The Use of Explicit Algorithms and Episodic Context
to Teach Subtraction to Students with Learning
Problems in Mathematics

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Abstract

This research attempted to address the question of the role of explicit algorithms and episodic contexts in the acquisition of computational procedures for regrouping in subtraction. Three groups of students having difficulty learning to subtract with regrouping were taught procedures for doing so through either an explicit algorithm, an episodic content or an examples approach. It was hypothesized that the use of an explicit algorithm represented in a flow chart format would facilitate the acquisition and retention of specific procedural steps relative to the other two conditions. On the other hand, the use of paragraph stories to create episodic content was expected to facilitate the retrieval of algorithms, particularly in a mixed presentation format. The subjects were tested on similar, near, and far transfer questions over a four-day period. Near and far transfer algorithms were also introduced on Day Two. The results suggested that both explicit and episodic context facilitate performance on questions requiring subtraction with regrouping. However, the differential effects of these two approaches on near and far transfer questions were not as easy to identify. Explicit algorithms may facilitate the acquisition of specific procedural steps while at the same time inhibiting the application of such steps to transfer questions. Similarly, the value of episodic context in cuing the retrieval of an algorithm may be limited by the ability of a subject to identify and classify a new question as an exemplar of a particular episodically
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defined problem type or category. The implications of these findings in relation to the procedures employed in the teaching of Mathematics to students with learning problems are discussed in detail.
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CHAPTER ONE: CHILDREN'S MATHEMATICAL THEORIES OF LEARNING

Introduction

Basic level (Ministry of Education, 1980) students were taught procedures for learning to subtract through either an episodic context, an explicit algorithm or an examples approach. The additive method of subtraction was chosen as a vehicle for teaching subtraction. This psychological study examined how students assimilated new information and were able to adapt to varying degrees of transfer problems. Retrieval procedures for solving mixed subtraction problems were highlighted by the verbal protocols of students.

Purpose of the Study

The purpose of this study was two-fold: 1) To determine if one form of an algorithm represented by a flow chart would facilitate the acquisition and retention of a new schema; 2) The use of language to create episodic context was expected to facilitate the retrieval of schemata, particularly in a mixed problem format.

The Rationale of the Study

It was hypothesized that basic level students who tried to master a set of
subtraction problems using a complex set of procedural algorithms or episodic schemata representing the additive method of subtraction would have marked success on similar, near transfer and far transfer problems. It was assumed that the role of explicit algorithms and episodic contexts would facilitate the retention of schemata in the acquisition and retention of procedural skills. As basic level students for the most part learn by rote when given rules to follow, the number of rules can result in working memory overload. In this respect the flow chart algorithm or the episodic story may serve as a cue for the student to recall the appropriate schema from long-term memory. Case and Bereiter (1984) realized that new schemata are usually not automatized and therefore take up a great deal of working memory space. As a result, students can have varying degrees of difficulty in recalling a complete algorithm and many errors are encountered by using incorrect procedures, rather than no schema at all. Brooks (1987) contended that forgetting was caused by interference from the episodic representations which were similar to the newly learned algorithms but not identical. In addition, students may lack a conceptual understanding of what they are solving mathematically and may not have the working memory space to accommodate or modify existing schemata to solve unique problems. Swanson and Rhine (1985) concluded that students often have stored information which is not easily accessible and this may hinder them from using more efficient
transformation strategies. Some strategies used may also be too complex for them to totally access from long-term memory. On the other hand, it may be that students can develop a taxonomy of strategies based on algorithmic or episodic schemata to aid in the solving of near and far transfer problems. Bilsky and Judd (1986) stated that if a student can use contextual and textual cues to identify a relevant schema or episodic representation, then that schema may be useful as a top-down processing framework to guide subsequent analysis. Authors van Dijk and Kintsch (1983) presented a model of discourse processing that assigned a central role to differentiated strategies. Bilsky et al. (1986) found that textual and contextual cues allowed the selection of purposeful schemata to guide the process and comprehension is facilitated. Although a near transfer problem or a far transfer problem would not utilize an identical schema for its solution, similarity to the original problem would allow the student to use the process of analogy for schema retrieval. Keeping in mind that analogy-based transfer is not based on a perfect match but rather degrees of similarity between old and new problems, it is nonetheless reasonable to assume that flow chart algorithms and/or episodic context would facilitate this process. This study was constructed based on the hypothesis that flow charting and episodic context may facilitate recognition of similarities in analogy-based transfer.
Theoretical Framework

It can be argued that the difficulties students experience in learning to subtract with regrouping are fairly representative of many problems encountered in learning new computational procedures in mathematics in general. Students tend to make different errors which vary with their level of comprehension or lack of it. These difficulties tend to fall into five basic categories. First, the students can fail to understand the concepts involved. The child learning to subtract in Grade Two; for example, may not understand the value principle (Resnick, 1983) and may regard a digit in the ten's column as no different from a digit in the one's column. Second, the student may make a mistake in computing the answer to a two-digit (subtracting) subroutine within the general algorithm (Engelhardt, 1982; Liedtke, 1982). Errors of this type are commonly thought of as number fact (computational) problems. Third, the student may have difficulty learning the general algorithm required in a task such as subtraction with regrouping (Anderson, 1983). This difficulty may be reflected in (a) a specific subroutine, (b) an inability to retrieve the algorithm, or (c) an inability to retain the algorithm in working memory while using it to solve a subtraction question (Hiebert, 1980; Romberg and Collis, 1985). Fourth, the student may have difficulty applying the algorithm in new contexts (Resnick, 1983). In one sense, this may be a retrieval problem which is exposed on near and far transfer questions. On the other hand,
this type of problem may be the result of the use of an algorithm that is undifferentiated and associated too closely with the initial stimulus questions from which it was derived. Fifth, the failure to automatize a particular subroutine may interfere with the ability of the student to retain an algorithm in working memory (Svenson and Sjoberg, 1981). A likely candidate for this type of difficulty may be the number of rules young children employ in adding (or subtracting) subroutines (Kaye, Post, Hall and Dineen, 1986). It is not unreasonable to hypothesize that a number of random numerals in a given problem may take up a considerable amount of working memory storage space and, as a result, displace part or all of a new algorithm. This assumes, of course, that new algorithms are usually not automatized and require a considerable amount of attentional capacity (Case and Bereiter, 1984). Although this list may not be exhaustive, it does suggest that faulty memory processes may underlie some of the difficulties students experience in subtracting. Specifically, it can be argued that working memory overload and long-term memory retrieval failure are processes which may contribute to more than their share of the grief experienced in a classroom. Unfortunately, both working memory overload and retrieval failure may occur for any number of reasons. Working memory overload, for example, can be caused by nonautomatic subroutines (Anderson, 1983), a new algorithm which is too complex (Anderson, 1983; Lewicki, 1988), a new algorithm which is poorly understood (Vakali, 1984),
subroutine repair generation and testing (Romberg and Collis, 1985), and extraneous factors, such as remembering where to put the decimal point when subtracting dollars and cents (Anderson, 1983). Similarly, retrieval failure may occur for a number of reasons. Prime among these may be encoding errors (Cooper and Sweller, 1985), forgetting, and the presence of new default rules (Svenson and Sjoberg, 1981). Encoding errors (Torgesen, Murphy and Ivey, 1979), such as a failure to notice the subtraction sign or a blank space place holder may result in the retrieval of the wrong algorithm or the failure to retrieve a default subroutine. Forgetting, on the other hand, is probably caused by an interference from the episodic representations of stimulus questions and their associated algorithms which are similar to the newly-learned algorithm (Brooks, 1987). Retrieval problems may occur when new default rules (Brown and Van Lehn, 1982; Resnick, 1983) are associated with fewer examples than the rest of the algorithm. As a result, the structure of the stimulus question may not always cue the retrieval of a new default rule along with the rest of the algorithm. Consequently, one would expect instances in which the student retrieved the algorithm for subtraction with regrouping minus the default rule subroutine for borrowing from zero.
Importance of the Study

This psychological study examined the process of learning and assimilating a new schema into an existing heuristic. The additive method of subtraction was chosen as a mode of delivery of the new material to a group of basic level students. Basic level students were selected for this study since many of them were experiencing difficulty with subtraction problems.

It was observed that many basic level students have poor organizational skills, weak problem-solving techniques, a low level of computational accuracy involving the four primary operations of addition, subtraction, multiplication and division. Possibly one of the reasons these students have so much difficulty is working memory overload. Geary, Widaman, Little and Cormier (1987) found that many students who have developmental lag in mathematics achievement clearly differed from academically "normal" students in terms of the elementary process involved in the mental solving of simple arithmetic problems. Since these procedures have not been automatized, the student must try to recall from long-term memory the heuristic that best fits the problem. These students tend to perform well when presented with a short lesson of approximately five to ten minutes in duration with a follow up characterizing similar type questions. Once mixed problems are presented or dissimilar questions are encountered, these students tend to make mistakes or simply stop working and give up.
A good teacher tries to anticipate difficulties that students might have and review these steps or procedures prior to presenting the new lesson. In some cases as the teacher is presenting the review procedures to the students this material can become a lesson in itself and the new concept can become lost in too many details. The concept of subtraction is clearly one area of difficulty. These basic level students have been working with an established algorithm of subtraction for several years. There may be varying degrees of understanding of the traditional form of subtraction by these students and some may have spent more time on this original concept than others. Perhaps a new approach, a fresh start, a different procedure than the one with which the students are used to may meet with more success. The additive method of subtraction (Sherrill, 1979) was therefore chosen because it provided the means of conducting a psychological study of basic level students. This equal addends method of subtraction was unfamiliar to all students in the study. This approach could be presented in varying formats to random groups of students. Moreover, as Myers and Thornton (1977) have argued, it may be necessary and advantageous to take children back to working only on addition.

Ginsburg and Opper (1969) tested Piaget's concept of conservation and showed that children are ready to learn arithmetic when they master conservation and that number conservation is a prerequisite to understand addition. A concept
of seriation occurred next in the preoperational stage of children. This shows the idea of comparison: shorter/longer, smaller/bigger. Reversibility of thought (decentration) involved seeing a relationship from two different points of view. Subtraction is two-directional: Two plus three equals five leads to two subtraction examples. However, it is easier to code an addition operation than to remove something from your original amount (Ginsburg and Opper, 1969). Rittner (1982) concluded that students may have more difficulty with subtraction since little emphasis is placed on the concept that subtraction is the inverse of addition. Vakali (1984) surmised that subtraction would appear to be a more complicated mental operation than addition based on the latency evidence. Even simple subtractions required on the average seven more seconds for a solution than simple additions (Vakali, 1984).

Protocols can be taken to match what the students said and what in fact they actually wrote (Duffin, 1986). This procedure has added insight into their thought processes, the contents of their working memory, their retrieval methods and the cues that the students used and how they assimilated new information. Comments like "You forget where you go; it's hard if you can't remember it; it's hard because you might forget the story" illustrated some of the difficulties the students were having. When students talk about mathematics it clarifies certain concepts and helps them to select those patterns which are of value and reject
those items which caused interference. This process enables the teacher to tune into the thought processes of the students and gain an insight into their attitudes, failures, bugs and successes.

When teachers understand how children think and interpret a concept, it will facilitate instruction. A teacher sends signals and the students receive these signals. The receiver must experience this signal and react to it in some way. Episodes or stories which are the basis of communication provide an overlooked tool for the presentation of new mathematical material. Students can interpret an episode or story into their existing framework or terms of reference. This is a unique perspective for each individual. A mathematical concept, may for a good mathematical student, be easily assimilated into his/her frame of reference whereas the same concept could easily be misinterpreted or misunderstood by a slow learner in mathematics. Two possibilities exist: The web to the frame of reference may be poorly established or the new concept may have so many parts that memory overload is experienced. The algorithm or episode hopefully will be utilized as a cue, perhaps differently in context, perhaps uniquely distinct in format but sufficient for the student to relate to this cue so that it triggers the retrieval of information which would lead to the thinking process to solve a problem involving a similar schema. Geary et al. (1987) argued that the time to retrieve a schema from memory may be inversely related to the strength of the
association is related to the frequency-of-problem presentation.

Terminology

Several terms will be discussed repeatedly in this thesis.

The term algorithmic mode will be used to refer to a flow chart depicting the schema required for the student to solve a subtraction problem correctly. Although an algorithm may take many different forms, this paper will only refer to flow charts. Each part of the arithmetic process is detailed into a minute number of individual steps that the student can trace through to complete the problem. The stages of a flow chart details a degree of order and logic that the student must follow in order to solve the problem.

The term episodic mode will be used to refer to the stimuli in the form of a story format that will be used by the students as a cue to facilitate the encoding of a representation of subtraction with regrouping. Many students first encountered episodes when they initially learned the shape of the formation of the numbers from one to ten:

1. A straight line down is fun
   That is how we make a one.
2. Around the track
   And then come back.
3. Around the tree, around the tree
That is how we make a three.

4. Down and across and down some more,
   That is how we make a four.

5. That old five goes down and around,
   Put a hat on top and see what you found.

6. Down with a loop,
   That six rolls like a hoop.

7. Across the sky and down from heaven,
   That is how we make a seven.

8. Start out to make a snake,
   Then come back to make an eight.

9. A ball on a stick looks fine,
   That is how we make a nine.

10. First a stick,
    Then a ball,
    That's a ten and that is all.

The term far transfer problem will be used to refer to problems that are in the same category as the initial problems but are so significantly different in structure that it would require some additional insight or extension to solve the
problem successfully. One would think that this type of problem would be easy to construct but, in fact, it turned out that this type was the most elusive to define and the most difficult to develop.

The term near transfer problem will be used to denote problems that have an operation step that is different from the steps already encountered but close enough in structure to the original questions not to make them remarkably distinct.

The term operational level will be used to denote the structure of problems by comparing the magnitude of each digit in each problem and that (L) represents larger, (E) equal and (S) smaller depending on their relative vertical placement.

For example: This would be the structure in this problem.

\[
\begin{array}{cccc}
42 & LS & 6742 & LSEL \\
-14 & -SL & -840 & -LES \\
\end{array}
\]

The term protocol will be used to refer to the verbal description outlining the mathematical processes as described by a subject.

The term similar problem will be used to refer to problems that were identical in structure but distinct because of different numerals than those previously presented to the students.
The term **structure** will be used to refer to the way the digits were placed together to form a subtraction question.

The term **worked example mode** will be used to refer to the students who are taught to solve subtraction questions by repeated examples. A worked example is a verbal representation of an algorithm and is usually reinforced by similar problems. This is exemplified in most mathematical texts (Norrie, Bernstein and Wells, 1975) illustrating the way that most students may be taught mathematics on a daily basis. Several examples are presented on the board and the teacher outlines the process for solving each. After that, the students are then given several similar type problems to solve.
CHAPTER TWO: LITERATURE REVIEW

Introduction

The purpose of this chapter is to examine the potential of basic level students to master tasks in arithmetic employing the additive method of subtraction. Areas of information processing will be reviewed and compared to the processing of mathematical concepts employed in solving problems. The role of schema theory and episodic theory each in turn will be examined in the understanding of how students experiencing difficulties learn. In addition, the emphasis on examples implicit in episodic theory will be considered. Examples in this framework will be viewed as both more central and as having more cognitive weight than in more traditional learning theories. In this respect Skemp (1965) found that it was just as important that the teacher be aware of what the examples did not include and which irrelevant attributes in examples may unconsciously influence the choice of examples and determine the schema.

Schema Theory

Students were taught to solve multi-digit subtraction problems by using a
complex set of procedural algorithms or flow charts. Many students have
difficulty in mastering computational algorithms and many of their problems arise
from incorrect processes, rather than not following any procedure at all (Brown
and Van Lehn, 1980; Engelhardt, 1977; Young and O'Shea, 1981). Carpenter and
Moser (1982) showed that children made different errors based on different kinds
of misunderstanding. Skemp (1965) stated that students cannot form more
advanced concepts unless they can handle the basic techniques effortlessly.
Linder (1985) presupposed that problem-solving activity may not only be
determined by characteristics of the current problem to be solved, but also by
characteristics of problems preceding and subsequent to the current problem
being investigated. Skemp (1961) declared

that as soon as a student had progressed
beyond simple routine processes- the problem
which can not be answered by routine application
of already known methods, required a new
combination or modification of existing methods.
To do this process effectively requires the
conscious awareness of the methods in one’s
repertoire and the ability to try various
combinations and modifications until the right
approach is found. (p. 49)

Near transfer and far transfer questions examined the students’ ability to
assimilate existing schemata and form a revised procedure to attack the problem
successfully.

