (Paris: Presses universitaires de France, 1969), 6.

relation to. If X has being independent of its relata then X is not a multiplicity but a relation-bearing unit.² I realise that it would be inconsistent to replace 'any relation whatsoever' with ' \in '—any symmetrical relation, for instance. But it is the nature of at least all mathematical relations to be reducible to certain configurations of set-membership. I understand that, to a certain degree, 'multiplicity' must remain an undefined term, and my relational definition of it is essentially circular (it uses the phrase 'the extent of beings that X is in a certain relation to'). I agree completely that, ultimately, the only way of rigorously defining multiplicity is through its axiomatisation. But so long as it is a matter of interpretation (of metaontology), the consequence of not identifying multiplicity and relationality completely is the privileging of the specific relation of 'belonging'. As soon as we leave axiomatic set theory to apply this concept to concrete situations, what seems to happen is that the intuitive or imaginary notion of belonging or containment takes over, and this is clearly the sort of thing that you are trying to avoid. Your ontology can offer a robust response to the 'ambiguous' and phenomenological ontologies of relation (Merleau-Ponty's, e.g.), but only if multiplicity subsumes relation rather than subtracts itself from it. (It handles quite nicely, for example, Merleau-Ponty's thesis that "it is of the nature of the real to compress into each of its instants an infinity of relations," and certainly brings this to a level of clarity that phenomenology cannot.)

AB: When I say that relation doesn't exist, I am naturally alluding to those metaphysics for which relation is primary, constitutive, etc. And which, consequently, reduce rationality to a theory of judgement whose norm is the exact character of the relations affirmed. Against the primacy of relation (for example, that of the intentional relation with Husserl), I declare that the entire being of relation, just like that of being in general, lets itself be thought in the concept of pure multiplicity. In other words, a relation is a particular type of multiplicity. From this point of view, the relation of belonging effectively poses a question, which you very justly elaborate. I will willingly say that this relation is not a relation in the initial sense of the term. It is only the relational (or discursive) elaboration of a thought of the multiple which must confront two forms of immanence, or of being-in. Namely, belonging (the elementary logic of being) and inclusion (the 'mereological' logic of being). This platform of immanence is in reality more fundamental than any relation. It is on this basis that one can distinguish between multiplicities that are relations and those which are not.

ZLF: You write in "Platonism and Mathematical Ontology" that "the Platonist's desire is for maximal extension in what can be granted existence: the more existences, the better." What do you think, then, of using Aczel's hyperset theory ($ZF+AFA$) for ontology? Aczel's theory provides a much richer universe of sets, but a consequence of this is that the matheme for the event is no longer inconsistent and extra-ontological (since the set $e_x = \{x \in X, e_x\}$ is not inconsistent in $ZF+AFA$). As I understand it, $ZF+AFA$ is consistent if and only if ZF is consistent. (I'm not certain about this. I read it somewhere recently, but I haven't seen the proofs.) If we identify $ZF+AFA$ with ontology, what is the event? If $ZF+AFA$ cannot be called ontology because it does not foreclose the matheme of the event (EE, 212), what is it that $ZF+AFA$ speaks of? Is the event such that it is inconsistent with any axiomatisation of ontology (so that if the ontology is reconfigured, the event's inconsistency is refracted differently)?

AB: The question of systems irreducible to ZF or ZFC is a very interesting and important question. I have not closely studied Aczel's system, as I have, for example, studied Quine's (NF)

² This is the basic intuition that was eventually developed into the definition of multiplicity with which I have framed the thesis.

or NBG [*sc.* the Neuman-Bernays-Gödel axiomatic for set theory]. Even if, after all, the event turns out to be a singular type of being (characterised by self-membership, the exposition of a site, etc.), that would not be a disaster! You will see that in *Logiques des mondes*, I maintain the ‘illegal’ dimension of the event, but you will also see that it is less important, that that is not is ‘power’. The definition of the latter depends on an entirely different register: that of the ‘sublation’ [« *relève* »] of an inexistent. That said, it is necessary to look at things very closely. Could you point me to where I may study the system AFA in detail, as well as where I could find the relative consistency proof? All the same, I should point out that I find the admission of classes repugnant, whatever conveniences they may offer, because as I see it, they are nothing but the projection of logical, or linguistic, categories, and have no pertinence on the plane of the intuition of multiplicities (or, if you like, on the level [« *au ras* »] of their inconsistency).