It can be argued that learning to subtract consists of constructing a schema or
an algorithm consisting of a set of procedural rules which are linked to operators (MacKay, 1987) and can be recoded as verbal statements. These rules may be acquired by identifying a set of procedures and descriptions or explanations of procedures which are common to a number of (worked) examples in a given lesson. The schema or algorithm then, consists of a set of abstractions, coded as general procedural statements, which are organized in some sort of action sequence. Individual procedural statements within this sort of structure may in turn act as the control nodes for a set of action operators (MacKay, 1987). Thus, the procedural rule (Move one column left in top numeral. Mark it as a borrow column) (Resnick, 1982) would be linked to control operators or action nodes in long-term memory for shifting visual attention to the left, perhaps putting a finger or pencil on the top numeral. The statement (Mark it as borrow column) may be linked to a semantic node (Resnick, 1982) or it may activate an action such as writing 10 above the column. Although these procedural rules would in some sense represent generalizations, it is important to keep in mind that this type of system would also require default rule branches (Brown and Van Lehn, 1982) in order to handle subtraction questions which contained zeros, blank space place holders. For example,

\[
\begin{array}{ccc}
708 & -235 & 4003 -2104 \\
\end{array}
\]
In fact, according to the repair theory (Brown and Van Lehn, 1980, 1982; Van Lehn, 1983), the origin of errors is described in terms of a lack of procedural skill. Rather than give up when they come to a dilemma, students tend to improvise and search for a better way to complete the solution. Or, to quote Van Lehn (1983)

> When a constraint or precondition gets violated, the student unlike a typical computer program is not apt to just quit. Instead he will often be inventive, involving his problem solving skill in an attempt to repair the impasse so that he can continue to execute the procedure, albeit in a potentially erroneous way. (p. 215)

In this respect learning would involve both abstracting general procedures from worked examples and attaching default rules to these procedures in those instances in which the general procedures were insufficient. In the case where the procedural error is trivial, the teacher may point out the appropriate or nonsequenced step and its inconsistency. In cases where the error is accompanied by lack of understanding for the process, reconstructing the procedure in a more concrete form would facilitate further understanding. Of course, learning when to apply a default rule may involve acquiring a second set of rules for identifying stimulus features which flag the default system and may be quite difficult if the general procedure the default rule is attached to has not been completely learned itself. One key aspect of the bug theory (Brown and Van
Lehn, 1980, 1982) is the conjecture that systematic errors are determined by fixed internalized procedures and the schemata that are employed in solving a problem are determined exclusively by the structure of the problem. A systematic bug or error is the outcome of incorrectly applying the core procedures. The instruction by the teacher is encoded as a set of facts about the skill of subtraction (Anderson, 1982). Anderson (1983) stated that one of the reasons why instruction is often so inadequate is that the teacher, likewise, has a poor conception of the flow of control in the student. The greater the level of difficulty in subtraction problems was due to extra steps required in computing that operation (Vakali, 1984). Hitch (1978) and Vakali (1984) both concluded that carrying or borrowing in a subtraction problem takes extra time and imposes extra mental load on working memory. Anderson (1982) determined that the composition of multiple steps into one procedure produced the speedup in time of students' solutions and led to unitary rather than piecemeal application. These small productions can be combined to form a larger one and the limit of this new production was determined by the capacity of working memory of the student (Anderson, 1982).

The theory referred to as the repair theory portrays subtraction problem solving as involving two unique components: a series of operations that generate an incomplete process, and a separate series of procedures that repair the process
so completion can be accomplished (Brown and Van Lehn, 1980, 1982). The first step involves writing an incomplete procedure which is not all encompassing or generalized in nature to account for all the differentials in the problem. This procedure will require repair during the problem-solving process so that the students can solve the near transfer and the far transfer problems. The resulting incomplete program works remarkably well on the similar problems but a quandary is encountered when solving transfer problems. To repair this procedure the student must generalize or apply certain strategies to the existing procedure to accommodate new types of problems not before encountered.

Young and O'Shea (1981) argued that arithmetic problem solving was analogous to a computer language production system. A production memory contains a collection of production rules (for example, If A then B). These are the program lines and when input data are added, the computer can run these procedures.

Brown and Burton (1978) assumed that students learned and processed a fixed set of production rules. From this procedural network a solution is simply a collection of subroutines and a bug is the breakdown of one of these sub-procedures. For example:

63  
-7 The bug in this question is not adding a 1 to the  
66 ten's column in the subtrahend before subtracting.

Anderson (1983) has presented a strong case for the argument that in
attempting to solve an unfamiliar math problem, the student first retrieves a declarative (verbal) representation of a procedural rule and then, if it is judged suitable, converts it into a set of actions. This is supported by the observation that students can verbally report much of what they do when working through a math question (Anderson, 1983; Schoenfeld, 1983; Van Lehn, 1983), although it should not be assumed that every procedural representation in an algorithm or schema may be accessible to consciousness (Ericsson and Simon, 1980). Lewicki (1986) demonstrated that subjects were able to process algorithms not only without being able to explain what they had learned, but even without being aware that they had learned anything. Subjects were exposed to a series of frames consisting of one of four possible targets on a computer screen. The subjects' task was to locate the target and press a button corresponding to its location. The sequence of locations followed a complex pattern that the subjects were expected to acquire nonconsciously. The gradual increase in the subjects' performance indicated that they acquired the processing algorithm for predicting the subsequent locations of the target. From follow up interviews it became clear that the subjects were neither aware that they had learned anything nor were they aware of how the acquired knowledge affected their performance. The students were able to perform a task that was relevant to a pattern but, in fact, they were unable to describe the pattern manipulated in the learning phase stimulus
material even though they had acquired some working knowledge about it.

The advantage of retrieving declarative information first may be that it allows the student to assess consciously the appropriateness of a given procedural rule before applying it. Presumably, procedural rules coded as actions would have to be applied first before the learner could reflect on their value (Anderson, 1983; Piaget, 1976). Anderson (1983) has pointed out that this would be a less efficient and, in some contexts, less adaptable strategy. This does not mean, of course, that students always retrieve every procedural rule in its declarative format before applying it. Automatic procedures, for example, would not fall into this category. More to the point, there is evidence which suggests that young children (Piaget, 1976) and learning-disabled students (Brown and Campione, 1986) frequently do not retrieve the declarative representation of a procedure first and have difficulty retaining it in working memory when they do. Before students can understand a procedure and apply it, they must formulate a structurization of many of these structurizations which are based upon logical mechanisms. These are not necessarily transmitted by language (Piaget, 1964). Rather, these two groups of subjects tend to apply directly the first procedure that is cued by the stimulus question and evaluate it on the basis of its results. Evaluation and repair or debugging may simply involve the retrieval of another procedure based on cue-procedure association strength. However, both Piaget’s theory of reflective
abstraction (1976) and repair theory (Van Lehn, 1983) suggest that students may attempt to recall or reconstruct their actions and generate alternatives in the form of declarative representations. Furthermore, both theories imply that this strategy would require a considerable amount of attentional capacity. The retrieval of schemata from long-term memory and the comparison of these schemata with a transfer problem in short-term memory would leave little room in working memory for students to find the similarities or differences in these representations. Piaget and Inhelder (1969) state that cognitive development results from an interplay of four factors, one of which is self-regulation. Self-regulation is the process of actively generating new mental schemes when existing schemes are insufficient and comprises the complementary phases of assimilation and accommodation whereas Skemp (1971) states that "the 'intuitive-before-reflective' order may be partially true for each new field of mathematical study (p.66)."

One of the cognitive processes in understanding mathematics is learning (Byers and Erlwanger, 1985). Many students have the ability to learn explicit algorithms and in fact understand mathematics. Lehman (1977) described understanding mathematics as any one of three kinds of knowledge - applications, meaning and logical relationships. Examine for a moment a framework of schemata as outlined by Skemp (1971):
A concept requires for its formation a number of experiences which have something in common. Being associated with a concept is a name which helps to classify it. A concept is a way of processing data which enables the user to bring past experience usefully to bear on the present situation, thus language is an essential part of formulating a concept. The higher the order of the concepts which symbols represent the greater the stored experience the concept brings to bear. Lower order concepts must be present before the next stage of abstraction is possible. Contributory concepts needed for each stage of abstraction must be available. There must be a certain degree of backtracking. (p. 28)

A schema has two main functions: it integrates existing knowledge and it is a mental tool for the acquisition of new knowledge. Almost everything we learn depends upon knowing something else first. Schematic learning may take longer than memorizing rules for a certain operation. If one only wants to perform a certain operation memorization of the rules may be the faster way. (p. 39)

Students can efficiently learn to do similar type subtraction problems. If the student wanted to progress, then the schema which incorporated many rules into a procedure reduces cognitive strain. This enhances the possibility of students being successful in solving near and far transfer problems in subtraction. Schemata are not stable but constantly changing in structure to accommodate to the new situation. This process is difficult since the students perceive new situations where the old schemata are not adequate and where variations occur. Instead of rejecting the old schema the new concept enables the student to
understand the new experience and react appropriately where the earlier experience is a special case (Skemp, 1971). The original schema is not overthrown but incorporated into the new framework. Better internal organization of the schema improves understanding.

Resnick (1981) stated that the part-whole schema had been shown at the base of the ability to perform subtraction with regrouping. It seemed probable that children's mathematical development could be aided by explicit instructional attention to schema and its applications. Wolters (1983) found that after an instructional program in subtraction problems the students were able to mentally perceive the transitive nature of the relation. However, many students find it difficult to relate mathematical expressions to their experience and their response is that they have memorized symbols and processes. As a result these students find it difficult to transfer ideas or skills to new and unfamiliar situations (Liedtke, 1982). Positive transfer is probably one of the most important areas of instructional methodology. Transfer allows a student to learn more difficult material because of what has been mastered previously. De Corte and Verschaffel (1981) found that in teaching using traditional methods in mathematics (worked example condition) students acquired a set of specific isolated solution methods that were almost not transferable at all to new and unfamiliar tasks. For example, a student who has learned that 3-0=3 and that 7-
0 = 7 generalizes that 0 is the identity element for subtraction. Frostig and Maslow (1973) called this type of example a stimulus generalization, whereas differentiation is also associated with transfer. It is essential that students can find differences or attributes which are not the same from one set of subtraction problems to another. Gagne (1968) discussed conditions for transfer and stated that learning consisted of building learning sets that are hierarchically related. The acquisition of subordinate learning sets facilitated the subsequent learning of higher learning sets. In essence, then, the teacher must guide the student to develop more complicated skills after first mastering earlier ones.

Skemp (1971) further contended:

Communication has a direct influence on reflective intelligence. This involves the necessity to link ideas together with symbols. (p. 65)

Once a student is able to analyze material for himself he is able to fit it into his existing schema in a way which is meaningful to him. The role of the teacher is to fit the mathematical material to the state of development of the learner's schema: the delivery of material must be received by the learner at the mode of thinking ability and concrete reasoning level that the student is capable of and thirdly the teacher must gradually increase the analytic ability of the students so that they no longer depend upon him to filter the material. (p. 67)

Once a mathematical process is understood with its many separate and
integral parts, it would be of great advantage in the future not to have to repeatedly go through all steps involved to get to the final outcome. It is essential that all elementary processes become automatic, thus freeing attention to concentrate on new ideas (Skemp, 1971). Thompson and Babcock (1978) believed that the use of any concrete material be parallel to the mental process required to solve the problem in a more abstract manner.

Cooper and Sweller (1985) demonstrated: 1) That worked examples were of assistance to students solving similar problems but not for dissimilar problems; 2) When similar problems and dissimilar problems were interpolated into a single set of problems the worked example group differences were diminished; 3) It was further observed that the schemata were easily forgotten with the worked examples. Cooper and Sweller (1985) concluded for worked examples, novice problem solvers have considerable difficulty abstracting general rules from specific schemata that incorporate them and unless abstraction had occurred attempts to use the rule under conditions where acquired schemata cannot be applied, resulted in errors. Skemp (1971) thought that what one stores is a combination of conceptual structures with associated symbols and that it is easier to catch hold of by symbols what we are trying to retrieve from this store.

Flow chart

The principle for solving a problem starts with an external memory aid,
followed by a reconstructive process then a reproductive process. Swanson and Rhine (1985) hypothesized that learning-disabled students might benefit from the use of a prompt designed to facilitate the encoding and retention of declarative procedural rules when learning a new procedure such as subtracting with regrouping. For example, if the student could learn to refer to a flow chart designed to represent an algorithm for subtraction with regrouping while actually attempting a question, both working memory overload and retrieval problems might be minimized. Consider the example: 

\[
\begin{array}{c}
42 \\
-17 \\
\end{array}
\]

and refer to Figure 1 (Algorithm One). Students would trace through the flow chart with their finger to the first decision box and exit on the "no" trail to reach the box Write a small 1 to the left of the unit's top digit, then proceed to the ten's column and find the bottom numeral. at the next decision box follow the "yes" path to increase the bottom numeral by 1, proceed to the unit's column and answer the question in the next box indicated What number added to the bottom number makes the top number? Answer expected 5 and write the result below the line, move one column to the left and repeat the process by proceeding through the first decision box again. Anderson (1982) argued that the more items required to be held active in working memory, the lower the activation of each and the slower the recognition judgement. Whether or not the flow chart is the exact representation of the algorithm or schema in the student's long-term
memory might not be critical.

Provided the student can follow its sequencing and understand the verbal formatting of the procedural rules, the steps or boxes in the flow chart might act as pointers to more complex cognitive subroutines. The flow chart would in effect make a portion of the declarative representation of the algorithm explicit and visible and the student would attempt to use it like a recipe. Like a recipe, it would specify what to do and when to do it. Unlike a recipe, the ingredients or stimuli would vary with each application. For example, students may find it hard to choose between an easy short-term schema or a more intricate long-term schema that must be assimilated to accommodate this particular situation (Skemp, 1971). For example, a short-term schema for solving subtraction problems would be Figure 1. A long-term schema would be the generalization of Figure 2 to include "n" columns of numbers. For a student to solve a simple subtraction problem (33-26), he/she has the choice of using the simpler schema for which Figure 1 will suffice or he/she may elect to overreact and choose a more generalized form of Algorithm Two. Both of these schemata will produce the correct response in this instance, the former will take up less working memory, hence a shorter processing interval, whereas the generalized algorithm will require further discrimination of attributes, the pace will be slower and the same question will be harder to solve. It is the task of the student to choose and select the
schema which most closely fits the criteria given.

Memory

Greeno (1973) proposed that the concept of working memory is essential for problem solving, for it is here that the solution is organized from the data supplied in short-term memory and the information retrieved from long-term memory. Baddeley (1976) referred to long-term store as semantic memory. Greeno (1973) stated that working memory is as important facet in learning mathematics in the classroom and that certain teaching techniques may in fact overload this memory component. Greeno (1973) proposed that two kinds of knowledge were stored in semantic memory: propositional knowledge which contained knowledge in the form of rules or operations, and algorithmic knowledge which translates readily into action, in the sense of performing operations. There is probably also a difference between visual short-term memory and auditory or verbal short-term memory (Baddeley, 1976).

Baddeley (1979) argued that short-term or working memory played a central role in normal cognitive functioning. Current information is stored in short-term memory, whereas previously stored information about cognitive operations is retrieved from long-term memory. In addition, short-term memory is composed of two components, an articulatory loop in which material is verbally rehearsed and a central executive system that integrates incoming information with past
Figure 1. Algorithm One
knowledge. In arithmetic tasks, the articulatory loop may store the operation sign of subtraction and the numbers in the problem whereas the executive may retrieve the algorithm necessary to solve the problem. Conrad (1972) showed that a speech-based coding system may be used in the articulatory loop. This may be important in that Webster (1979, 1980) observed that the performance of children with arithmetic disabilities was significantly poorer than that of normally achieving children on auditory short-term memory tasks. Siegel and Linder (1984) also demonstrated that short-term memory is important for the performance of arithmetic operations. This suggests that not only are numbers and signs retained in short-term store but also that the operations combining them are carried out in short-term memory.