ZLF: *I’m curious as to how you would see the difference between your understanding of the thesis that mathematics is ontology, and Husserl’s, given that he makes the same claim in Formal and Transcendental Logic. It’s clear enough that you each treat the thesis quite differently, but I would like to hear what you have to say about your specific points of divergence with Husserl over this shared thesis.*

AB: It’s been so long since I’ve read all that! But it seems to me that Husserl’s fundamental category is that of *formal* ontology. This supposes that another approach to the thinking of being is possible, even necessary. That ontology cannot be exclusively formal is what drives Husserl’s genesis [*devenir*], and also explains the ‘passage’ from phenomenology to Heidegger. In his seizure of the ‘*Lebenswelt*’, with accents similar to those of Bergson, which one finds very early in Husserl, he seems to me to exercise the philosophical will to simply go beyond the formal dimension of ontology, to show that this formalism is itself a restrictive sedimentation. As for myself, in a sense more Platonic than Husserl, I think that it is of the destiny of a rational ontology to be a theory of Forms, and to have no other available paradigm than the *matheme*. And so we come back to where we started: insofar as the project is the appropriation by thought of the being what is given, seized in its being ‘as such’ [« *tel quel* »], then in its historical dimension this project has to be strictly mathematical.

Part II: Excerpt from “*The Concept of Model, 40 Years Later: Interview with Alain Badiou (2007)*”

The following is an excerpt from an interview that Tzuchien Tho and I conducted with Badiou in the Spring of 2007. The full text will be appearing as an appendix to my translation of Badiou’s 1968 text, *Le Concept de modèle* (re.press, forthcoming). Many of the questions posed in the interview were formulated by Tzuchien and I beforehand, shortly after I completed the body of my thesis. The interview provided me with the opportunity to pose several questions that had stubbornly survived all the efforts towards a solution that I had made while working on my thesis. The excerpt presented here contains a handful of these, including, most notably, a question bearing on the awkward relation between the ontological situation and the situations allegedly schematised by it in ontological form, a problem tackled with limited success in Chapter III of my thesis.

The questions themselves were put to Badiou by Tzuchien, who was fortunate enough to be in Paris at the time. My questions are marked **ZLF**, Tzuchien’s, **TT**. The footnotes to this text have all been provided by Tzuchien.

ZLF: The topic of formalism has long been central to your work, and has recently been revitalized in *The Century* and *Logiques des mondes*. *The Century* concludes with the directive to overcome the 'animal humanism' [l'humanisme animal] that characterizes our current ideology by way of a 'formalised in-humanism' [in-humanisme formalisé].³ *Logiques des mondes* defines the subject as being, in general, 'un formalisme porté par un corps'.⁴ Among other things, *The Concept of Model* offers us an extended analysis of what mathematical formalization is, shedding light on the way in which this operation is thoroughly bound up with mathematics' relation to its own historicity. Formalization, in *The Concept of Model* and other early texts, however, is understood as an important dimension of the a-subjective process that is mathematics, whereas the concept later reappears as a dimension of the subject itself.

My questions here are very general. First of all: what has changed in your understanding of the subject and formalism, such that they have ceased to be mutually exclusive and come to be implicated in one another? Second: how far can we generalize the key theses in *The Concept of Model* on formalism—are they specific to mathematics, or can they be brought to bear on the other modalities of subjective formalism addressed in your recent work? What are the essential differences between, say, mathematical, artistic, and political formalism?