Anderson (1982) proposed that the number of small arithmetical productions that can be combined to form a large production is determined by the capacity of working memory. Svenson and Sjoberg (1981) found that switches in strategy were characterized by changes toward increased sophistication in terms of memory representation during the problem solving process. According to Svenson and Sjoberg (1981), a reproductive process meant that the solution is directly retrieved from storage in long-term memory and the reconstructive process meant that the answer is reached through a series of conscious manipulations in working memory to get the answer.
Siegel, L. and Linder, B. (1984) studied the role of phonemic coding in short-term memory of students with arithmetic disabilities and concluded that the use of a phonemic code in short-term memory appeared to develop more slowly in learning-disabled than "normal" children.

Although the student would not have to retain subroutine sequences in working memory, he/she would still have to allocate attentional capacity to applying general declarative procedural rule representations to specific numbers, computing the answers to two-digit subtraction subroutines, and generating repair strategies. In that a flow chart would not necessarily specify the semantic or conceptual features of a procedural rule (Resnick, 1982), attentional capacity might also have to be allocated to understanding the application of a given procedure, particularly when it had to be combined with a new default rule. Nevertheless, the reduction in the working memory load experienced when learning a new algorithm or schema might be considerable when a flow chart is used successfully and should result in an improvement in performance because of (a) less forgetting of procedures, and (b) the freeing up of attentional capacity for other factors.

Episodic Theory

It can also be argued that learning to subtract is instance based and does not rely on the abstraction of a generalized schema or algorithm (Brooks, 1978). This
approach is suggested by models of learning which stress the role of episodic memory (Kintsch, 1974; Tulving, 1983) and parallel distributed processing (Rumelhart and McClelland, 1986). Within the framework of these models, learning to subtract would consist of forming an episodic representation of a lesson which included the example subtraction question, the instructor’s actions, the products or outcomes of these actions, semantic interpretations of the lesson, and the context in which the lesson was experienced. It would also include the students’ actions and their outcomes if the students participated in part or all of the lesson. New lessons would involve the construction of new episodic representations. Although past learning (sometimes referred to as semantic memory)\(^1\) can inform and enter into the construction of a new episodic memory (Kintsch, 1974), an encoding that is common to both a previous and new episodic representation is not abstracted out in the form of a new higher level context independent schema, such as an algorithm. Rather, procedures and the declarative representations of procedures are seen as remaining firmly embedded in the context of the episodic representation within which they were originally encoded. This does not mean, however, that abstraction in the sense of isolating and encoding stimulus and/or procedural regularities can not occur. Two possibilities exist. First, Kintsch’s (1974) model suggests that a procedure

\(^1\)This is Tulving’s (1983) use of the term "semantic memory"
encoded as an action, embedded in an earlier episode would remain unchanged but the new episode would contain a more abstract or generalizable coding of procedure. With time, the learner might develop multiple episodic memories, each with a slightly more abstract coding of the same procedure. Studies involving recall have also supported the argument that these memories may be tagged and retrieved in an order of perceived importance (Meyer, Schvaneveldt and Ruddy, 1975). Hidi and Baird (1986) concluded that mental structures are hierarchically organized, with the most important propositions being at the upper levels and the extent to which a proposition can be recalled is determined by its structural position. Thus, the higher a proposition in structure the greater the likelihood in it being recalled. Furthermore, PDP (parallel distributed processing) models predict that all of these memories would be accessed when the student attempted to solve a new problem and that encoding specificity (the degree of match between the encoded features of the new question and a previous episode) would determine which one was used (Rumelhart and McClelland, 1986; Tulving, 1983). Second, Brooks (1987) has suggested that in those cases which stimulus (or procedural) regularities are too complex to code in short-term memory, the learner may store them in a distributed fashion across many episodic representations without any higher-order recoding. Hintzman (1986) modelled schema-abstraction and proposed that there was only one memory system which
stored episodic traces, and that abstract knowledge does not have to be stored but can be derived from the number of traces of specific experiences at the time of retrieval. Each event gives rise to its own memory trace and that repetition of a stimulus does not strengthen a prior representation but produces a new trace that coexists in memory with traces of other occurrences of the same item. Only traces of the individual episodes are stored and that aggregates of traces acting in unison at the time of retrieval represent the category as a whole. A memory trace is a record of an experience or episode and it preserves the properties making up the experience. Communication between primary and secondary memory is restricted to two operations: 1) a retrieval cue or probe can be sent from primary memory to all traces in secondary memory; and 2) primary memory can receive a single reply or "echo" that emits back from secondary memory. Hintzman further contends that the greater the similarity of traces to the probe and the greater the number of such traces, the greater the intensity of the echo. Information of various degrees of abstraction can be quickly retrieved from a store of many episodic memory traces. For trace activation, each trace is compared to the number of attributes that are similar to the probe and the echo intensity is the sum of the number of traces that have a high correlation to the probe, whereas the echo content is derived by only selecting those traces which are similar to the probe but not necessarily identical in content. Many of the
traces may contain information not present in the probe so that the sum of the activated traces may, in fact, contain additional information, and associative recall is produced. Hintzman's model is capable of retrieving an abstract idea even though the abstraction was never stored. Both Brooks (1987) and Rumelhart and McClelland (1986) argue that when the learner encounters a new exemplar of the regularities (or is asked a higher-order question if the new stimulus) a temporary local average is computed across the various episodic memories which allows the learner to respond as if he/she had formed and stored an abstract rule.

The advantage of this approach is threefold. First, and perhaps most important, it accounts for the variability that can be introduced into learning as a function of (context) cuing (Tulving, 1983). For example, repeated exposures to a stimuli produce not only traces of the occurrence in an episodic memory system but also a unitary abstract representation of the classification in a separate generic memory system.

Students may solve problems by applying procedures that are similar to proceeding problems even though some of the characteristics of the target problem have been altered from the original problems (Linder, 1985). Lockhead and King (1983) studied sequential effects and found that numerical scaling given a particular stimulus along some physical continuum depended not only on the stimulus but also on the values of the stimuli which were arranged to precede that
stimulus in the sequence of stimulus trials. Furthermore, sequential dependency effects are of two forms: assimilation and contrast. **Assimilation** refers to a tendency for a response to be positively correlated with the preceding responses and **contrast** refers to a tendency for a response to be negatively correlated with the preceding responses. Subjects appear to use the similarity information implied in these sequential dependencies to infer general rules or problem-solving schema. This process may relieve the subject from abstracting rules and from the burden of over-generalization and the proliferation of job default rules (Brooks, 1987). It may also account for the implicit learning of complex stimulus and procedural regularities without conscious rules (Brooks, 1978). As outlined earlier, (Lewicki, Hill and Bizot, 1988), the subjects in the computer target study had acquired a specific working knowledge about the patterns on the screen and this influenced their performance, but they were unable to articulate these processing algorithms. This seems to imply that the learner may encode implicit information in addition to procedural rules from a given lesson and that instructional methods which take advantage of this "richer" encoding model might see improvements in learning. Specifically, encoding tasks which encourage the learner to form episodic representations which are more elaborated, more integrated, and have more semantic memory associations than is normally the case, tend to result in better recall (Tulving, 1983). Less interference may result
during retrieval in solving mixed sets of subtraction questions in subsequent sessions. This hypothesis is based on the assumption that one of the difficulties students have lies in retrieving a distinct and complete representation of an algorithm or schema on a task such as subtraction with regrouping. Students need retrieval practice in this area. In this respect, student verbal reports tend to suggest that this type of interference may occur from the subtraction without regrouping and addition algorithms when attempting to learn subtraction with regrouping. Elaborated representations are seen as facilitating retrieval by increasing the probability of an overlap between context cues and the features of episodic memory. Put simply, the more elaborate a given episodic representation, the more likely some component of it will be part of the retrieval context. In addition, elaborated traces may be more distinctive and, as a result, less prone to retrieval interference (Klien and Saltz, 1976). This suggests that contextual processes are operative in students' information processing. Distinctiveness in this sense would be in terms of fewer overlapping components between two or more episodic traces and would result in a lower probability of a given retrieval cue activating traces which were similar to the target trace. Lewicki et al. (1988) suggests "subjects are able to acquire specific working knowledge (i.e., processing algorithms) not only without being able to articulate what they had learned, but even without being aware that they had learned anything" (p. 25). From memory
traces left by the repeated experiences there may arise a cumulative trace which contains only those properties which are common to the experience. Repetition has a cumulative effect and at a final stage it has become a concept (Skemp, 1961). Associations with semantic memory may also increase the probability of a given context cue making contact with an episodic memory trace. In this case, however, retrieval would be indirect in that the cue would first retrieve the associated semantic trace which would then be used to retrieve the target episodic memory. Presumably, semantic associations act to increase the amount of potential cue-trace overlap in a system. For example, establishing a semantic link between a given episodic component (I looked at the one's column and saw that the top number was smaller than the bottom one) and a second less obvious component (I looked at the ten's column to see if I could borrow)\(^2\) should increase the probability of the first component retrieving the second one (Resnick, 1982). Increasing the degree of integration or internal cohesion of an episodic representation, on the other hand, may increase the probability of one component of the representation acting as a retrieval cue for another, less well learned component.

Karplus (1977) has developed the concept of a learning cycle based on a Piagetian-based teaching model designed to foster self-regulation. The learning

\(^2\)The term "less obvious" here refers to the possibility that this procedure may not be associated with a stimulus cue such as a relationship between the top and bottom digits.
cycle consists of three aspects, exploration, concept introduction and application. Many learner-centred mathematics programmes follow this model. In essence, this model parallels the discovery lesson in mathematics where the students experiment with concrete materials to develop a pattern or rule based upon their intuitive thinking. During exploration the learner is involved in experience with familiar or concrete materials. These experiences can be structured and designed so learners experience information slightly beyond their understanding. At this stage the student enters the concept introduction phase. This is when the learner accommodates and grasps a new level of understanding. The instructor provides the students with a set of directed questions to focus on the concept being developed. The third aspect or application phase necessitates the extension of the newly acquired concept to other appropriate ideas. Karplus’ model provides a path for teaching self-regulation.

Within this framework, episodic models suggest that embedding the procedures of a subtraction lesson on regrouping in the context of a story might be of value. The purpose of the story would not be that of a mathematics word problem. Rather, a story format might be designed to facilitate the encoding of a distinctive and integrated episodic representation of the algorithm for subtraction with regrouping. Schema-abstraction (Hintzman, 1986) provides a framework for the learner to acquire the potential for abstraction through episodic structure.
The episodic model in this study of the additive method of subtraction consists of two sets of core processes, each one with many similar attributes but also with some differences. The episodes provide stimuli to the student to aid in the successful solution of subtraction problems. Each time the student hears, reads or even thinks about this episode which parallels the procedure for the additive method of subtraction, a new memory trace may be stored in memory.

This possibility is supported by Greeno (1979) who discussed the concept of perceptual relativity in relation to information processing. Perceptual relativity effects involve demonstrating changes in the perception of a stimulus feature embedded in a single array as a function of changes in the remaining attributes of that array. This perception depends upon the entire "sensory field" present at the time at which the stimulus is perceived. Baron (1978) also showed that letter recognition is influenced by contextual factors by demonstrating that the speed in which a letter is identified is enhanced when the letter is embedded in a word as compared to when presented in a random sequence. Skemp (1962) presented one hundred symbols to two groups of students; one group learned the symbols by rote and the second group had learned a schema into which the new symbols and their meanings could be assimilated. The difference between the schematic and the rote learning was dramatic. For schematic learning, twice the number of symbols were recalled immediately after learning and seven times as many, after
four weeks, could be retrieved than the rote learners. In fact the rote learners forgot more in one day than the schematic learners did in four weeks.

Recently, investigators recognized that structural importance is not the only variable that determines how discourse is remembered. The role of interestingness in text comprehension and recall is now considered to be a factor (Luftig and Johnson, 1982). If students’ attention is sustained, it will enhance the learning process in a marked manner and enable the students to organize and create a structure for the learning task. Hidi and Baird (1986) suggested that both cognitive and emotional interest invoke an increase in cognitive effort - information search and inferencing. Thus it may be that retrieval is enhanced when the learner is motivated to increase intellectual activity to cope with incoming information. Anderson, Shirley, Wilson and Fielding (1983) deduced that interest had a powerful effect on children’s learning and recall of single sentence materials. Hidi and Baird (1986) concluded that interest was a process responding to the significance of information. This statement suggests that material may be recalled better by subjects if the knowledge fits into their personal reality. Taken together then, this literature suggests that the use of an episodic (story) format should facilitate procedural learning and transfer in mathematics.
Poor Learners

For learning-disabled students at the secondary level, Skrtic (1980) recommended that to remediate mathematical deficiencies, concrete and graphic representations of concepts, relationships and operations should be employed. Liedtke (1985), after working with slow learners or learning-disabled students, argued that in order for an operation to be understood, students have to be able to visualize the operation and relate it to their task experience. Liedtke's transition board (1985) provided students who had difficulties with a subtraction algorithm a visual framework to transcend from the concrete objects to recording abstract symbols in the operation. This transition was a piece of cardboard which enabled the student to simulate the subtraction process. Bugelski and Alampay (1961) argued that visual perception is not determined solely by physical characteristics but also by the trial context in which the figure was presented. Smith (1979) showed that the contextual stimuli were incidental to the encoding of the study items. Subjects were to learn words in one room and they were taken to a distinctively different room. Half of the subjects were taken back to the original room where the learning took place and the other half of the students were taken to another location. The context group in the original room recalled twenty-five percent more words than the second group in a different room. The incidental contextual stimuli of the original learning setting had been encoded and
became a part of the learning experience. Secondly, Smith (1979) showed that the subjects, by simply thinking about the setting of the original room where the learning took place, improved their retrieval of the test items and their level of recall. Students who had difficulties with an algorithm may have memorized the process but had forgotten some of the steps. If the student can visualize the flow chart or picture the concrete representation on a transition board (Liedtke, 1985), then the likelihood for errors is diminished. For these students, to understand subtraction facts, the addition facts and their properties must be retaught first to enable the student to realize that for each addition fact there are two related subtraction facts. Students who have difficulty with an algorithm likely have memorized steps or procedures without fully understanding them. It is further suggested that learning-disabled students appear to make the same adjustments as nondisabled students in strategies but not to the same degree of proficiency. Many learning-disabled students do not know the basic procedures for subtraction and have not acquired other skills like direct modelling or counting strategies that would help them solve problems, hence they try to use algorithms (as often as "normal" children) but unsuccessfully (Romberg, T. and Collis, K., 1985).

Swanson and Rhine (1985) argued that the reason that learning-disabled children do not function at the same level as "normal" children at the same chronological age and socioeconomic background is that they do not employ the same level of
strategies to solve problems. For example, children with learning difficulties at the early stages of arithmetic acquisition showed insensitivity to intralist phonemic similarity, presumably because of the difficulty with the speech-based coding system that is part of short-term memory. The use of a phonemic code in short-term memory appears to develop more slowly in learning-disabled children than in normally achieving children, but it does develop (Shankweiler, Liberman, Mark, Fowler, and Fischer, 1979). Alley and Deshler (1979) and Wong (1984) argued that learning-disabled children utilize different strategies than their nondisabled control group. The learning-disabled children relied on the time-consuming and developmentally less mature counting algorithm (Ashcroft, 1982) for a problem solution whereas a shift from the counting strategy to a memory-retrieval process was evident for "normal" children (Geary et al., 1987). However, Neches and Hayes (1978) found that disabled children initially learn the same strategies as "normal" children but the latter group modifies their strategies to become more efficient. Torgesen, Murphy and Ivey (1979) argued that learning-disabled children do not approach certain kinds of cognitive tasks in an efficient manner and that they fail to engage goal-directed activities that are related to the efficient use of cognitive resources. For example, subjects were asked to study the stimuli of twenty-four pictures over a three-minute interval and after a time delay of ten minutes were tested for recall of the stimuli. Subjects were encouraged to do
anything during the study period that would help them remember the pictures better. Many subjects classified the pictures by sorting them into subgroups. However, the learning-disabled subjects failed to classify and approach tasks in an efficient manner.

Liedtke (1982) concluded after working with slow learners that even if errors of similar types are encountered, the reasons for these errors are likely to differ from individual to individual. However, even the learning-disabled student will have responded on the basis of some reasoning even though it may be incomplete or incorrect (Sadowski, B. and McIlveen, D., 1984).

One of the reasons that subtraction is more difficult than addition is that the subtraction algorithm contains a greater number of rules than the addition algorithm - the comparisons at each step to see if subtraction is possible immediately or only after a regrouping, the requirement of remembering the borrow when the attention shifts to the tens column (Vakali, 1984). The more complex the procedure, the greater the chance of more bugs in the rules. Students appeared to invent many new strategies in order to adapt the taught algorithms to their own conceptual framework. Vakali (1984) speculated the fact that subtraction word problems caused more idiosyncratic strategies to be used shows that as the complexity of the problem increases, the mental effort and nature of solution strategies also becomes more complex.
Byers and Erlwanger (1985) remarked that "understanding mathematics is...the ability to do mathematics". What is required is the ability to do valid mathematics (Byers and Erlwanger, 1985):

A good student in mathematics can organize his material in such a way to minimize cognitive strain and is able to strike a balance between memory and deduction. He knows which formulas have to be remembered, which partially remembered and partially deduced and which can be left to be derived as needed. He uses mnemonic devices successfully, and is able to devise his own. A poor student cannot do this; so he tries to remember by brute force a multitude of rules, facts, and procedures. (p. 277)

In an unpublished study Brooks (1989a) found that students who tried to master a set of subtraction problems which were presented in a episodic format with story titles, learned the additive method of subtraction and had success on similar, and transfer problems. Brooks (1989a) felt that forgetting for these students may have been caused by interference from the episodic representations which were similar to the newly learned algorithms but not identical.

For students who have difficulty mastering a particular concept, it is sometimes desirable to try a different form of delivery or a fresh approach. All of these students had been exposed to the traditional form of subtraction for at least seven years and still had not mastered this process. The equal addend method of subtraction provided the ideal vehicle for a new start for these students.
Although this additive method of subtraction is not usually taught in North America, it is the process used to teach subtraction in some countries, one of which is Greece (Vakali, 1984). Some of the subtraction errors identified by Van Lehn (1983) that students made in the United States using the decomposition method of subtraction did not occur using the equal addition algorithm. For this reason the equal addend method of subtraction was selected to help some weak mathematics students who still had difficulty in subtracting. The concepts of episodic representation, schema, transfer and worked examples rendered the framework for this present study of retrieval of mathematical information employing algorithms (flow charts), episodes (stories) and numerical problems.
CHAPTER THREE: METHODOLOGY

Hypotheses

1. It was hypothesized that the use of flow chart algorithms to instruct students in the procedural steps in a new subtraction method (the Equal Addends Method) would result in a clearer differentiation of these steps in terms of their representation and understanding in working memory and, as a result, fewer trials to criterion in initial and transfer training than would be observed in a control worked example training condition.

2. It was hypothesized that the use of story-based episodic presentation formats to instruct students in the acquisition of a new subtraction method (the Equal Addends Method) would result in a more distinctive representation of the method in long-term memory and hence (a) greater retrievability on training questions relative to a control worked example condition and (b) greater resistance to interference and hence greater retrievability on transfer-training questions relative to a control worked example training condition. Greater retrievability would be reflected in the number of trials to mastery and the level of success on the operational level of the additive method of subtraction.
In order to test these hypotheses, a sample of forty-two basic level secondary school students was randomly sorted into (1) a worked example group, (2) an algorithm (flow chart) group, and (3) an episodic (story) group. These students were then individually taught to subtract with regrouping using the equal addend method. The instruction took place in four phases or separate sessions. In the first two phases, the student had to achieve mastery on a set of eight questions. On Phase 3 after the student achieved mastery he/she was given a set of twelve near transfer questions. Phase 4 consisted of a test of thirty-two mixed subtraction questions. The dependent variables were (a) the number of correct responses on similar, near and far transfer questions and (b) the number of questions to criterion or mastery. In addition, error types and verbal protocols were analyzed.

Subjects

From a sample of six hundred twelve students from a suburban high school which is classified as a Basic Level school in Ontario, Canada, one hundred students were matched as closely as possible by I.Q., mathematics, socio-economic background and sex. Forty-two students were randomly selected from this matched sample of one hundred students. Using stratified random sampling, the students were assigned to one of three groups, each containing fourteen subjects: an episodic group, an algorithm group, and a worked example group.
For admission to this school, all students are screened by an admission board, which examines I.Q. and mathematics and reading ability. All students must have an I.Q. in the range between 85-110, must be operating at the Grade Five/Six reading and mathematics level of proficiency, and must be at least fifteen years of age. The mean and standard deviations for each category were as follows: algorithm group, mean I.Q. = 92.1, S.D. = 6.39; Episodic group, mean I.Q. = 95.3, S.D. = 5.11; worked example, mean I.Q. = 96.6, S.D. = 6.65.

All I.Q.s were based on Weshsler Intelligence Scale of the (WISC-R). A pretest of thirty-five mixed subtraction problems was given to each of the students. The Student's t Distribution test was employed to establish that there were no significant differences between the three groups. (algorithm group, mean X = 27.0, S.D. = 6.3; episodic, mean X = 29.1, S.D. = 5.6; worked example, mean X = 27.2, S.D. = 5.4).

Stimuli

The ability of students to solve subtraction problems requires students to perceive numbers and their relationships. A transition from similar problems to near and far transfer problems requires generalization of the concepts of the operation of subtraction.

The procedures of the additive method of subtraction are schematically
represented in Figure 1. This process does require the students to know the mathematical meaning of words and expressions such as column, unit’s column, and ten’s column. This study is interested in the transparent processes underlying the acquisition of the process of the additive method of subtraction; inherent in this process are base ten concepts. A set of productions for performing subtraction is schematically represented by the flow chart in Figure 1. To trace the productions for the task specific example of:

\[
\begin{array}{c}
23 \\
-7
\end{array}
\]

For the first few steps, refer to Figure 1. The student looks at the right hand column and compares the digits 3 and 7. Since 3 is less than 7 the "no" decision is taken and the student proceeds to write a small 1 to the left of the 3. Moving to the ten’s column on the bottom there is no number beside the 7, hence the next step is to write a 1 beside the digit 7 to make the number 17. The student continues to trace through this flow chart until reading the final answer. Figure 2 is an extension of Figure 1 paralleling subtraction problem involving a hundred’s column. This process of generalization requires an awareness of the operations and concepts but also the inter-relationship between the two schemata. A further generalization requires a modification of Figure 2 procedures by the student.
A second mode of stimuli was episodic context. The content of the episode paralleled the structure of the subtraction problem and the analogy of the episode may produce a separate memory trace. The retrieval cue is the episode itself as a representation of the schema of the additive method of subtraction. Episode One demonstrates the verbalization of the structure and abstraction of the subtraction problem 23-7. The student may use the episode as a reflective interpretative schema to model his/her cognitive processes to abstract the procedures for a solution to the subtraction problem. Episode Two (Appendix XI) is a modification of Episode One and is representative of the generalization of the additive method of subtraction. An episodic view does not propose that generalization relies on an abstract representation, rather a generalization from memory (Brooks, Jacoby and Baker, 1989b). Brooks (1987) reflected that performance reflects differences in the encoding and retrieval of presented instances and that encoding-retrieval interactions are important for the influence of prior experience on supposedly semantic-memory tasks (Brooks et al., 1989b). As outlined in Chapter One, operational level will be used to denote the structure of problems by the positioning of the magnitude of each digit in each problem and was classified as large (L), equal (E) and small (S) depending on their relative vertical placement. The structure of the similar problems for each of the instructional conditions in Phase 1 was:
Procedures

The systematic teaching study was conducted over a four-day interval. A collective subtraction pretest (Appendix M) was administered to all students in the study. The pretest yielded data about the initial level of arithmetic competence in the area of subtraction before the teaching program was applied to the experimental groups. During the instruction phase each student left his or her classroom during the regular school day and went into another small room adjacent to the library. After having entered into this experimental room the subject was taught on an individual basis procedures for the additive method of subtraction by algorithms (flow charts) or by episodes or by worked examples. The subjects were tested on four distinct days. The experiment was subdivided into four phases. In Phase 1 the subject was taught the additive method of subtraction (regrouping to 1's) and worked on a series of similar problems until a mastery level was attained. A prompt was available at all times. In Phase 2 and Phase 3 on successive days students were pretested on similar and transfer subtraction problems, and taught schemata for subtracting with regrouping to 10's, 100's and 1000's. Verbal protocols were also taken. When the experimental.
Figure 2. Algorithm Two
Figure 3. Phase 1 and Phase 2 of procedures for regrouping in subtraction
Figure 4. Phase 3 and Phase 4 of procedures for regrouping in subtraction
teaching program was concluded, in Phase 4 the posttest was administered to the students in the experimental and the control groups. The posttest consisted of thirty-two mixed subtraction problems (Appendix K).

**Phase 1**

In Phase 1 the subjects were taught the additive method of subtraction (regrouping to 1's) and worked on a series of similar problems until a mastery level was attained. A prompt was available at all times during this phase. The visual prompt varied across all three conditions, episodic, algorithmic and worked example, but remained the same within each condition for each subject.

**Episodic condition.** After entering the library seminar room the subject was seated at the table beside the instructor and was asked if he/she were interested in football. A sheet of paper with Episode One was placed on the table in front of the subject. The following episode was presented orally by the instructor while the subject read the story. The subject was asked if he/she understood the episode and from the positive responses to the two questions, "Which team is going to the Grey Cup and by how many points did it win?", the instructor was able to ascertain this fact. Subjects were permitted to refer to Episode One at any time during Phase 1. **Episode One:**

*The football team which advances to the Grey Cup each year must win a two-game series by total points. In Game One, the Toronto Argonauts (23) beat the*
Hamilton Tiger Cats (7). The Argos have a 16 point advantage going into the second game. In Game Two, the score tied 10-10. Which team is going to the Grey Cup and by how many points did it win?

The instructor then demonstrated the solution to the subject in the following expanded form by adding 10 onto the ten’s column in the subtrahend and by adding 10 onto the one’s column in the minuend:

Add 10

Argos 23 --------> 20 + 13

Add 10

Cats -7 --------> 10 + 7

10 + 6

(Note how the episode parallels the structure of the solution.)

The subject was then given a second problem of 42-14 and instructed to solve it using the same procedure.

Add 10

42 --------> 40 + 12

Add 10

-14 --------> 20 + 4

20 + 8
Episode One was made available to the subject as a reference while he/she attempted to achieve mastery of this method. The criterion for mastery was defined as correctly responding to at least six out of eight problems from Series A. If less than six problems were correct, the student was given a second set of eight problems (Series B) and instructed to use Episode One in solving them.

If the student were unable to achieve mastery with Series B questions, the same procedure was repeated with Series C and D questions. Once mastery was achieved, however, no further questions were given.

Algorithm condition. After entering the library seminar room the subject was seated at the table beside the instructor. The subjects in this condition were presented with the same problem used in Episode One, (23-7) and required to point the unit's column and the ten's column. A sheet of paper with Algorithm One was placed on the table in front of the subject (Figure 1). The algorithm was presented orally by the instructor while the student simultaneously traced through the problem. The subject then completed the problem by using the algorithm as a guide in computing the correct response. The subject was then given a second problem of (42-14) and instructed to solve it using Algorithm One, which was made available as a reference. The criterion for mastery was again defined as correctly responding to at least six out of eight problems from Series A. If less than six problems were correct, the subject was given a second set of
problems (Series B) and instructed to refer to Algorithm One while working through them. This procedure was repeated with similar problems from Series C and D if mastery was not achieved. Once mastery was achieved, however, no further questions were administered.

**Worked example condition.** Each subject was instructed on an individual basis on the equal addend method of subtraction. No procedures were written down for the subject to refer to and all the methodology was verbal.

For example:

23  Q: Can you subtract 7 from 3?

-7  E.S.R.: No.

Q: What do you think you should do now?

E.S.R.: Add ten ones to the 3 to make 13.

(Instructor crosses out the 3 and writes 13 above the 3.)

23

-7

(Instructor demonstrates step two by adding 1 ten onto the number being subtracted and writes it in the ten's column -17 beside the 7.)

Q: What number added to 7 makes 13?
E.S.R.: 6

23

-17

6  (Digit 6 is written directly below the 7.)

Q: What number added to 1 makes 2?

23

17

16  (Digit 1 is written to the left of 6.)

Q: What is your final answer?

E.S.R.: 16

The subject was asked if the procedure was understood. If the procedure was misinterpreted, it was retaught. The subject then completed the following example with the aid if the instructor, if and when assistance was required to produce the correct answer of 28:

42

-14

The instructor again asked if this new method of subtraction was understood.
If the subject replied in the affirmative, then he/she was given the subtraction Series A as described earlier and instructed to complete eight problems employing this additive method of subtraction and to show the appropriate changes of the values of the digits as outlined in Example One. The criterion for mastery was again defined as correctly responding to at least six out of eight problems from Series A. If less than six problems were correct, the subject was given a second set of problems (Series B) and instructed to refer to the worked examples while working through them. This procedure was repeated with similar problems from Series C and D if mastery was not achieved. Once mastery was achieved, however, no further questions were administered.

**Phase 2**

In Phase 2 the subjects were pretested on two similar and two near transfer subtraction problems to determine how well they recalled the procedures of the additive method of subtraction and if they were able to adapt the schema to solve successfully the near transfer problems. If students made errors on the similar problems, they were still directed to solve the near transfer problems. The near transfer subtraction problems involved the operation of regrouping to 10's. The actual time delay between Phase 1 and Phase 2 was a minimum of twenty-four hours. Each session lasted between fifteen and twenty minutes and each subject
was interviewed individually.

**Task one.** This task was administered to all of the subjects in all three treatment conditions. Verbal protocols were taped while the subject simultaneously solved an initial four problems. Each individual was presented with two similar questions and two near transfer questions at the beginning of the session. The form of the instructor's statement was: "Why don't you do these questions and tell me what you are doing while you are subtracting?" If the subject correctly answered both similar problems, he/she was allowed to proceed to Task Two. However, if the subject made an error on these similar problems, then the instructor assisted the student using the algorithm, episodic and worked example methods respectively. On a series of similar questions (Appendix C, D, E), the student was brought up to the required level of mastery before proceeding to Phase 2-Task Two.

The first two questions are similar problems to those in Phase 1 and the second two questions are near transfer problems involving regrouping to 10's.

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>92</td>
<td>33</td>
<td>362</td>
<td>726</td>
</tr>
<tr>
<td>-57</td>
<td>-15</td>
<td>-81</td>
<td>-43</td>
</tr>
</tbody>
</table>

The structure of the similar problems used here was identical to the structure of the problems which the subject had encountered in Phase 1 and reached the
required level of mastery.

**Task two.** In Task Two the subjects were taught the additive method of subtraction (regrouping to 10's) and worked on a series of similar problems (regrouping to 10's) until a mastery level was attained. A prompt was available at all times during this phase. The visual prompt varied across all three conditions, episodic, algorithmic and worked example, but remained the same within each condition for each subject.

**Episodic condition.** Episode Two (Appendix L) was made available for the subject to refer to while solving a series of eight similar problems. A sheet of paper with Episode Two was placed on the table in front of the subject. The following episode was presented orally by the instructor while the subject read the story. The subject was asked if he/she understood the episode and from the positive responses to the two questions, "Which player had more yardage in the series and by how much?" the instructor was able to ascertain this fact. Subjects were permitted to refer to Episode Two at any time. The subject continued to solve a subsequent series of eight subtraction problems (Appendix F) as described earlier and instructed to complete eight problems employing this additive method of subtraction and to show the appropriate changes of the values of the digits as outlined in Episode Two. The criterion for mastery was again defined as correctly
responding to at least six out of eight problems from Series A. If less than six problems were correct, the subject was given a second set of problems from Series B (Appendix G) and instructed to refer to Episode Two while working through them. This procedure was repeated with similar problems from Series C (Appendix H) and Series D (Appendix I) if mastery were not achieved. Once mastery was achieved, however, no further questions were administered.

**Algorithm condition.** Each subject was given Algorithm Two (See Figure 2) in this condition. The procedure was the same as described in the Episodic Condition, the only difference being that the subject had Algorithm Two to refer to instead of Episode Two.

**Worked example condition.** The same problems of regrouping to 10's that were used with Episode Two and Algorithm Two were presented to each student in this group. As in Phase 1, each individual had the opportunity to ask questions at any time about any part of the new process. The subject responded and completed the series of similar questions for Task Two as outlined earlier, until mastery was reached for these problems.