AB: This is a very just, very good question. I think you have posed a central question. This is a question of the relation between formalism, that is, form, and the theory of the subject.⁵ It is clear that in the first part of my work, the general orientation was one of separation, of delimitation, between formalism, considered as completely objective activity, perhaps even as objectivity itself, and the subject, which I interpreted, in those days, a bit like Althusser, that is, as an ideological given (I think because I was still an Althusserian at the time even if he later talked about the subject as ideological interpellation). I think in that period I was freeing myself from Sartre, existentialism and phenomenology, but unfortunately it was not a focused or deliberate effort and thus the subject was rejected as ideology. This brought me to a critique of Lacan that we see in "Marque et Manque", an article published around the same time, where I said that the thesis of the suture is a thesis which does not permit us to account for mathematical formalism, which I considered as principally non-subjective. This is to say that mathematical formalism is neither sutured nor subject but, in fact, non-subjective.⁶ Here I was at a preliminary point of departure. I would say that what first seduced me in my mathematical education was the non-subjective, the making possible of a capacity to think outside of all intentionality and subjectivity. I reconsidered this point when I came to understand that, even if it is not right to consider formalism as something constituted as intentional subjectivity, it was necessary to take and maintain some aspects of subjectivity in the elements of formalism itself. I am not saying this for political reasons but I remain convinced that every philosophy that eliminates the category of the subject becomes unable to serve a political process. That is not to say that a subject should be identified from the outset as the working class or the like. Certainly, there is subjectivity in politics and there is subjectivity in art and subjectivity in love and subjectivity in science itself. As such, I attempted to completely rethink the relation between formalization and subjectivity.

To answer the first question, I attempted to recommence, as one should always do, with the theme of separation, radically and absolutely. From *Théorie du Sujet* and onwards in the

³ Alain Badiou, *The Century*, trans. Alberto Toscano, Polity, Cambridge, 2007, pp.177-78.

⁴ 'A formalism carried by a body.' *Logiques des mondes*, p. 593.

⁵ Badiou develops the relation between form and formalism in *Being and Event* especially with respect to his notion of subjectivity in meditation twenty-two, 'The form-multiple of intervention'. *Being and Event*, pp. 223-231.

⁶ Alain Badiou, 'Marque et Manque: a propos de zéro', *Cahiers pour l'analyse*, no. 10, 1969.

1980s, I began again to re-knot these terms in a different way.⁷ But that is not to say that I abandoned the primacy of formalization because it remained a thought. I found it necessary to re-knot this formalism with the figure of the subject. But, from that, what do I conserve of the separation or what do I still hold of the separation? I think the separation I maintain is the idea that the relation between the subject and formalism is on the side of formalism and not on the side of the subject. In the rigorous examination of formalization, one can dispose or place the subject, ultimately, as an effect and not as a cause. Thus, finally, an event is that which renders possible a new formalism for such a relation. The subject will be the subject of the formalism or with respect to this formalization. Where there is an effect of puncture in the particular underlying structure, the subject will be defined as a new process of formalization. It is a bit messy, but there you go. If we want to think the subject, one should begin by thinking through formalization.

On the second point, formalization, in its essence, is not only mathematical or logical. While mathematical and logical formalization is a paradigm for formalization, formalization is not identical with this. The question of knowing what is, or how one should analyze the formalization of artistic or political formalization comes down to a question of analyzing those very sequences. If we have an event that makes possible a new formalization, we should study this possibility of formalization for itself. There, the principles of formalization, even if they can be comparable to or analyzable with mathematics, would not be identical. Definitively, the study of the different types of generic procedures is truly the study of the different types of formalization. You know, as I described in *Logiques des mondes*, if we are to begin a study of a particular formalization, it would be necessary to return to the position of the difference between formalization and the eventual *énoncé*, the stakes are always there.⁸ Well, what follows from this are the enquiries particular to the process. Thus the general schema is that every event is an opening of a new possibility of formalization, carried forth by a new body. This new body always supports the formalization with respect to formal articulations.⁹ This is maintained for every truth procedure, and the study of particular formalizations will be the study of a regional world.

ZLF: *Mathematics has always held a privileged place in your thought, from your earliest published works to the most recent. Beyond this rather general interest in mathematics, however, model theory—the topic of your first book—seems to stand out as of utmost importance. From The Concept of Model to Being and Event, several of the canonical mathematicians in your work are those whose greatest breakthroughs have been in the theory of models—Gödel and Cohen leap to mind here.*

In the work of these mathematicians, we see model theory employed in a manner that strikes a profound resonance with several of your own writings; it combines, in an almost paradoxical fashion, an attentive examination of the effects of formalization with a sort of underlying Platonism which aims to locate the precise points where a given formalized theory transcends itself, or opens onto a point of undecidability. I say that this combination seems paradoxical because we find, often together in the same texts, a commitment to the (formalistic) identity or immanence of mathematical thought and its mode of expression, but also the espousal of the Platonistic idea that the reality accessed by mathematical thought transcends its mode of expression. Without the former conviction, one could not seriously take, say, a denumerable

⁷ Alain Badiou, *Théorie du sujet*, Paris, Éditions du Seuil, 1982.

⁸ Badiou discusses the relation between formalization, subjects and the types of truths in eventual announcements (*énoncé*) in *Logiques des mondes*. A schematic presentation is given in book I, section 8. pp. 81-87.

⁹ A theory of bodies is developed in book seven of *Logiques des mondes*. pp. 471-526.

model as offering a genuine interpretation of the axioms of set theory, even if Löwenheim's theorem proves to us that such an interpretation is possible. (Bernays dismissed the importance of Cohen's findings along these lines.¹⁰) Without the latter conviction, neither Cohen nor Gödel would have grounds to see several of their results (the incompleteness theorems, the independence of the continuum hypothesis, etc.) as any indication of the limitations of mathematical thought.

My question is: what exactly is the importance of the theory of models in the context of your general understanding of mathematics? Is there a connection between your early theses on model theory and the formalistic Platonism that you espouse in your later works (notably in *Court traité d'ontologie transitoire*)?

AB: Evidently, the concept of model is absolutely central because it is the heart of what follows from formalization. The fact that I began with the concept of model is not a coincidence. It turned out well because, at base, a model is a concept I investigated in an attempt to focus on what one could call the dialectic of formalization. I call it the dialectic of formalization due to the fact that every creation of thought is in reality a creation of a new formalization and at the same time this new formalization establishes a relation or takes part in an interaction with the particularity of what we are trying to express. In this case, we determine the formalization as a universality, but it is ultimately a particularity that carries universality in the model. Because, at base, we can say that, even if we take, for example, a painting by Picasso, that is, if we are taking a cubist painting by Picasso or by Braque in 1913, we find the creation of a possibility of a new type of pictorial formalization.¹¹ That is to say, it renders possible a way to formalize in the space of painting something that was previously unacceptable before. On the other hand, it realizes itself in a particular context, with respect to the materials used or in the sorts of cultural references that render the painting a particular painting. It is a model. Picasso's work is a model of this possibility of formalization. It is not that the formalization is drawn abstractly from thin air. It is rather something that was realized in a particular time and place. And thus the model, since my earliest reflections, has been something that assumes the particularity or the singularity of a region of being or of a world, and at the same time, raises the universality of a possible new formalization. This is why it is effectively a concept that describes the dialectic of formalization. That is to say that it is a dialectic of the truth procedure as such.

It is entirely situated then, in what at times can be called "situations" and other times "worlds" and, at the same time, it transcends its situation because it proposes a new type of formalization which has the power to be summoned up in the history that follows. Thus, if we return to the dimension of mathematics, we will see that we have been given completely bare, as one might say, a particularity with least possible particularity. Thus while mathematics is a particularity just like everything else, mathematics will be the least particular or that which carries with it the least particularity.

TT: Right, as you said in the Duchamp lecture, 'the least particular particularity'.¹²

¹⁰ See Paul Bernays, 'What Do Some Recent Results in Set Theory Suggest?' in Imre Lakatos (ed.), *Problems in the Philosophy of Mathematics*, Amsterdam, North Holland Publishing Co., 1967, p. 82.

¹¹ In his lengthy introduction to *Logiques des mondes*, Badiou takes Picasso's *Deux chevaux traînant un cheval tué* (1929) and *Homme tenant deux chevaux* (1939) as artistic examples of the logical treatments of truth procedures in worlds. In this context, he relates these works to the prehistoric grotto paintings of horses at Chauvet-Pont-d'Arc, Ardèche, see *Logiques des mondes*, pp. 25-29.