**Phase 3**

In Phase 2 the subjects were pretested on two similar and two near transfer subtraction problems to determine how well they recalled the procedures of the
additive method of subtraction and if they were able to adapt the schema to solve successfully the near transfer problems. If students made errors on the similar problems, they were still directed to solve the near transfer problems. The near transfer subtraction problems involved the operation of regrouping to 10's. The actual time delay between Phase 2 and Phase 3 was a minimum of twenty-four hours. Each session lasted between twenty and thirty minutes and each subject was interviewed individually.

Task one. This task was administered to all of the subjects in all three treatment conditions. Verbal protocols were taped while the subject simultaneously solved an initial six problems. Each individual was presented with two similar questions and two near transfer questions and two far transfer questions on a sheet of paper at the beginning of the session. The form of the instructor's statement was; Why don't you do these questions and tell me what you are doing while you are subtracting? If the subject correctly answered both similar problems, he/she was allowed to proceed to Task Two. However, if the subject made an error on these similar problems, then the instructor assisted the student using the algorithm, episodic and worked example methods respectively. On a series of similar questions (Appendix VI, VII, VIII), the student was brought up to the required level of mastery before proceeding to Phase 3-Task Two.

The first two questions are similar problems to those in Phase 2 involving
regrouping to 10's and the second two questions were near transfer problems involving regrouping to 100's and the third two questions were far transfer problems.

<table>
<thead>
<tr>
<th>642</th>
<th>526</th>
<th>4286</th>
<th>6742</th>
<th>7643</th>
<th>9283</th>
</tr>
</thead>
<tbody>
<tr>
<td>-186</td>
<td>-235</td>
<td>-1934</td>
<td>-840</td>
<td>-2708</td>
<td>-4095</td>
</tr>
</tbody>
</table>

The structure of the similar problems used here was identical to the structure of the problems which the subject had encountered in Phase 2 and reached a level of mastery. The time lag was a minimum of twenty-four hours from Phase 2. As outlined earlier a near transfer problem was one that had an operation step involving adding a factor of ten to one column in the minuend and the same value to the subtrahend in another column other than the type already encountered. The structure of the near transfer problems was:

```
<table>
<thead>
<tr>
<th>LSL</th>
<th>LSEL</th>
</tr>
</thead>
<tbody>
<tr>
<td>-SLS</td>
<td>-LES</td>
</tr>
</tbody>
</table>
```

The structure of the far transfer problems was:

```
<table>
<thead>
<tr>
<th>LSS</th>
<th>LLSS</th>
</tr>
</thead>
<tbody>
<tr>
<td>-SLS</td>
<td>-SSL</td>
</tr>
</tbody>
</table>
```

Verbal protocols were taken while the subject solved these six problems. As in Phase 2-Task One, if the subject correctly answered both similar problems, he/she was allowed to proceed with Phase 3 Task Two. If the subject made an error on the similar problems, then the instructor made the respective Episode Two or Algorithm Two or Worked Example Two instructions available to the subject.
before proceeding to Task Two. The subject was then given a series of similar questions (Appendix VI, VII, VIII). As before, each subject was brought up to the required level of mastery before proceeding with Phase 3 Task Two.

**Task two.** The purpose of Task Two was to see how well students were able to assimilate schema previously learned and accommodate near transfer problems into their existing framework. Each subject was presented with a sheet of paper containing twelve near transfer problems (Appendix IX) from the similar problems solved in Phase 2. The subject was not permitted to refer to any written materials or any visual prompts and no verbal protocols were taken during this part of the session. The structure of near transfer problems was:

<table>
<thead>
<tr>
<th></th>
<th>LSLL</th>
<th>LSLS</th>
<th>LSEE</th>
<th>LSLLE</th>
<th>LSEL</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>SLSS</td>
<td>SLSL</td>
<td>SLEE</td>
<td>SLSSE</td>
<td>SLES</td>
</tr>
</tbody>
</table>

**Phase 4**

The posttest was administered to all forty-two subjects in the study. Each subject was individually given thirty-two mixed subtraction problems on a sheet of paper containing similar, near transfer, and far transfer types (Appendix K) to solve without the aid of any written materials or visual prompts and instructed to show the process used while answering each problem. This session lasted approximately twenty-five minutes. The time lag between Phase 3 and Phase 4 was a minimum of twenty-four hours.
CHAPTER FOUR: ANALYSIS AND EVALUATION

Results

The accuracy on the pretest of thirty-five mixed subtraction problems was 74.9% for the episodic group, 77.3% for the algorithm and 75.7% for the worked example group. (Appendix M) There was no significant difference in the ability of the subjects across the three groups. On basic operations such as subtraction, the observer would expect a mastery level of ninety percent from basic level students.

As can be seen in Table 1, the number of problems that were answered correctly in attaining mastery did not vary much across the three treatment groups. However, the worked example group required an additional 80 questions to achieve mastery. This indicates that the worked example group made more errors in achieving mastery. In Phase 4 the percentage of correct mixed problems was 81.7% for the episodic group, 77.7% for the algorithm and 40.6% for the worked example group. Moreover, the achievement level of worked example group was not as high as the other two groups on the final mixed problems test in Phase 4. This suggests that either the amount of learning associated with mastery performance was less in the worked example group or that these students were experiencing retrieval problems on the Phase 4 questions.
Table 1

Number of Problems to Mastery across Phase 1, 2, 3

<table>
<thead>
<tr>
<th>Group</th>
<th>Total number of problems correctly answered to reach mastery</th>
<th>Number of problems attempted to reach mastery</th>
</tr>
</thead>
<tbody>
<tr>
<td>Algorithm</td>
<td>515</td>
<td>604</td>
</tr>
<tr>
<td>Episodic</td>
<td>517</td>
<td>604</td>
</tr>
<tr>
<td>Worked</td>
<td>531</td>
<td>684</td>
</tr>
</tbody>
</table>
Table 2

Percentage Correct of Similar and Near Transfer Problems in Phase 2

<table>
<thead>
<tr>
<th>Group</th>
<th>Percent Correct Similar Problems</th>
<th>Percent Correct Near Transfer Problems</th>
</tr>
</thead>
<tbody>
<tr>
<td>Algorithm</td>
<td>96.4</td>
<td>71.4</td>
</tr>
<tr>
<td>Episodic</td>
<td>82.1</td>
<td>17.9</td>
</tr>
<tr>
<td>Worked Example</td>
<td>78.6</td>
<td>60.7</td>
</tr>
</tbody>
</table>
The data in Table 2 suggest that by the second session (Phase 2) the students in all three groups were responding at a high level for the similar problems. However, the episodic group appeared to have some initial difficulties with the first sets of near transfer problems. As it turned out this was a transient problem, perhaps caused by the structure of Episode One since it was not obligatory to use the subtraction algorithm to make a correct conclusion but rather a desired process by the instructor.

A Student's t Distribution was used to compare the performance of the three treatment groups: \((n_1 + n_2 - 2) = 14 + 14 - 2 = 26\) degrees of freedom in Phase 2 on the similar problems. This analysis showed that there were no significant differences between these three groups on these problems.

A similar analysis of the near transfer problems in Phase 2 indicated that here was no difference between the algorithm and worked example groups. However, this analysis yielded a significant difference between algorithm and episodic groups \((t = 5.01, p<.01)\) and between episodic and worked example groups \((t = 2.74, p< .01)\).

By Phase 3, as can be seen in Table 3, the performance of the worked example group on the similar problems was significantly less than both the algorithm and episodic group (algorithm vs. worked example \([t = 3.64, p<.01]\); episodic vs. worked example \([t = 3.85, p<.01]\). In the case of the near transfer
Table 3

Percentage Correct of Similar and Near Transfer Problems in Phase 3

<table>
<thead>
<tr>
<th>Group</th>
<th>Percent Correct Similar Problems</th>
<th>Percent Correct Near Transfer Problems</th>
<th>Percent Correct Far Transfer Problems</th>
</tr>
</thead>
<tbody>
<tr>
<td>Algorithm</td>
<td>78.6</td>
<td>75.0</td>
<td>25.0</td>
</tr>
<tr>
<td>Episodic</td>
<td>92.9</td>
<td>85.7</td>
<td>53.6</td>
</tr>
<tr>
<td>Worked Example</td>
<td>46.4</td>
<td>50.0</td>
<td>25.0</td>
</tr>
</tbody>
</table>
problems, on the other hand, the performance of the episodic group began to stand out (episodic vs. worked example \( t = 2.67, p<.01 \)). On far transfer problems in Phase 3 there were no significant differences in performance between the algorithmic and the worked example group; however, there were significant differences between the episodic and the worked example group \( t = 3.98, p<.01 \). It would appear, then, that the results of the algorithm group and the worked example group were identical in nature; by operational level they were distinct. In ten instances the subjects from the worked example group demonstrated interference with the traditional form of subtraction but only one such instance occurred in the algorithm mode. Moreover, seven of the subjects in the worked example mode reverted to their old processes of subtraction and did not successfully execute the additive method of subtraction. It should also be noted that performance stayed at the same level on the similar and near transfer for all three treatment groups, but dropped off considerably on the far transfer problems.

The additive method of subtraction is quite unique in its basic structure as compared with other subtraction practices. The thought process of each student was recorded simultaneously while the problem at hand was being solved. The students did not always use the correct mathematical word in the proper context
Figure 5. Percentage correct: Phase 3 - Task One
Table 4
Operational Level of the Additive Method of Subtraction

<table>
<thead>
<tr>
<th>Phase 2 Task One</th>
<th>Phase 3 Task One</th>
</tr>
</thead>
<tbody>
<tr>
<td>Algorithm</td>
<td>Algorithm</td>
</tr>
<tr>
<td>100%</td>
<td>50%</td>
</tr>
<tr>
<td>Episodic</td>
<td>Episodic</td>
</tr>
<tr>
<td>42%</td>
<td>50%</td>
</tr>
<tr>
<td>Worked Example</td>
<td>Worked Example</td>
</tr>
<tr>
<td>17%</td>
<td>19%</td>
</tr>
</tbody>
</table>
but upon examination of these responses it was easy to trace the logical or illogical thoughts as the problems were solved in sequential fashion. From the protocols it was not difficult to determine if in fact the students were using the additive method of subtraction exclusively.

Turning to Phase 4 performance, the results of a comparison of the three treatment groups on the total number of (Mixed) questions administered indicated that there was a significant difference between the worked example group and the algorithm group ($t = 7.14, p<.01$) and between the worked example and the episodic group ($t = 7.20, p<.01$). There were no significant differences, however, between the episodic and algorithm groups. In general terms then, these data suggest that both the algorithm and episodic treatments represent some sort of improvement on the traditional worked example approach for learning-disabled students.

As can be seen in Table 5, this difference appears to be the result of performance on the near and far transfer problems and not on the similar problems. Statistical analysis showed that there were no significant differences between the three treatment groups on the similar questions. On the other hand, performance on the near transfer problems was significantly poorer in the worked example group than in the episodic group ($t = 4.58, p<.01$) and the
Figure 6. Percentage correct: Phase 3 - Task Two
<table>
<thead>
<tr>
<th>Group</th>
<th>Total</th>
<th>Similar</th>
<th>Near/Far Transfer</th>
<th>Near/Far Transfer Revised</th>
</tr>
</thead>
<tbody>
<tr>
<td>Algorithm</td>
<td>77.7</td>
<td>90.8</td>
<td>73.0/75.3</td>
<td>77.1/64.3</td>
</tr>
<tr>
<td>Episodic</td>
<td>81.7</td>
<td>91.8</td>
<td>78.6/79.2</td>
<td>79.3/77.4</td>
</tr>
<tr>
<td>Worked Example</td>
<td>40.6</td>
<td>76.5</td>
<td>32.7/28.0</td>
<td>34.2/19.0</td>
</tr>
</tbody>
</table>
algorithmic group ($t = 3.93, p < .01$). In Phase 4 there were significant differences for the far transfer problems (worked example vs. episodic, $[t = 4.80, p < .01]$ and worked example vs. algorithm $[t = 4.79, p < .01]$). In both of these cases the differences between the algorithm and episodic groups were not significant. By Phase 3, subjects in the episodic condition had a pool of two episodes to retrieve, the results of near transfer jumped dramatically from 17.9% in Phase 2 to 85.7% and 78.6% in Phase 3 and 4 respectively. Since the nature of far transfer problems were not well defined up to this point, an investigation was conducted to identify those problems which gave the subjects the most difficulty. The mixed problems were rescored by categorizing the various types of errors for each problem in Phase 4. There were six problems which produced the greatest number of errors. These six problems were then reclassified as far transfer revised problems. A far transfer revised problem was one that had two or more operational steps involving adding to the subtrahend and correspondingly to the minuend in a column already encountered or a process which had not been introduced to the student, such as adding quantities where the digit zero appears, and adds further complication. The remaining near and far transfer problems were reclassified as near transfer revised.

The reclassification produced no changes in the overall pattern of the data.
For both the near transfer revised and the far transfer revised questions there was a significant difference between the percentage correct of the worked example group and the percentage correct of the other two groups: near transfer revised worked example vs. episodic, \( t = 4.57, p<.01 \); worked example vs. algorithm \( t = 4.38, p<.01 \); far transfer revised worked example vs. episodic \( t = 5.06, p<.01 \); and worked example vs. algorithm \( t = 3.97, p<.01 \). Similarly, there were no significant differences in performance between the algorithm and the episodic groups on either type of reclassified problem.

On the thirty-two mixed problems in Phase 4; the algorithm group accumulated 135 operational errors, the episodic group 109 errors, and the worked example group 497 errors.

An examination of the types of errors made by each treatment group (see Table 6) showed that (a) the worked example group was still making the most number of errors, (b) that the failure to increase the subtrahend resulted in the most number of errors for all three treatment groups, and (c) that decreasing a number as though carrying was required was also problematical for all three groups. Basic fact errors and increasing a subtrahend when it was unnecessary also appeared to be problem areas. In addition, the pattern of the errors in Table 6 suggests that the algorithm and episodic groups were experiencing similar kinds of difficulties.
A final set of comparisons was done to simply look at performance across Phase 2, 3, and 4 for each problem type. These comparisons are shown in Tables 7, 8, and 9. Taken as a whole, the pattern of these results suggests that the worked example group was unable to cope with the increasingly difficult near and far transfer questions in Phases 3 and 4. They also indicate that the episodic group was at first making more errors than the other two groups on the near transfer questions. The verbal reports of the subjects in this group suggested that transferring or generalizing the episodic representation was initially quite difficult.

**Procedural Errors**

According to Van Lehn's (1983) repair theory, students tend to improvise and create unique alternatives when they come to an impasse. It also points to the observation that the results of the worked example group would not be as high if the students were exposed to a brand new topic; thus, they would not be able to revert to traditional methods to solve these problems. In this context the operational level is more indicative of actually learning than the correct response results. In Phase 4 on the thirty-two mixed problems the students in the worked example group had a total of 182 questions correct out of a total of 448 problems and 497 operational errors, the episodic group had 366 questions correct and 109 operational errors and the algorithm group had 348 questions correct and 135
operational errors. Carpenter and Moser (1982) affirm that children make different errors based on different kinds of misunderstanding. The protocols suggested that the students had an inability to express clearly the type of operation or procedure that they were employing. Typical protocols demonstrate the subjects' thought processes. For example:

\[
\begin{align*}
6742 \\
-1840 \\
\end{align*}
\]

Put 1 in front of the 7, make it 17, put 8, 9 to make 17. That's 6, okay, I don't know so I'll do it now, Okay, the 7, 7 I think 9 makes 16, no, yeah, that's right, so 7 9 makes 16. Take 2 away from 7 is 5.

\[
\begin{align*}
92 \\
-57 \\
\end{align*}
\]

Subtracting 92 from 57 and what I did first was in the ten's column it's a 2, so I put a small 1 at the top and then the 7 take 12 is 5 and move over to the hundred column-ten column and is 9 take away 5 and I increased the 5 by 1 it makes 6 and
6 take away 9 is 3.

There was a marked difference between the episodic responses and that of the algorithmic and worked example modes. Consider the question:

\[
\begin{array}{c}
92 \\
-57
\end{array}
\]

**Response in the Episodic mode:** Add to the top number that becomes 90, and you add 10 to 2 and that becomes 12. You add 10 to 50 that becomes 60 and the 7 stays the same. So 90 from 60 is 30, 12 from 7 is 5 which is 35.