¹² Badiou used this expression during a lecture on the work of Marcel Duchamp at L'École Normale Supérieure, March 3 2007, seminar series of the *Centre Internationale d'Etude de la Philosophie Française Contemporaine* (CIEPFC).

AB: Exactly. Thus, what we can see here is that the model is that which permits the study, on one hand, of the power of formalization, but also on the other hand, at its limits, that it becomes something that permits us to arrive at a dialectical point, the most concentrated point. These are specific points or certain times, at which the infinite power of formalization and its limitations are irreducible and present a point of undecidability distinct from the others. Thus, what interests me in particular is something that in fact supports my peculiar Platonism. I should say that Platonism, in the end, is the knowledge of ideality. But this is also the knowledge that we have access to ideality only through that which participates in ideality. The great problem of Platonism is not really the distinction between the intelligible and the sensible, but the understanding that sensible things participate in the intelligible. What interests me in Plato is participation and what would otherwise be very obscure becomes very clear. It is central to note that, in the end, the eternal truths as well as ideas would be nothing if they were incapable of being accessed from what is given in the sensible. We talk about the idea of the table, to take a classical example, that there would be no idea of a table if there were not a table. We would have no idea of the idea of the table if there were not tables. We could take other examples with things that are on a level a bit more technically formulated. The model is thus that which allows us to conceive formalization; conceived after the fact, given mathematical inventions are not simply formal inventions but rather an invention of models. It is that which permits us to access formalization or to access the universality of things, at the same time it permits us to determine the particular point of limitations. To put it more directly, the model is that which allows us to think through participation.

TT: *So it seems that there is an interesting history here, your personal history. As you said, you discovered Lautman after this period, but for Lautman the question of Platonism was exactly that of metaxu or participation.*¹³

AB: Absolutely, that is right. Lautman directly expresses this point. As he put it, in a certain sense, mathematics is the *model* of the dialectic; but for Plato, mathematics was an *introduction* to dialectics. The Lautmanian interpretation centres on the dialectic of ideas in the history of mathematics to which we finally gain access with Gödel.

TT: *So it seems to me that there are two sides of Platonism here, of which we have already spoken. On the one hand, with Gödel, we have a sort of classical Platonism or, as it were, an Anglo-Saxon version; and on the other hand, there is something else that you seem to be closer to, the Platonism of Lautman.*¹⁴

AB: Yes, absolutely. I think that while Gödel's Platonism is a Platonism of ideal objects, of formalization as the construction of ideal objects, Lautman's Platonism is a Platonism of participation. What I would say is that we have a Platonism that's a bit too dogmatic versus a dialectical Platonism (or something like that). Gödel himself struggled against the American

¹³ See Albert Lautman, *De la réalité inhérente aux théories mathématiques* and *Essai sur les notions de structure et d'existence en mathématiques* for short but precise discussions of platonic participation in modern mathematics. Lautman, *Les mathématiques, les idées et le réel physique*, pp. 65-69, 125-223.

¹⁴ For Badiou's discussion of his opposition to a standard Anglo-analytic formulation of Platonism in mathematics as represented by Fraenkel and Bar-Hillel—a camp in which Gödel is often included—see *Briefings on Existence*, pp. 89-99 (also published as 'Platonism and Mathematical Ontology,' trans. and ed. Ray Brassier and Alberto Toscano, *Theoretical Writings*, London, Continuum, 2002, pp. 49-58). However, in this text Badiou nuances his interpretation by distinguishing Gödel from a standard expression of mathematical Platonism.

trend of empiricism and against this American empiricism; he said, no, mathematical objects exist in themselves. Thus he maintained an over-idealised Platonism against this trend of empiricism. For us, in France, we had much less of this confrontation with empiricism; instead the question of dialectic has always been central. Thus, there is more development in my work apropos of mathematics that engages in a dialectical interpretation of Platonism. As such, the heart of dialectics in Plato is the question of participation. It is not so much the distinction between the sensible and the intelligible. Rather, I found it more useful to think through the problem of the intersections between the sensible and intelligible that Plato called participation, something which is finally formally realized in the concept of model. A model is developed from a particular world, and it participates through the idea of formalization.