**Response in the Algorithmic mode:** Subtracting 92 from 57 and what I did first was in the ten’s column it’s a 2, so I put a small 1 at the top and then the 7 take 12 is 5 and move over to the hundred column-ten column and is 9 take away 5 and I increased the 5 by 1 it makes 6 and 6 take away 9 is 3.

**Response in the Worked example mode:** I added a 1 to the 12. I got it from the 5 on the bottom. 1 away from 5 make it a 6, 1 higher. From the 5 got like 12 I went 12 subtract like 7. I got 5 and I went 9 subtract 6 and got 3. That makes 35.

The episodic group was also more verbal and applied a strategy quite distinct from the other two forms. The reader can determine this fact by reading the protocols of Phase 2 Task One and Phase 3 Task One. The algorithmic group was very concise and pointed in its response, the method was predisposed to be
quite methodical logical and concise.

O.K. um I can subtract 5 from 6 which gives me 1, and I can’t subtract 4 from 2 so I’ve got to bring the 1 up, which gives me 12, and 12 from 3 is 9 and I bring a 1 down to the 2 to make it 3 and subtract 3 from 5 which is 2 which gives me 291.

In contrast, the worked example group was very nonmathematical and used verbiage which did not accurately describe what was transacting in verbal terms:

O.K. cross out this 1 and put a 2 and uh out a 1 up here you cross the 7 cross the 8 and turn that into a 6 to make it a 9. You put a 1 by the 2 works out by 614 subtracted from 9 is 6 and 6 divided by 2 is 4.

Many of the phrases used did not literally describe mathematical operations. For example, students in this group talked about dividing and multiplying while
actually subtracting.

\[ 526 \]
\[ -235 \]

Cross the 2 make it 1 put on by 2, 12 divided by 3

is 9 and 5 subtract the 1 is 4.

Some students tended to be incoherent in describing the operations that they
were simultaneously performing:

\[ 642 \]
\[ -186 \]

O.K. um 642 minus 186 so this number is smaller

then the bottom number so 6 from 2 you cancelled
to be. You bring 1 from the 4 which is 12 so 6 from

12 which is 6. O.K. so 8 from 4 you cancel this
would be 14 minus 9 you've gonna want the 8 which is
9 so that would be ummm 5, so that would be 5 and

over here you put the 1 to 2 so 2 from 6 which is 4

and then the answer is 456.

Many of the responses did not relate to the operation at hand:

\[ 7643 \]
\[ -2708 \]
7643 subtract 2708 so 8 from 3 you can’t so you bring a 1 from the 4 which would 30 minus 3 minus 8 would be 5 and 4, 0 from 4 you can’t Oh O.K. 1 from 4 so that will be 3 and then 7 from 6 that’d be 1 and 3 from 7 would be 4. So the answer is 4135.

526

-235

O.K. lst me try this, how was the way I was doing it before. If I borrow from here 2 for this. Like I do have to borrow all the time, you always borrow no. But that will be 300 then or will it be a 300 oh I’m not sure, wait a minute 16 that’s a lot 50 2 hundred no but look at the answer its not supposed to be 11 is it?

It may be a fact that students do not interpret words literally and use them correctly when describing an operation:

526

-235

526 subtract 235 um 5 from 6 leaving 1. 3 from 2 you
can't so ummm this is 4, 3 from 2 you cancel 12
and would be 4. 3 from 12 so 1 would be 9 and here
you bring 1 from the 2. 2 from 5 so it would be 3,
3 from 5 which is 2 so the answer would be 291.

Children have some difficulty in dealing with function words and mathematical words used in context. It is most emphatically demonstrated in the description of the protocols of the algorithmic mode and the episodic mode in contrast to the worked example mode. Perhaps these words and the students' understanding of these words should be incorporated within the framework of the development of the models being developed for presentation to the learner. Although students do not know the general format of story problems, they rarely state that they do not know what to do but, in fact, create an erroneous solution.
### Table 6

**Phase 4: Error Types**

<table>
<thead>
<tr>
<th>Types of Errors</th>
<th>Algorithm</th>
<th>Episodic</th>
<th>Worked Example</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Error Basic Fact</td>
<td>17</td>
<td>16</td>
<td>24</td>
<td>57</td>
</tr>
<tr>
<td>Disregard of Symbol</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Inversion</td>
<td>2</td>
<td>0</td>
<td>5</td>
<td>7</td>
</tr>
<tr>
<td>Neglect of Extra Digit in Minuend</td>
<td>1</td>
<td>0</td>
<td>25</td>
<td>26</td>
</tr>
<tr>
<td>Decreasing a number as though Carrying was required</td>
<td>8</td>
<td>14</td>
<td>106</td>
<td>128</td>
</tr>
<tr>
<td>Failure to increase Subtrahend</td>
<td>75</td>
<td>58</td>
<td>263</td>
<td>396</td>
</tr>
<tr>
<td>Multiple Exchange Difficulty</td>
<td>9</td>
<td>8</td>
<td>13</td>
<td>30</td>
</tr>
<tr>
<td>Increase Subtrahend when unnecessary</td>
<td>14</td>
<td>9</td>
<td>40</td>
<td>63</td>
</tr>
<tr>
<td>Failure to increase Minuend</td>
<td>8</td>
<td>1</td>
<td>23</td>
<td>32</td>
</tr>
<tr>
<td><strong>Total errors</strong></td>
<td>135</td>
<td>109</td>
<td>497</td>
<td>741</td>
</tr>
</tbody>
</table>
Table 7

Percentage Correct of Similar Problems

<table>
<thead>
<tr>
<th></th>
<th>Phase 1</th>
<th>Phase 2</th>
<th>Phase 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Algorithm</td>
<td>96.4</td>
<td>78.6</td>
<td>90.8</td>
</tr>
<tr>
<td>Episodic</td>
<td>82.1</td>
<td>92.9</td>
<td>91.8</td>
</tr>
<tr>
<td>Worked Example</td>
<td>78.6</td>
<td>46.4</td>
<td>76.5</td>
</tr>
</tbody>
</table>
Table 8
Percentage Correct of Near Transfer Problems

<table>
<thead>
<tr>
<th></th>
<th>Phase 2</th>
<th>Phase 3</th>
<th>Phase 4</th>
<th>Phase 4 Revised</th>
</tr>
</thead>
<tbody>
<tr>
<td>Algorithm</td>
<td>71.4</td>
<td>75.0</td>
<td>73.0</td>
<td>77.1</td>
</tr>
<tr>
<td>Episodic</td>
<td>17.9</td>
<td>85.7</td>
<td>78.6</td>
<td>79.3</td>
</tr>
<tr>
<td>Worked Example</td>
<td>60.7</td>
<td>50.0</td>
<td>32.7</td>
<td>34.2</td>
</tr>
</tbody>
</table>
Table 9

Percentage Correct of Far Transfer Problems

<table>
<thead>
<tr>
<th></th>
<th>Phase 3</th>
<th>Phase 4</th>
<th>Phase 4 Revised</th>
</tr>
</thead>
<tbody>
<tr>
<td>Algorithm</td>
<td>25.0</td>
<td>75.3</td>
<td>64.3</td>
</tr>
<tr>
<td>Episodic</td>
<td>53.6</td>
<td>79.2</td>
<td>77.4</td>
</tr>
<tr>
<td>Worked Example</td>
<td>25.0</td>
<td>28.0</td>
<td>19.0</td>
</tr>
</tbody>
</table>
Figure 7. Percentage correct of similar problems
Figure 8. Percentage correct of near transfer problems
Figure 9. Percentage correct of far transfer problems
Figure 10. Percentage correct: Phase 4
CHAPTER FIVE: GENERAL DISCUSSION

Information Processing

Generally, the worked example group appeared to experience more difficulty in learning and retrieving the near and far transfer question procedures than the other two groups. That this difference was not found with the similar questions suggests that the difficulty was not one of acquiring a set of procedures for subtraction with regrouping. Rather, it appears to be associated with either (a) developing a new or modified procedural representation for a slightly different and more complicated set of questions, (b) learning to retrieve and use two or more procedural representations with two or more types of questions or (c) some combination of both of these requirements. Greeno (1978) found qualitative differences in problem solving between two groups of students who had been taught by methods involving instrumental and relational teaching. Skemp (1978) described two types of mathematical understanding:

Instrumental understanding is "rules without reasons" and is illustrated by the term "borrowing" in subtraction and when the teacher asks a question that does not quite fit the rule, of course the students will get it wrong. Instrumental understanding necessitates memorizing which problems a method works for and
which not, and also learning a different method for each new class of problems. (p. 12)

Relational understanding in mathematics involves knowing and truly understanding the concept. By students knowing not only what method worked but why, would enable them to relate the method to the problem and possibly adapt the method to new problems. (p. 13)

It is apparent that the worked example group did not have the same degree of capability of solving near and far transfer problems as the other two groups who were taught employing relational techniques. Algorithm One (Figure 1) and Episode One both illustrate how the additive method of subtraction is taught with understanding (relational understanding) whereas the worked example condition typifies rules for subtraction without understanding (instrumental learning).

Algorithmic knowledge relates to actions and performing operations in a particular sequencing fashion. Remembering episodes are personal experiences and will relate differently to each individual. Both the episodes and the algorithms may act as mnemonic devices prompting the learner to recall and organize mathematics that has been learned in a unique way so that the learner can interpret some new problems that have not been taught explicitly but are related through near and far transfer problems. These resulting structures may improve his/her understanding since he/she must assimilate and apply the thinking process to solve these resulting new structures.
One point of view to stress is that the latter possibility is the most likely because of the intimate connection that has been shown to exist between encoding and retrieval (Brooks, 1987; Tulving, 1983). The conclusion here is that both the algorithm and episodic presentation methods facilitate the formation of a more differentiated procedural representation than the worked example approach. As a result, these procedural representations are less likely to be subject to retrieval interference, particularly when they must compete with highly similar representations. Similarity or overlap, of course, should be quite high in the case of the procedural representations for similar, near, and far transfer questions. Thus, there is a high probability in such cases that the learner may not encode the critical feature differences between an initial and transfer procedural representation.

Consider a stimulus problem (Brooks, 1990) that has two attributes $S_1$, $S_2$: $S_1 \rightarrow P_1$ and $S_2 \rightarrow P_2$. The first attribute implies Procedure 1 and the second attribute implies Procedure 2. Thus a correct response would be the combination of $P_1$ and $P_2$. Another problem which is similar in structure to the stimulus problem would contain these two identical attributes $S_1$, $S_2$. A student who recognizes these two attributes to be identical to the stimulus problem could find a successful solution by processing $P_1$ and $P_2$. The structure of a transfer problem would contain the original attributes $S_1$, $S_2$ but in addition have attribute $A_1$. 
Case 1: The student recognizes that two of the three attributes $S_1, S_2$ of the transfer problem are similar to the two attributes $S_1, S_2$ in the stimulus problem and begins to solve the problem - $S_1 \rightarrow P_1$ and $S_2 \rightarrow P_2$ by processing $P_1$ and $P_2$, but for attribute $A_1$ an additional process $A_1 \rightarrow R_1$ is tried. The student has classified this problem as a version of the stimulus problem.

Case 2: The student does not recognize that the attributes $S_1, S_2, A_1$ of the transfer problem are similar to the attributes $S_1, S_2$ in the stimulus problem and treats it as a new problem. The failure to transfer is based on the failure to recognize the similarity between the original stimulus problem and the transfer problem. This problem is treated as entirely new and the learned procedures $P_1$ and $P_2$ are not retrieved. In fact it is as if $S_1$ is now $S_a$ and $S_2$ is now $S_b$ and the attributes of this problem resemble $S_a, S_b, A_1$ with processing $S_a \rightarrow R_1$; $S_b \rightarrow R_2$, and $A_1 \rightarrow R_3$.

Analogy theory infers relationships between entities. Consider the process involved in an analogical transfer task involving isomorphic problems. First, in order to use an analogy as a basis for transfer, the solution of the original problem, or base, must be learned and represented in terms of the structure of a generalizable mental model (Brown et al., 1986; Johnson-Laird, 1980) rather than in terms of specific surface details, such as object attributes. Second, the correspondence between the known solution (base) and the target problem must
be noticed (Gholson, Eymard, Long, Morgan, Leeming, 1988). Next, the base must be retrieved in terms of its structure, rather than in terms of its surface details (Gholson et al., 1988). Finally, the one-to-one correspondences between the base and the target problem must be mapped and any indicated activities carried out (Brown et al., 1986; Holyoak, 1984).

Consider a stimulus problem: A:B as C:D. A maps to B as C maps to D. If the subject can find a relationship between A and C, then there would be an identical transformation of this mapping between C and D. Let \((S_1S_2S_3) \rightarrow (R_1R_2R_3)\) as \((S_4S_2S_3) \rightarrow (R_4R_2R_3)\). The subject must first learn the relationship between \((S_1S_2S_3)\) and \((R_1R_2R_3)\) and realize that there is a similarity between \((S_1S_2S_3)\) and \((S_4S_2S_3)\). From this similarity \(S_2S_3\), the subject can map \(R_2R_3\) and \(R_4\) must be inferred on the basis of \(S_4\). It is the failure to notice the similarity between \((S_1S_2S_3)\) and \((S_4S_2S_3)\) that the episodic model is attempting to address. Multiple stimuli producing many:one mapping (Episode One and Episode Two) produced a high level of transfer between the two episodic stimuli problems and the target transfer problems; whereas one:many mapping did not have the same degree of success. There was a 4:1 ratio improvement in performance of the subjects in the episodic condition on near transfer problems after the second episode was presented to the subjects.

Learning a given procedural representation to a mastery criterion on a
homogeneous set of questions may not necessarily involve acquiring the degree of differentiation necessary to identify the similarity between the initial and transfer problems. Put another way, the students in the worked example condition may not have differentiated and integrated the procedural representations they were constructing enough to meet the retrieval demands of mixed questions sets, in spite of the fact that they achieved criterion on each phase of this experiment. This strongly supports the notion that categorically based retrieval is an important part of transfer in mathematics and that experimental or pedagogical methods which fail to take this factor into account may seriously underestimate learning. Although students do not know the general format of story problems, they rarely state that they do not know what to do, but in fact create an erroneous solution.

In addition, it can be argued that there is a possibility that both the algorithm and episodic formats promoted the integration of subroutines within the procedural representation. In this respect, it is difficult to conceptualize how a given procedural representation would be retrieved unless the subroutines within it were organized as a coherent unit. Clement and Lockhead (1976) concluded that many students, when solving problems, operated with an inconsistent system - a collage of newly learned principles and old concepts which are resistant to change. Initially, it was anticipated that the episodic (story) format would facilitate more integration than the algorithm format. However, to the degree
that either of these approaches promoted integration, there were no observable differences between them. Although the observer can only speculate at this point, it may have been that the arrows of the flow chart diagram in the algorithm format made it very easy to identify and encode the relationships between the subroutines. On the other hand, it is entirely possible that both the episodic and algorithm formats somehow made it easier for the learner to construct a verbal description of the procedural representation and/or for each subroutine and that this factor facilitated indirect retrieval through semantic memory. Again, although one would have expected this to occur more in the case of the episodic format, the observer can think of no reason why it should not have occurred in the algorithm format.

Implications of Findings Based on Protocols

Greeno (1979) argued that different types of inferences may be drawn in a story context as opposed to a problem-solving context. Examination of some example protocols supported this notion. It may have been that qualitatively different representation formats were employed. The following protocols illustrate this point:

Episodic mode
You add 20 to the top and it becomes 90 and you add 10 to the 12...10 to the 2 and that becomes 12.

For the bottom you add 10 to the 5 and that becomes 60 and that becomes. You subtract 90 from 60 and it becomes 30.

12 from 7 which is 5. That's 35.

Algorithmic mode

OK I'm subtracting 92 from 57 and what I do first is um the ten's column there is a 2 I put a small 1 at the top and then 7 take away 12 is 5

And move over to the hundred's column, that's ten's column, 9 take away 5.

I increase the 5 by one which makes 6 and 6 take away 9 is 3 or vise-versa.

Worked Example mode

I added the 1 to the 12 and I got it from the 5 on the bottom, I took 1 away from the 5 and made that a 6. It made it 1 higher than the 5 and O got like 12. And I went 2 subtract 7 and I got 5 and I subtract 6 and got 3. That makes 35.

Unfortunately, although the worked example protocol feels more concrete and less generalizable than the other two, the precise difference between these three protocols is not entirely clear at present. Nevertheless, the hypothesis that there
is a difference between these representations is supported by the performance data. The protocols were scrutinized by examining the operational level of the Addend Method of Subtraction for each student interviewed. In Phase 2 Task One the episodic group of students were the only ones that clearly gave up and admitted that all did not know how to proceed further and at this stage they were unable or unwilling to complete the near transfer problems. The episodic group mastered forty-two percent of the procedures required for the Additive Method of Subtraction, the algorithmic group, in comparison, ranked one hundred percent on correct methodology whereas, in contrast, the worked example group scored only seventeen percent. These items consisted of both the similar and near transfer problems.

While comparing these results with the number of correct responses in Table 2, the algorithmic group then committed sixteen percent mechanical errors. The episodic group at the operational level was only forty-two percent efficient but the number of correct responses was approximately fifty percent. This indicates that some of the students in this group may have assimilated and combined the additive method of subtraction with the traditional form of subtraction. This suggests that some interference may have occurred between the additive method of subtraction and the traditional form of subtraction, whereas the worked example group only utilized the additive method of subtraction seventeen percent
of the time at the operational level but accomplished seventy percent efficiency at the correct response level. This figure suggested that the worked example group did not successfully master the additive method of subtraction but rather reverted to the traditional form to solve many of these problems.

The protocols described in Phase 3 Task Two indicated that the algorithmic group dropped from one hundred percent to fifty percent at the operational level, but the episodic group increased from forty-two percent to fifty percent at the operational level. Based on correct responses, only the algorithmic results were sixty percent and the episodic group was seventy-seven percent. There was little difference between the worked example group at the operational level from seventeen percent to nineteen percent. In contrast, the correct response was forty percent but only nineteen percent at the operational level. This is a strong indication that the students in this group never adequately applied the Additive Method of Subtraction and reverted to the traditional form of subtraction.

Limitations of the Study

In the study reported here, there existed a marked difference between the episodic group and the worked example group and also between the algorithm group and the worked example group. Although in the discussion there were variances between the episodic and the algorithm groups, it was difficult to determine why the marked differences occurred. This study was focused at the
secondary level of basic level students and the findings should be confined to this age range and this basic level. Further research is needed to validate these results on a more normal population sample at this age level and also at a younger age level. There may exist a unique correlation between the episodic and the algorithm groups. Students in the episodic and algorithm groups demonstrated a high degree of accommodation of existing schemata to solve correctly the near transfer problems in Phase 3 and Phase 4 (Table 8) and developed the ability to generalize to solve far transfer problems in Phase 4 (Table 9).

Another limitation is the unknown degree of interference created by the old schema of the traditional form of subtraction with the additive method of subtraction across the three groups. It was demonstrated emphatically by the worked example group who reverted to existing schemata as evidenced by the discrepancy in the operational level and the number of correct responses in Phase 4. It was difficult to determine what was identified as far transfer problems and on second examination several were redefined as far transfer revised.

How do verbal protocols affect the inner thought process of the subjects? This was a new experience for all subjects and what effect it had on the information processing and the retrieval process is unknown. However, the observer felt that the verbal protocols enhanced the linkage of retrieval of existing
schemata from short-and long-term memory.

**Educational Implications**

In conclusion, this was a preliminary study that, at best, opened up some interesting possibilities. The use of algorithms (flow charts) and episodes (stories) helps students with learning problems in Mathematics acquire new procedures. It is also very important to keep in mind that the locus of the effect of these approaches was in retrieval learning and that this may be where many learning-disabled students experience much of their difficulty. Whether or not our treatments resulted in better subroutine integration, more procedural representation differentiation or both, must remain a question for future research. According to Skemp (1971) a student does not know what future mathematical concepts he/she will be required to recall. Therefore, when learning new schemata it is necessary to decide whether or not to incorporate them into existing schemata or retain them as unique representations. It is the role of the teacher to build an infrastructure and introduce the fragmented skills which enhance this infrastructure to assimilate into revised schema. If two related formulae are duplicated in a particular application and there exists a linkage between them, good mathematical students may synthesize and evolve a third formula, thus reducing the number of stages in the former two related forms.
The learner must be aware of the occasions when it is necessary to revise appreciable skills and accommodate new concepts and replace existing schemata with better more generalized ones. Further research is needed to discover exactly what students know about the format of story problems. And within the educational framework it may be feasible to gear instruction in such a manner that the students have a cue to relate to when tackling new and transfer problems. Once students understand that they have to relate what they know and extend their knowledge by generalization, then interpretation and solving of transfer problems will become part of their knowing domain. Skemp (1978) acknowledges that instrumental learning is usually easier to understand because it is based on easily remembered rules, but relational learning consists mainly in relating a task to an appropriate schema. Piaget and Inhelder (1969) conclude that what a child remembers depends on the level of cognitive development he or she has attained. Furthermore, they contend that a child’s recall will improve if between one test and the next he or she has progressed in the development of an appropriate schema. This could account for the fact that the students in the algorithmic and the episodic groups were able to develop their strategies and integrate and modify what was stored from one day to the next. Metaphoric linkage may be the cable that the episodic group of students are using in their transfer of learning from one level of understanding to a deeper level of processing. Old strategies and
new concepts tend to be confused and substituted for each other leading to confusion and misappropriate noise. Good mathematical students minimize cognitive interference and develop uncluttered retrieval lines to recall pertinent knowledge and skills. These students utilize formulas which may only be partially remembered but they have the ability to partially deduce and can complete open-ended schemata to decipher a problem; in fact they can take a mnemonic device like an algorithm and devise their own version to fit the problem. The worked example group of students can not do this, but must try and remember a horde of facts, rules, and procedures. This tends to lead to a multitude of errors as indicated by Table 1.

Lewicki et al. (1988) argues that students acquire procedural knowledge and maintain some form of working knowledge about patterns but are unable to articulate these processes. It is akin to learning the grammar of a foreign language; to the native tongue these syntactic rules are working knowledge.

Although the results of the study indicate the process of acquisition of knowledge, it was stored in such a manner that it made it difficult to retrieve and relate simultaneously in a comprehensive way. Brown and Campione (1986) maintain that blind training instructs students to follow a particular manner but does not help them to understand its significance. This is evident in the inability of the worked example group to master the far transfer problems. The students
are told what to do but not when and under what conditions to do it. Brown and Campione (1986) stated that students taught in this manner do not maintain and generalize learned strategies. If a learner can utilize an algorithmic structure or episodic story in the manifestation of a distinctive schema, then these cues may be useful in forming a net or framework from which the learner can weave the desired result. Cooper and Sweller (1987) maintain that worked example subjects may acquire a limited number of schemata allowing them to solve specific problems. Einstellung, as defined by Cooper and Sweller (1987), occurs when a previously acquired schema is inappropriately used because a problem is incorrectly perceived as belonging to a familiar category that requires the use of that particular schema. They conclude that worked examples are of assistance while facing similar problems but this luxury does not extend to dissimilar problems. The protocols indicate that students dissected the problem into a number of small steps. There were considerable individual differences between the verbal responses from group to group.

Further investigation is needed to validate if many:one episodic representations facilitate the ability of subjects to successfully solve transfer problems.

It is not an easy task for mathematics teachers, as was discovered during a summer teacher inservice course at the University of Toronto (1990), to create
episodes that parallel mathematical concepts. An episodic story that was created by these teachers is described as follows:

A long, long time ago, in ancient Egypt, there lived a Pharaoh by the name of VERTEX. The Pharaoh Vertex lived in a small suburb called CIRCUMFERENCE. One day while sitting in his spacious backyard, it was decided that he would build an outdoor swimming pool for his amusement. The shape of the pool had to be different from any other pool in the area and so he sent his servants out into the forest to find something which would form this unknown shape.

A day and a half later, the servants returned to Vertex with the branch from a tree. The Pharaoh was furious. "You've been gone for a day and a half and this is what you come back with, a branch off a tree?" The servants explained, "It's not just any branch, Vertex. This branch was taken from a RADIUS tree". They then proceeded to show Vertex how the branch worked.

Vertex was to stand in one spot, holding one end of the branch. At the same time, the servants held the other end of this magic branch and took turns walking around Vertex. After some time, the grass in the path of the servants began to wear down. Suddenly, the shape of the pool could be seen and a shout of jubilation left the lips of Vertex.

The Pharaoh's son, CHORD, even got into the act by following the servants as
they walked around Vertex. Chord was still quite small and got tired very quickly, so instead of walking all the way around with the servants, he would simply cut "corners" and wait for the servants to catch up while he rested. The Pharaoh's wife, Lady DIAMETER, came out to the backyard and looked at the CIRCLE. When she walked across the circle and met her husband at the CENTRE, she realized that she was twice the length of the branch. The Pharaoh's friend, SIR CUMFERENCE, walked around the entire circle and decided he'd build a fence around the pool so that young CHORD would not fall in while unattended.

Cousin SEMI-CIRCLE arrived and found he tired after walking half of the pool and so that was all he could manage. The pool was built shortly after and many people enjoyed the entertainment it provided.

Even to-day we remember the inventors of the CIRCLE by using the names of the Pharaoh, his wife, son, nephew, friend and that strange branch from that what-ya-ma-call-it-tree.

We have now come full circle.
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Phase 2 Task One:

<Today you have four different questions in front of you. I want you to do these questions and tell me what you are doing as you are subtracting.>

**Algorithmic Mode.**

- 92 O.K. I take 7 from 12 is 5 and 6 from 9 is 3.
- 57 That makes the answer 35.

- 33 O.K. 5 from 13 is 8 and 2 from 3 leaves 1.
- 15 That makes the answer 18.

- 362 O.K. 1 from 2 leaves 1. 8 from 16 leaves 8
- 81 and from 3 leaves 2. That makes the answer 281.

- 726 3 from 6 leaves 3. 4 from 2, from 12 leaves 8.
- 43 1 from 7 leaves 6. That makes 683.

**Episodic Mode.**

- 92 You add 20 to the top and that becomes 90 and you add 10 to the 2 and that becomes 12. For the bottom you add 10 to the 5 and that becomes 60 and that becomes 7.
- 57 You subtract 90 from 60 which is 30 and 12 from 7 which is 5 and that's 35.
33 The next one you add 30 to the top and 10 to the 3 and that's 13. For the bottom you add 10 to the 1, that's 20 and the 5 stays the same. You subtract and its 19. The correct answer is 18.

362 I don’t know how to do this. I haven’t done them yet.
-81 <TRY>
Add 30 to that...that becomes 40. [long pause] I don’t know, sir. I haven’t really tried them yet.

Episodic Mode

92 You take 92 make it 90 plus 2. 57 make is 50 plus 7.
-57 30 plus 5 make it 35.
33 Keep goin', eh. 33 became a 30 plus 13.
-15 15 become a 20 plus 5 equals um becomes 18.
362 O.K. 362 becomes...this stays the same right?
-81 300 O.K. make it 300 plus 160 plus 2 and then make this 100 plus 8 right? plus 1 becomes 1 because 152 and this becomes 200 and this 353 right?
726 That’s 726, 700 plus 120 plus 6 and 43 became 100
-43 plus 40 plus 3. um..719.

Worked Example Mode.

92 Take away 7 from 12 leaves 5 and 8 from 5 leaves 3.
-57
33 And this one 13 take away 5 leaves 8.
-15 2 from 1 is 1.
362 And take away 1 leaves 1. 16 from 8 leaves 8
-81 and you bring the 2 down.
726 And 3 from 6 leaves 3 and 12 from 4 leaves 8 and 6.
-43 Answer is 83.
Algorithmic Mode.

92 Subtracting 92 from 57 and what I did first
-57 was in the ten's column it's a 2, so I put a small 1 at the top and then the 7 take 12 is 5 and move over to the hundred column-ten column and is 9 take away 5 and I increased the 5 by 1 it makes 6 and 6 take away 9 is 3.

33 Now second question was um 33 take away 15
-15 so I was um the one's column and I added a 1 to the 3 and it makes 13. Take away 5 is 8 and I went over to the ten's column and I went down to the bottom numeral it was a 1 so I put another 1 and it makes 2. 2 take away 3 is 1.

362 Um is 362 take away 81. 2 take away 1 is 1 and
-81 its 8 take away 6 won't work so I put a 1 beside the 6 um equals 8 and at the hundreds column there's no, nothing there so I a 1 down there and then I took a 3 away from 1 is 2.

726 726 take away 43. um 6 take away 3 is 3, 2 take 4
-43 won't work so I added a 1 to the top beside the 2 which is 12 take away 4 is 8 and hundreds column. There is a 7 there nothing on the bottom 1 put a 1 there 7 take away 1 is 6. Answer is 63.

Worked Example Mode.

92 I added a 1 to the 12. I got it from the 5 on the bottom.
-57 1 away from 5 made it a 6, 1 higher. From the 5 got like 12 I went 12 subtract like 7. I got 5 and I went 9 subtract 6 and got 3. That makes 35.

33 I added 1 to the on the bottom made that 2 nd went up
-15 added 1 to the 13 and got 13 subtract 5 that made 8 and I went over and I subtract 3 subtract 2 and I got 1. So answer is 18.
Um, it was 2 subtract 1 so do nothing for that I made
that a 1. 8 was higher on the bottom so you have to
make that 16 and go over and carry 1 away from the 3
to make 2 and that's how I got 16. So 16 subtract 8
leaves 8 and 3 subtract 2 is 1. So I got 181.

2 subtract 1 stayed the same. 6 subtract 3 you had
nothing to change since 6 is higher on top. So 1 over
and then 2 is lower so 4 was higher on the bottom than
2 so had to go over and take 1 away from 7 so to make
6 and 1 to 12. 12 is higher than 4 so 12 subtract 4
makes 8 and just 6.

Algorithmic Mode.

92 9 take away 5 equals 3. Put a 1 at the 2. 12 take away
57 7 equals 8 and that's 38. Oh change 5 to 6 equals 3.

33 Put 33- 3 from 1 equals 2. 13 from 5 equals 8 so
15 that's 28. O.K. um, um 3 subtract 2 equals 1 so answer is 18.

362 I change the 6 no 8 to a 9. Put a 1 beside the 6.
81 Now 2 from 1 equals 1. 16 from 8 equals 8 and 3
from oh um 3 from 1 equals 2. Answer 281.

726 Put a 1 beside the 2 which equals 12 and I change the
43 4 to a 5 subtract now 6 from 3 equals 3. 12 from 5
equals 7 and 7 from 1 equals 6. 672

Episodic Mode.

92 Add to the top number that becomes 90, and you add
57 10 to 2 and that becomes 12. You add 10 to 50 that
becomes 60 and the 7 stays the same. So 90 from 60 is
30, 12 from 7 is 5 which is 35.

33 You add 30 and you add 10 to the 2 that becomes 13.
15 You add 10 to 1 that becomes 20, subtract and that becomes 18.
For the 1 add 300 and it becomes 160 and 3 and then the second 1 becomes 80 and 1 and then you subtract them and then it becomes 1 and you add those 3 so 381.

For the second 1 its 700, 3 becomes 120 and the 6 stays the same, 4 becomes 40 and 3 stays the same and the answer is 783.

**Worked Example Mode.**

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<td>And 2 take away 1 makes 1.</td>
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<td>12 take away 4 makes 8. 7 take away nothing makes 7.</td>
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**Phase 3 Task One:**

<Today we are going to look at six different problems that you have in front of you. I want you to do the questions that are in front of you and tell me what you are doing as you are subtracting.>
Worked Example Mode.

526 - 235=526 subtract 235 um 5 from 6 leaving 1. 3 from 2 you can't so ummm this is 4, 3 from 2 you cancel 12 and would be 4. 3 from 12 so 1 would be 9 and here you bring 1 from the 2. 2 from 5 so it would be 3, 3 from 5 which is 2 so the answer would be 291.

4286 - 1934=4286 subtract 1934. 4 from 6 leaving 2 and uh 3 from 8 so that would be um 5 and 9 from 2 you can't so this will be 12 and this will be 10. O.K. um sorry 9 from 12 so um 3 and this 1, 2 from 4 leaving 2 so the answer would be 2352 O.K.