ZLF: *In The Concept of Model, you make a thorough case against what you call the bourgeois-epistemological versions of the category of model, both in the 'vulgar' form you encounter in Lévi-Strauss and others, and the sophisticated form you find in positivist epistemologies such as Carnap's. The fault you find with the former is that, by envisioning science as the confrontation between formal models and empirical reality, it amounts to little more than the importation of the ideological opposition between thought and reality, or culture and nature, into the philosophy of science; an importation that more often than not operates by crudely analogical means.*

As an example, you cite a few passages from Von Neumann and Morgenstern's Theory of Games and Economic Behaviour, where the authors write that models 'must be similar to reality in those respects which are essential in the investigation at hand', and that 'similarity to reality is needed to make the operation significant'.¹⁵ Here, we get a depiction of scientific activity that amounts to little more than 'the fabrication of a plausible image', and the historical, productive and transformative character of science is effaced.¹⁶ You recognize positivist epistemology as attaining a far greater level of rigour and fidelity to the logical concept of model (i.e. positivism exhibits a true philosophical category and not a mere ideological notion), insofar as it both reverses the vulgar image of science and treats the empirical as the model of the formal, formulating a rigorous set of rules for the interpretation of the scientific syntax in an empirical model.

Nevertheless, one does not seriously break with the bourgeois depiction of science as an imitative activity, concerned first and foremost with drawing a correspondence between the formal and the empirical. Once again, the historicity, productivity and transformational capacity of science are ignored. Bearing this in mind, it surprises me that in Being and Event we find the situation of ontology (set theory) opposed to 'non-ontological situations' in a way that seems to mimic, down to the details, the bourgeois opposition between the formal and the empirical. Like Carnap, you seem to treat non-ontological situations as if they can legitimately be understood as 'models' of the ontological situation (for instance, in order for Cohen's methods to be of any use in understanding truth procedures, our situation must be understood as being analogous to a denumerable, transitive model for the Zermelo-Fraenkel axioms). Like Von Neumann and Morgenstern, you seem to leave the operation of correspondence between the ontological situation and its non-ontological 'outside' up to the vagaries of analogy, as when you write, at the beginning of mediation twelve in Being and Event:

Set theory, considered as an adequate thinking of the pure multiple, or of the presentation of presentation, formalizes any situation whatsoever insofar as it reflects

¹⁵ John Von Neumann and Oskar Morgenstern, *Theory of Games and Economic Behaviour*, Princeton, Princeton University Press, 1953, p. 32. Cited in *Le Concept de modèle*, p. 21.

¹⁶ *Le Concept de modèle*, p. 20.

the latter's being as such; that is, the multiple of multiples which make up any presentation. If, within this framework, one wants to formalize a particular situation, then it is best to consider a set such that its characteristics — which, in the last resort, are expressible in the logic of the sign of belonging alone, \in — are comparable to that of the structured presentation — the situation — in question.¹⁷

How does Being and Event avoid or respond to the criticisms voiced in The Concept of Model?

AB: Yes. In a certain sense, the difficulty that you point out in *Being and Event* is part of the origin of *Logiques des mondes*. You see, *Logiques des mondes* shows that the relation between a formal ontology of sets and the question of effective situations cannot be adequate to being a relation established through the form of analogy. This is true but I have never read a critique in the way you have posed it. But it is in a certain regard quite pertinent. There (in the passage quoted), I was concerned with the idea, perhaps not very well posed, that the ontology of the pure multiple can be a model of a concrete situation. In reality, this does not at all capture the real relation. While real situations are composed of pure multiplicity like everything else, there are nonetheless some different parameters. This is why the transcendental was introduced in *Logiques des mondes* as an element which localizes or which topologizes multiplicity and not as something which turns out to be a formal imitation.

What happened in *Being and Event* was that I left to one side the particularity of the situation, or, as I announced in the beginning, that I occupied myself with the ontological structure of multiplicity, an abstraction made full by the singularity of the situation (of mathematics) and thus I developed a theory of the ideality of the multiple and not a theory of participation. Hence I gave myself the goal in *Being and Event* to attempt a formal description of what a truth procedure could be.

I might clarify myself with respect to the critical passage here by saying that, since there is a diversity of situations as well as laws governing situations, the question might then be how these diverse situations participate in the same ontology. In *Being and Event* what interested me was to arrive at thinking the possibility of a process of truth as an exception from a general regime of the simple repetition of the multiple. If you like, I would say that, at each point of my approach to the question of the particularity of the situation in *Being and Event*, I treated them as something related to the situation as a multiplicity. Thus, to be able to say that *this* multiplicity is different from another, I had no other recourse than an analogy to set theory. The root of the problem is that when we want to pass from the theory of pure multiples to the theory of a composed world, we need resources that are not available in *Being and Event*. In *Being and Event*, we had that which one could strictly access from the structures of belonging or inclusion of the void, with a certain number of the formalizations of set theory. From this sort of approach, I gave an account of what an event is, what fidelity is, or what the consequences of the construction of generic truth are. In the end, *Being and Event* achieved its goal when it specified the ontology of truth as the being of the construction of a generic multiplicity. Certainly, this is valid for all truth procedures. This is all good, except for what concerns the particularity of each kind of truth procedure. In the last instance, the particularity of truth presupposes that we can think the particularity of a world and we could not think this particularity with the materials of *Being and Event*. In this sense we might say that the criticisms in *The Concept of Model* were an anticipation of *Logiques des mondes*. The positive dimension of the critique was realized more in *Logiques des mondes* than in *Being and Event*.

¹⁷ Badiou, *Being and Event*, p. 130.

TT: *It seems it would be a confusion to say that the four procedures of truth, from different situations, are all connected to the base of ontology or the base of mathematics. This would be a misunderstanding of your work.*

AB: Yes, certainly. While we might say that the four processes of truth, in a strictly formal sense, continue to be held together as generic multiplicity, by doing so we would not have said anything about their differences. As such, we cannot base the four truth procedures in set theory. For this reason, we need to engage, at one point or another, with that which gives some indication of singularities, that is, an investigation into the question of what is a “type” in truth procedures. This is a question that is not reducible to ontology. Ontology can only give what it can give, it cannot but be what concerns multiplicity as multiplicity. However, when a break is situated in a world, in a certain historical shift or an interruption as a particular event, we are in a different register and it will be a different question.

TT: *So perhaps we can reformulate this general confusion as a confusion concerning the terms situation, presentation, multiple, and now... a world.*

AB: Yes. (Laughs)... and now a world.

TT: *Situation, presentation, multiple, world.*

AB: I should clarify this point. They are not at all identical. Presentation is a particular qualification at the level of situation; presentation is a synonym of belonging (*appartenance*).¹⁸ In a situation, there is not only presentation but also representation because we should not forget that there is also a state of the situation. They are not at all the same, presentation and situation. On the other hand, the entire effort of *Logiques des mondes* has been to ask another question. Why is it that I speak about worlds? A world is not reducible to a situation because, if we look at *Being and Event*, a situation is a multiple. That is all. As such, with respect to a world, we have something that is already much more complicated. First, there is not simply a multiplicity, but there is an inaccessible infinity of multiplicity and there is also a transcendental map of connections and relations.¹⁹ You are absolutely right to point this out. Situation, presentation, multiple and world are concepts that should remain distinct.

ZLF: *In “Marque et manque”, a text published around the same time as The Concept of Model, you level what is, in my view, a decisive critique of Jacques-Alain Miller’s attempt to connect the psychoanalytic concept of ‘suture’ with the mathematical zero.²⁰ The main thrust of your argument is that mathematics lacks nothing that it does not produce, that it organizes these lacks according to a process of stratification, and therefore never encounters the sort of uncontrolled lack such as Miller posits in “Suture”. In Being and Event, however, the concept of suture returns, and you say that the empty set (\emptyset) is, in truth, set theory’s ‘suture to being’. What is unsettling about this return is that nowhere do you provide a defence against your earlier attacks on this concept, neither in Being and Event, nor in your brief discussion of Miller’s text in Le Nombre et les nombres, despite the prominence of the notion of ontological suture in both*

¹⁸ This is a central thesis of *Being and Event*, meditations three and seven, pp. 38-48, 81-92.