6742 - 840=0 from 2 you can't, 4 from 4 you cancel and this will be 14 minus 4 so that will be 10 and bring 1 over so this would be 8. 8 from 8 you can't so you take 1 from the 6 and put it here would be 70 and that would be um 9 and here it would be 5. So the answer would be 5920.

7643 - 2708=7643 subtract 2708 so 8 from 3 you can't so you bring a 1 from the 4 which would 30 minus 3 minus 8 would be 5 and 4, 0 from 4 you can't Oh O.K. 1 from 4 so that will be 3 and then 7 from 6 that'd be 1 and 3 from 7 would be 4. So the answer is 4135.

9283 - 4095=9283 subtract 4095, so 5 from 3 you can't so you add 1 there 5 from 13 so that would be 7 and 8 from... 9 from 8 you can't so that will be umm 10 so that will be 10 and 9 from 10 is 1 and 2 from 0 you can't 4 from 9 put 1; make it 5 from 9 which is 4, so then it's 4017.
Episodic Mode.

642  Do I have to talk into that. If I borrow from this
-186  that's gonna be 200 not 100, if I borrow from
there um 6 can make 12 because 6 and 6 is 12, right?
Its 5 and 2 its 4, 456.

526  O.K. Ist me try this , how was the way I was doing
-235  it before. If I borrow from here 2 for this. Like
I do have to borrow all the time, you always borrow
no. But that will be 300 then or will it be a 300
oh I'm not sure, wait a minute 16 that's a lot 50
2 hundred no but look at the answer its not supposed
to be 11 is it?

Algorithmic Mode.

642  I gotta put a 1 above the 2 to make it 12 since I
-186  can't subtract. 6 from 12 gives me 6. Bring down
the 1, change 8 into 9, you can't subtract 9 from 4
so I put 1 above the 4 to make it 14 and I get 5.
And I bring the 1 down to another 1, to make 1 a 2.
Subtract 2 from 6 which gives me 4.
The answer is 556.

526  O.K. um I can subtract 5 from 6 which gives me 1,
-235  and I can't subtract 4 from 2 so I've got to bring
the 1 up, which gives me 12, and 12 from 3 is 9
and I bring a 1 down to the 2 to make it 3 and
subtract 3 from 5 which is 2 which gives me 291.

4286  I can subtract 4 from 6 which gives me 2 and
-1934  subtract 3 from 8 which I can do and that gives me
5. And I got to bring 1 up to the 2 to make it 12
and 9 from 12 is 3 and bring down the 1 to the other
1 to make it 2 and 2 from 4 gives me
2 which gives me 2252.
6742  O.K. 0 from 2 is 2 and 4 from 4 gives me 0 and I
-840  can't subtract 4 from 7 so I bring the 1 up which
gives me 9. I gotta bring down 1 cause there's
nothing there. And 1 from 6 gives me 5 which
gives me 5902.

7643  And I gotta bring 1 above the 3 to give me 13,
-2708  and 8 from 13 is 5 and bring down 1 to 0 to
give, makes it 1 and 1 from 4 is 3 and bring 1
above the 6 to make it 16. 7 from 16 is 9 and bring 1
down to the 2 to make it 3 and 7 from 3 is 4 which
gives me 4935.

9283  And I gotta bring 1 above the 3 to make it 13 and 5
-4095  from 13 is 8 and I gotta bring the 1 down to the 9
to make it 0. And I can't subtract 10 from 18 so
I gotta make it, bring it up to 1 so 10 from 18
gives me 8 and 2 from 0 is 2 and 9 from 4 is
5 which gives me 5288.

Worked Example Mode.

4286  Cross out 1 turn it into a 2, put a 1 up there,
-1934  put the 1 by the 2, cross out that 9 turn it into
an 8, two times 8 comes to a 3, 5 times 1 into 8
4 times and into 2 is 2 times.

6742  Put a 1 under 6, cross the 1 to make it into a 2
-840  put a 1 up there and 2 subtracted by 0 is 2.

7643  4 three's are 4, 0, 73 by 84 and 6 bring away
-2708  2 is 4. O.K. cross that 2, 2 by 2 is 3, bring
by 6, cross out that 7 and put an 8 up there.
Episodic Mode.

642  O.K. 600 plus 140 plus 2, 200 plus 80 plus 6
-186  now what do I do do? This isn't right.
   Because how can I divide 6 into 2? 12? 180?
   Count em 2 times, there I got 456 and I added it.

526  O.K. make the 2 in 526 into a 12 and I made the 2
-235  in 235 a 3 and my answer is 291.

4286 O.K. I changed the 2 in 4286 to a 12 and I changed
-1934  the 1 in 1934 to a 2 my answer is 2252.

6742 O.K. I changed this 7 in 6742 to a 17 and 840
- 840  to 1840 and my answer is 5902.

7643 O.K. the 7643 I changed the 6 to a 16 and the 3
-2708  to a 13 and 2708 I changed the 2 to a 3 and the
   0 to a 1 and my answer is 4935.

9283 For the top number 9283 I changed the 3 to a 13
-4095  and the 8 to an 18 the bottom number 4095 I
   changed 9 to a 10 and there's 0 to a 1
   and my answer is 5188.

Episodic Mode.

642  All right I added a 10 to the 4 in 314, added
-186  a 100 to 186 to make it 286. O.K. I borrowed uh,
   14 to make it a 12, 2 to a 12 and changed the 486
   to 286 to 296 now. Alright the final answer is 456.

526  All right I added 10 to a 2 to 526 and I added
-235  a 100 to the 200 to make it 315 now and
   the final answer is 291.

4286 O.K. you have 4286 I added uh, 10 to a 12 I
-1934  need 1934 add a 1000 to uh 1000 to make it 2000
   and the final answer is 2352.

6742 O.K. I added uh 10 to a 7 became 17 I added
- 840  uh 1000 to 840 the final answer is 5902.
O.K. here I had to borrow from, uh, 0 to make it 13, I added 10 to the 6 to make it 16 and uh, 1000, to the two to make it 3000. The final answer is 4935.

O.K. I added uh, 10 to the 12, to make uh, to pay for it. I added 10 to the 2 to make 12, I had to borrow 4 from the 9 to make it 13, and I changed the 4000 to 5000. I had to borrow from, uh, I have to make it a 2, I can do it uh, and here I add 10 to the 2 to make 12 and I had to borrow from a 9 to make 13 and I added a 1000 to 4000 to make 5000. And I added a 10 to the 8 to make 18, and the answer is 5288.

Worked Example Mode.

The one here, uh, I don’t understand this now.
If I have a 1 here, a 1 here, well I really don’t know.
I’ll just, ah, this one stays the same, that’s a 3, take away 2, 5. Add 1 on the 2, equals 3.
Put 1 in front of the 7, make it 17, put 8, 9 to make 17. That’s 6, okay, I don’t know so I’ll do it now, Okay, the 7, 7 I think 9 makes 16, no, yeah, that’s right, so 7 9 makes 16. Take 2 away from 7 is 5.

I put 1 in front of 13, no, 5 and 8 make 13, put 1 in front of 18, 9 and 9 are 18, 1,2,5 ... Got you let’s check.

Episodic Mode.

Made the 2 into 12, the 2 to 3 and subtracted it. The answer is 291.
Ah, make the 2 to 12 and the 1 to a 2, and subtracted the answer was 2352.
I, uh, made the 7 a 17 and uh, made a thousands column and subtracted and 5902.
7643  Made the 3 into 13, put a 1 where 0 was,
-2708  made the 6 to a 16. And made the 2 into a 3
and subtracted and the answer is 4935.

9283  Made that 3 to 13, 8 to 18, the 2 to 12,
-4095  the 0 to 10, the 4 to 5, and subtracted the answer is 4298.

Algorithmic Mode.

642  Number 4, you can’t subtract, so you put a 1
-186  up there, subtract from 6, 5 take away 6, the number
in the tens you bring it up to 9.

4 is smaller than 9 so you have to put a 1
up to the 4. 9 from 14 is 5, the number is the
hundred’s place is 1 bring it up to 2, subtract 2
from 6 is 4.

526  The number is bigger so you can subtract, it’s 1,
-235  the number is in the ten’s so you bring up the 4,
and the number is too small so you put a 1 beside it.
I made a mistake, its supposed to be a 3.
Put a 1 by the 2, you get 12. 3 from 12 is 9.
5 is bigger than 2, so you subtract that would be 3.
The answer is 391.

4286 The number is bigger than the 4 so you can subtract
-1934  just 2. That would seem, the number is bigger again.
And you subtract, it’s 5. Put a 1 above the 2
which makes it 12, 9 from 12 is 3. The number is
bigger again, you could make it 3. The answer is 3252.

6742  The number is bigger on top so you make the same
- 840  for put 1 on the top by the 6.

7643  7 and 16...9. 2 from the 7 equals 3,
-2708  3 from 7 is 4...4945.

9283  Can’t subtract from 3 so I have to put a 1,
-4095  you get 13. 8 is exactly the same number,
you wanna put 8, 18, 9 from 18 is 9, and bring
this up to 1. 1 from 2 is 1. 4 from 9 is 5.
5 and 1, 9, 8.
Algorithmic Mode.

642 We put the 1 in front of the 2 because the 2 is not
-186 bigger than the 6. You go up to the next column,
and you make a 9 into a 4 because 4 is smaller than an
8, and you put a 3 at the top. Since this is not as
great as the 8, then you put a 1, that makes it 13.

Then you go up the other column and then you put,
umm, you go one more up to 6, put a 1, then you put 6
take away 2, cross out the 1 and put a 2. You subtract
the 12 take away 6 equals 6. Then 13 take away 8 is 5.
Then you subtract 6 take away 2 and its 4.
The answer is 456. Ok.

526 For the second one, you don’t need to put a 1 in
235 front of the 6 because the 6 is bigger than a 5.
So all you do is just subtract 6 take away 5, subtract
the 6 take away 5 equals 1. You go to the next column,
you put a 1 in front of the 2, because a 2 is smaller
than a 3. Then you go to the next column and put a 3
in front of the 2 to make it more large than the 2, so
that you come up. Cross out the 2 and put a 3 at the top,
and you subtract 12 take away 3 is 9. Then you from 1 and
subtract 12 take away 3 is 2.

4286 Okay, since the 6 is bigger than a 4, all you do is
-1934 subtract 6 take away 4 and its 2. Then you go to the
next column and subtract 8 take away 3 because the 8 is
bigger than a 3 and that’s 5. So you go over to the next
column, so it’s a 2 is smaller than 9, put a line through
the 2 and make it a 1, since the 1 is not bigger than 9
you put another 1 beside it and that’s 11, then you go and
subtract 11 take away 9 and that’s 2. You go up to the next
column, you put 1 more, 1 more ahead of the 1 to make it a 2,
then you subtract the 4 take away 2 is 2.

6742 The next question has a 2 is bigger than a 0, all you do
-840 is just take away 2, take away 0 and put the 2. Then you
go to the next column and it’s since a 4 and 4 are even,
then you just subtract 4 and 4 that’s 0. Since the 7 is
smaller than the 8, you cross that out and make that 6
then you put a 1 in front of the 6 because the 6 is smaller than the 8, then you subtract 16 take away 8 that’s 8, then put 1, just the number 1 below the 6, so there’s nothing below the 6 then you subtract 6 take away 1 is 5.

Next question since 3 is smaller than the 8 put a 1 in front of it, subtract 13 take away 8 is 5, now since the 4 is bigger than the 6 you don’t have to make a cross because 4 is greater than 0 you just subtract 4 take away 0 is 0 and equal to the other column and scratch out the 6 to make it a 5, because the 6 is smaller than a 7, so you put the 5 at the top, so its a 5 smaller than a 7 you put a 1 in front of it and you subtract 18 take away 7 is 8 then you go up to the next column subtract you would put a 1 in front of the 2 to make it a 3 and then you subtract 7 take away 3 is 4.

5 below that that’s a 3 is more than a 5 you put a 1 in front of the 3 then you go and subtract that 8 then you go to the next column, see its smaller than a 9 you cross it out and make it a 7, since a 7 is smaller than a 9 you put a 1 in front of it to make it 17 then you subtract 17 take away 9 that’s 8 since a 2 is bigger than a 0 you don’t have to scratch it out, just subtract 2 take away 0 and that’s 2 then for the next column you put a 1, you put 1 in front of the 4 to make it a 5 and subtract 9 take away 5, 4

Worked Example Mode.

O.K. first you see 6 from 2 you can’t so you go over to the 4 you take 1, as 3 so you say 6 from 12 that leaves 6 you go over to 8 you go over the other side and you see 8 from 3 you can’t and so you go over 1. 8 from 13 leaves 5, 1 from 5 leaves 4.

O.K. Next question O.K. 5 from 6 that leaves 1.

3 from 9 3 from, 2 you can’t so you go over to 5 and you borrow 1 and take 3 from 2. 12 leaves 9. then you go over to the other side and say 2 from 4. because remember you take 1 from the 5 and put out at
the 2 that makes 12. so you say 3 from 4 leaves 3.
What 3 from 4 leaves what 2 from 4 leaves .2. 6.

4286 4 from 6 leaves 2. 3 from 8 leaves 5. 9 from 12
-1934 leaves 3. 1 from 4 leaves 3.

6742 0 from 2 leaves 2. 4 from 4 leaves 0, 9 and 5.
-840 O.K. 5 oh,

7643 See 8 from 13, 8 from 3 you can’t so you go over 4
-2708 borrow 1 and put at the 3 and make it 13. And so 8
from 13 leaves 5, 0 from 4 leaves 4. So 0 form 3
because borrow 1 leave 3, 7 from 6 you can’t so you
go over 1 and you borrow 1, 7 from 16 leaves 9
that’s right? So that leaves it 6, 6 from 2,
2 from 6 leaves 4.

9283 O.K. 5 from 13 leaves 8, 9 from 18 leaves 9,
-4095 0 from 1 leaves 1. One sec. O.K. 9 starting all over,
5 from 3 you can’t so you go over borrow 1 make it 13,
so 5 from 13 leaves 8, um, 9 from 17 leaves 8,
2 from 1 no, no, no, 0 from 1 leaves 1
and 4 from 9 leaves 5.

Algorithmic Mode.

642 Take 1 from the 4 is 3. And you put 1 out of 12,
-186 2 is 4, so it would be 9. 6 this must be 2, no,
um, yeah, 4 from 8 is 6, 2 from 6 is 4.
Your answer is 466.

526 5 from 6 is 1. It’s 12 and 3 umm, 3 3 from 12
-235 is 9, 2 from 5 , 3 from 5 is 2.

4286 4 from 6 so 2, 3 from 8 is 5. 9 from 12 is 3,
-1934 2 from 4 is 2, the answer is 2352.

6742 0 from 2 is 2, 2 from 4 is 2. 17 is 8 from 17
-840 s 9, 1 from 6 is 5. The answer is 5902.
This is 13. 1 from 8 from 13 is 5. 1 from 4 is 3.
7 from 16 is 9, 3 from 7 is 4, is the answer 4935.

This 13, this is 10, 3 from 5 and 13 is
5 from 13 is the 7 um is 18 is 16, 8 from 10 is 8
8 plus 12 is 5, 12 from 10 is 2,
5 from 9 is 4. That answer is 4487.
### APPENDIX B

#### SUBTRACTION PHASE 1 SERIES A

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### Appendix C

#### Subtraction Phase 1 Series B

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APPENDIX D

SUBTRACTION PHASE 1 SERIES C

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APPENDIX E

SUBTRACTION PHASE 1 SERIES D

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#### SUBTRACTION PHASE 2 SERIES A

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### APPENDIX G

#### SUBTRACTION PHASE 2 SERIES B

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# APPENDIX I

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APPENDIX J

NEAR-TRANSFER PROBLEMS

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### APPENDIX K

**SIMILAR, NEAR TRANSFER AND FAR TRANSFER PROBLEMS**

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APPENDIX L

EPISODE TWO

The leading halfback in the Grey Cup semi-finals carried the ball for the Argos 345 yards in game one compared the leading Cat’s player with 182 yards. In game two, both men carried the ball only 100 yards each. Which player had more yardage in the series and by how much?

(Add one hundred)

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(Add one hundred)

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### APPENDIX M

#### PRETEST SUBTRACTION QUESTIONS

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## APPENDIX N

**SIMILAR, NEAR TRANSFER AND FAR TRANSFER PROBLEMS**  
(* INDICATES FAR TRANSFER PROBLEMS REVISED)

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