¹⁹ In *Logiques des mondes*, Badiou gives an account these terms in book four, section one, entitled ‘mondes et relations’, pp. 319-342.

²⁰ Jacques-Alain Miller, ‘La suture (elements de la logique du signifiant),’ *Cahiers pour l’analyse*, no. 1, pp. 39-51. Available in English as ‘Suture (elements of the logic of the signifier),’ *Screen*, 18.4, pp. 24-34.

books.²¹ I am aware that much has changed in your understanding of mathematics between “*Marque et manque*” and *Being and Event* — mathematics, for instance, goes from being understood as a process, even a ‘psychosis’, without a subject, to being at once the science of being and a subjective procedure *par excellence* — but it is unclear how these changes might allow you to evade your earlier critique.²² Is the notion of suture, as it functions in these later texts, still subject to the critique put forth in ‘*Marque et manque*’? If not, why not?

AB: Yes. I understand the complexity of your question. However, the problem here is that the word suture changes its meaning between Miller’s text and my usage in *Being and Event* when I discuss the suture to being. Why is there this change of meaning? In Miller’s text, suture designates the point of absolute lack which accommodates the heterogeneity of the subject. At the point where there is lack, one can discern the symptomatic of the subject. Here I would repeat my disagreement with Miller, while clarifying that this problematic does play the same role in *Being and Event*. For me suture designates a juncture between ontology and its ‘object’. To hold mathematics as ontology, the very limits of being as such will be touched by the void. But, as such, the void is not the point where we discern subjective heterogeneity. The void is the point on which we found the constructible sets which allow us to unfold the characteristics of pure being. In this sense the void also represents inconsistency.²³ If we admit both that multiplicity is inconsistent and that ontology makes itself consistent in that possibility. With inconsistency (of the void), we are at the point where it is equivocally consistent and inconsistent. That is the void. Since the void is the multiplicity of the nothing, the question of knowing whether it consists or not is split by a pure mark (\emptyset). Suture carries its importance in my work in this way in relation to this difference in meaning. For Miller, it is a dynamic according to which all repetitions are conditioned by the marking of lack as hidden subjectivity. In *Being and Event* however, I simply provided a justification of the fact that mathematics, since it is consistent, is an ontological discipline that measures its (suture’s) connection with the void. Whether the void is consistent or inconsistent is undecidable. Thus, it is both that (in the void) there is nothing that consists and which can yet be considered to consist in not consisting.

That is what I have to say on suture but I would like to return to another part of the question that was supposed in what we spoke of earlier. There is an important transformation in my work that is not really addressed in the question of suture by itself. Here, I think there has been an important transformation in my work between *Marque et manque* and *Being and Event*: between the two, I have reintroduced the category of the subject. By consequence, even in thinking of mathematics as a truth procedure, there is a sense of the subject in this procedure. In doing this, I did not put the subject on the side of the void. In what I developed elsewhere, I showed that the fundamental difference between Lacan and my position is that the central concern for mathematics in Lacan is his suggestion that the void is on the side of the subject, while for me the void is on the side of being. So this is precisely the gap between the two senses of the word suture. On the one side, we have a suture to being and on the other, a suture of subjectivity. But at the same time it remains true that the construction of this interpretation of mathematics makes possible the existence of the subject in the mathematical procedure insofar as the subject of mathematics will not be localized in the void. Rather, the void is not at all the subject but rather the ‘object’ of the procedure. This is the important shift.

²¹ Alain Badiou, *Le Nombre et les nombres*, Paris, Éditions du Seuil, 1990.

²² Badiou’s use of the term ‘psychosis’ comes out of the critique of Jacques-Alain Miller’s use of the term to describe the structure of science. Jacques-Alain Miller, ‘Action de la structure’, *Cahiers pour l’analyse*, no. 9, pp. 93-105.

²³ In *Being and Event*, many of Badiou’s central theses on the identity of ontology and mathematics hinge on the point that “The void is the name of being,” *Being and Event*, p. 56. Badiou gives an account for the void’s linking of consistent and inconsistent multiplicity in meditation four, pp. 52-59.

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